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Robust Optimization for Multiobjective Programming Problems with Imprecise Information

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Abstract

A robust optimization approach is proposed for generating nondominated robust solutions for multiobjective linear programming problems with imprecise coefficients in the objective functions and constraints. Robust optimization is used in dealing with impreciseness while an interactive procedure is used in eliciting preference information from the decision maker and in making tradeoffs among the multiple objectives. Robust augmented weighted Tchebycheff programs are formulated from the multiobjective linear programming model using the concept of budget of uncertainty. A linear counterpart of the robust augmented weighted Tchebycheff program is derived. Robust nondominated solutions are generated by solving the linearized counterpart of the robust augmented weighted Tchebycheff programs.

Keywords: Multiobjective Programming; Imprecise Coefficients; Robust Optimization; Interactive Procedures

1. Introduction

This study develops an interactive procedure to solve multiobjective linear programming problems with imprecise information. Robust optimization is used in solving robust augmented weighted Tchebycheff programs formulated from the multiobjective linear programming model using the concept of budget of uncertainty [1]. Tradeoffs among the objective functions are made in the interactive procedure through progressive articulation of the preference information from the decision maker (DM) so as to locate the most preferred solution.

Interactive methods are the most promising approaches for solving multiobjective programming problems [2]. Many interactive multiobjective programming procedures have been developed in the last 50 years. Researchers are continuing to develop new interactive procedures for different types of multiobjective programming problems. Two phases, a solution generation phase and a solution evaluation phase, are

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performed alternately in an interactive procedure. A set of representative trial solutions, usually efficient or nondominated, are generated in the solution generation phase. The set of representative trial solutions is presented to the DM in the solution evaluation phase. The DM articulates his/her preference information through the evaluation of the representative solutions. In the next iteration, the solution space is reduced using the preference information from the DM, new representative trial solutions are generated from the reduced solution space and presented to the DM, the DM articulates his/her preference information and so on. This process continues until a satisfactory solution has been found or a predefined stopping condition has been reached.

The majority of mathematical programming formulations in the literature are based on deterministic data. However, in many real world applications, the input data are very difficult to estimate. Consequently, stochastic [3, 4] and fuzzy [5, 6] approaches are introduced to the classical multiobjective programming formulations to address the issue of imprecise and incomplete information. However, both of these approaches assume distribution details for the coefficients in the model, an assumption which is sometimes inapplicable for ground-breaking endeavors, such as in R&D project portfolio selection and in new product (*e.g.*, cancer treatment drug) development.

Robust optimization is a relatively new approach which addresses imprecise and incomplete information by way of set inclusion, *i.e.*, the true value of data is contained within an interval without any assumption on its distribution. Robust optimization addresses the problem of data uncertainty by guaranteeing the feasibility and optimality for the worst instances of the problem. Since it is naturally a worst case approach, feasibility often comes at the cost of performance and the solutions obtained are usually overly conservative [7]. Bertsimas and Sim [1] developed an approach called "the budget of uncertainty" to control the cumulative conservativeness of uncertain coefficients in the problem.

The current study develops a robust optimization approach to generate robust nondominated solutions for multiobjective linear programming problems with interval uncertainties in the coefficients of the objective functions and constraints. The concept of the budget of uncertainty is used in robust optimization in this study. The developed approach can then be embedded in interactive procedures when representative nondominated solutions need to be generated. This approach is the first method that uses robust optimization for multiobjective programming problems contaminated with uncertainties.

2. Problem definition

Let K , m and n represent the numbers of objective functions, constraints and decision variables, respectively, in the multiobjective programming model. The multiobjective linear programming model is stated formally in the following

$$\begin{array}{lll} \min & z_k = f_k(\mathbf{x}) & \forall k \\ \text{s.t.} & g_i(\mathbf{x}) \le b_i & \forall i \\ & x_j \ge 0 & \forall j\,. \end{array} \tag{1}$$

