Corrigendum

Corrigendum to “Pre-torsors and equivalences”

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Received 5 December 2007

Communicated by Susan Montgomery

Abstract

The following incorrect claim occurred: An $A$-coring $\tilde{C}$ is a left, equivalently, right extension of an $A$-coring $C$ if and only if there exists a homomorphism of $A$-corings $C \rightarrow \tilde{C}$. Here we present a corrected statement and claim that the error does not influence other results in the paper.

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Instead of the incorrect claim on page 548, the following lemma holds. Let $A$ be an associative and unital algebra over a commutative ring $k$ and $C$ and $\tilde{C}$ be $A$-corings. Consider the right comodule categories $\mathcal{M}^C$ and $\mathcal{M}^{\tilde{C}}$, and the forgetful functors $F : \mathcal{M}^C \rightarrow \mathcal{M}_A$ and $\tilde{F} : \mathcal{M}^{\tilde{C}} \rightarrow \mathcal{M}_A$ to the category of right $A$-modules.

Lemma 1. For two $A$-corings $C$ and $\tilde{C}$, the following assertions are equivalent.

(i) There is a $k$-linear functor $U : \mathcal{M}^C \rightarrow \mathcal{M}^{\tilde{C}}$ such that $F = \tilde{F} \circ U$.

(ii) Considering the $A$-bimodule $C$ as a left $\tilde{C}$-comodule via the coproduct, there is a right $\tilde{C}$-coaction $\tilde{\rho} : C \rightarrow C \otimes_A \tilde{C}$, making $C$ a $C$-$\tilde{C}$ bicomodule.

(iii) There is a homomorphism of $A$-corings $\kappa : C \rightarrow \tilde{C}$.
Proof. (i) $\Rightarrow$ (ii). By property (i), there is a right $\tilde{C}$-coaction $\tilde{\rho}$ on the right $A$-module $\tilde{C}$. Since under assumption (i) $\tilde{C}$ is a right extension of $C$, $\tilde{\rho}$ is a left $C$-comodule map by [1, Theorem 2.6].

(ii) $\Rightarrow$ (iii). The map $\kappa$ is constructed as $\kappa := (\epsilon_C \otimes_A \tilde{C}) \circ \tilde{\rho}$, where $\epsilon_C$ denotes the counit of $C$.

(iii) $\Rightarrow$ (i). The functor $U$ is given by the corestriction functor along $\kappa$, cf. [2, 22.11].

A symmetrical statement holds for the categories of left (co)modules. Note that, in order for the construction in the proof of the implication (ii) $\Rightarrow$ (iii) in Lemma 1 to yield a well-defined and right $A$-linear map $\kappa$, the $A$-module structures of $C$, as an $A$-coring on the one hand and as a $\tilde{C}$-comodule on the other hand, need to be the same, as in Lemma 1(ii). This condition is missing in the original formulation of the claim.

As a consequence of this omission, part (1) of Lemma 3.7 needs to be modified as follows.

**Lemma 2.** Let $T$ be a faithfully flat $A$-$B$ pre-torsor and $C$ and $D$ be the associated $A$- and $B$-corings, respectively. Let $\tilde{C}$ be an $A$-coring for which $T$ (via the canonical right $A$-action determined by its $A$-ring structure) is a $D$-$\tilde{C}$ bicomodule. Then there is a homomorphism of $A$-corings $\kappa : C \to \tilde{C}$.

Note that the additional assumption about the $A$-action on $T$ is needed for the proof given in the original article to be correct. Observe that the right $A$-action on $T$ has the required form in part (2) of Lemma 3.7 by definition: In a $\tilde{C}$-Galois extension $B \subseteq T$, $T$ is an $A$-ring and a $\tilde{C}$-comodule via the same right $A$-actions.

In the proof of Theorem 3.4, part (2) of Lemma 3.7 is used, so it holds without modification.

**References**