In the model, $\mathbf{x} \in \Re^n$ is the vector of decision variables, $f_k(\mathbf{x}) = \sum_{j=1}^n c_{kj} x_j$ is the k th objective function, and $g_i(\mathbf{x}) = \sum_{j=1}^n a_{ij} x_j \le b_i$ is the i th constraint. The above model is called the nominal model assuming the values of a_{ij} , $\forall i, j$, and c_{kj} , $\forall k, j$, are exactly known. Since the objective functions are usually in conflict, model (1) usually does not have a single feasible solution that simultaneously minimizes all objective functions. The optimal solution is defined to be a feasible solution that maximizes the DM's value function [2]. Because the

DM's value function is not readily available, the solution process to model (1) is the searching for a solution which is most preferred by the DM.

The following concepts are from Steuer [2]. The set of solutions satisfying all constraints, *i.e.*, $X = \{\mathbf{x} \in \Re^n \mid g_i(\mathbf{x}) \le b_i, \forall i, x_j \ge 0, \forall j\}$, is the feasible region, and a point $\mathbf{x} \in X$ is a feasible solution in decision space. The set $Z = \{\mathbf{z} \in \Re^K \mid z_k = f_k(\mathbf{x}) \mid \mathbf{x} \in X\}$ is the feasible region in criterion space. A point $\mathbf{z} \in Z$ is a feasible solution in criterion space or a feasible criterion vector. A point $\bar{\mathbf{z}} \in Z$ is a nondominated criterion vector if there does not exist any other criterion vector $\mathbf{z} \in Z$, such that $\mathbf{z} \le \bar{\mathbf{z}}$ and $\mathbf{z} \ne \bar{\mathbf{z}}$. $\bar{Z}$ is used to represent the set of all nondominated solutions in criterion space. A point $\bar{\mathbf{x}} \in X$ is an efficient solution in decision space if $\bar{\mathbf{z}} \in \bar{Z}$ such that $\bar{z}_k = f_k(\bar{\mathbf{x}})$, $\forall k$. $\bar{X}$ is used to represent the set of all efficient solutions in decision space. A criterion vector $\hat{\mathbf{z}} \in Z$ is optimal if it maximizes the DM's value function. An optimal solution must be nondominated, *i.e.*, $\hat{\mathbf{z}} \in \bar{Z}$. A point $\mathbf{z}^* \in \Re^K$, such that $z_k^* = \min\{f_k(\mathbf{x}), \mathbf{x} \in X\}$, $\forall k$, is the ideal point. For most multiobjective programming problems, $\mathbf{z}^* \notin Z$, *i.e.*, $\mathbf{z}^*$ is infeasible. A point $\mathbf{x}^* \in \Re^n$, such that $z_k^* = f_k(\mathbf{x}^*)$, $\forall k$, usually does not exist [8]. A point $\mathbf{z}^{**} \in \Re^K$, such that $z_k^{**} = z_k^* - \varepsilon_k$, where $\varepsilon_k > 0$ and small, is called a utopian point.

When a multiobjective programming problem is solved, especially when an interactive procedure or an approach requiring *posteriori* articulation of the DM's preference information is used, many nondominated solutions are generated as trial solutions. These trial nondominated solutions are usually evaluated by the DM as a means to elicit preference information. Nondominated solutions are usually generated by solving augmented weighted Tchebycheff programs derived from the nominal multiobjective programming model (1) [2].

The weighting vector space is defined as

$$\mathbf{W} = \{\mathbf{w} \in \Re^K \mid w_k > 0, \textstyle\sum_{k=1}^{K} w_k = 1\}. \tag{2}$$

Any $\mathbf{w} \in \mathbf{W}$ is a weighting vector. For a given $\mathbf{w} \in \mathbf{W}$, an augmented weighted Tchebycheff program for the nominal model (1) is formulated as in (3) in the following

$$\begin{array}{lll} \min & \alpha + \rho \sum_{k=1}^{K} (z_k - z_k^{**}) & \\ \text{s.t.} & \alpha \ge w_k (z_k - z_k^{**}) & \forall k \\ & z_k = f_k(\mathbf{x}) & \forall k \\ & g_i(\mathbf{x}) \le b_i & \forall i \\ & x_j \ge 0 & \forall j \\ & z_k \ \text{unrestricted} & \forall k \\ & \alpha \ge 0, & \end{array} \tag{3}$$

where $\rho > 0$ is a small scalar. Usually $\rho = 0.001$ is sufficient.

The augmented weighted Tchebycheff program (3) is a single objective linear programming problem. If its optimal solution is represented by the composite vector $(\mathbf{x}_\mathbf{w}, \mathbf{z}_\mathbf{w}, \alpha_\mathbf{w})$, then $\mathbf{x}_\mathbf{w} \in \bar{X}$ and $\mathbf{z}_\mathbf{w} \in \bar{Z}$, *i.e.*, $\mathbf{x}_\mathbf{w}$ is efficient and $\mathbf{z}_\mathbf{w}$ is nondominated. For a given $\mathbf{w} \in \mathbf{W}$, the augmented weighted Tchebycheff program (3) generates a given nondominated solution. By using a widely dispersed set of weighting vectors in $\mathbf{W}$, a widely dispersed set of representative nondominated solutions are generated.

As previously stated, the values of the coefficients in the multiobjective linear programming model (1) are not known with certainty and, hence, the solution obtained for the nominal model (1) may not be close to the true most preferred solution of the DM or, even worse, could be infeasible for some realizations of these imprecise coefficients. Given that a_{ij} , $\forall i, j$, and c_{kj} , $\forall k, j$, are imprecise and their exact values are unknown but within certain intervals, the focus of this study is on finding a solution to model (1) such that the solution not only is feasible with a very high probability, but also is very close to the most preferred solution of the DM. An interactive procedure, such as the interactive weighted Tchebycheff procedure [2, 10], is proposed for this purpose.

3. The robust optimization approach for linear programming problems

Consider the following standard linear programming problem

$$\begin{aligned} \min \quad & f(\mathbf{x}) \\ \text{s.t.} \quad & g_i(\mathbf{x}) \le b_i \qquad \forall i \\ & \mathbf{l} \le \mathbf{x} \le \mathbf{u} \end{aligned} \tag{4}$$

where $g_i(\mathbf{x}) = \sum_{j=1}^{n} a_{ij} x_j \le b_i$, as in (1), is the i th constraint and $f(\mathbf{x}) = \sum_{j=1}^{n} c_j x_j$ is the single objective function of the problem. The standard linear programming model in (4) with precise a_{ij} , $\forall i, j$, is the nominal formulation. Now assume that each a_{ij} is an imprecise coefficient with unknown exact value in the interval $[\overline{a}_{ij} - \hat{a}_{ij}, \overline{a}_{ij} + \hat{a}_{ij}]$ where $\overline{a}_{ij}$ is the nominal value and $\hat{a}_{ij}$ is the half-interval width of a_{ij} . The precise value of c_j , $\forall j$, is assumed to be known. The purpose of robust optimization is to find an optimal solution, called the robust solution, which remains feasible for almost all possible realizations of the uncertain problem coefficients. We quantify this concept by reformulating the nominal model in (4) as follows. The absolute value of the scaled deviation from its nominal value of the uncertain coefficient a_{ij} , denoted by δ_{ij} , is defined as

$$\delta_{ij} = |(a_{ij} - \overline{a}_{ij}) / \hat{a}_{ij}| \qquad \forall i, j . \tag{5}$$

Apparently, δ_{ij} takes values in the interval $[0,1]$. A budget of uncertainty Γ_i is imposed to the i th constraint in the following sense

$$\sum_{j=1}^{n} \delta_{ij} \le \Gamma_i \qquad 0 \le \Gamma_i \le n , \tag{6}$$

where $\Gamma_i = 0$ and $\Gamma_i = n$ correspond to the nominal and worst cases, respectively. Bertsimas and Sim [1] showed that letting the budget of uncertainty Γ_i vary in the interval $[0, n]$ makes it possible to build a model where performance is appropriately adjusted against robustness. When each a_{ij} is treated as a variable, the nonlinear robust formulation of the nominal model in (4) can be stated as

$$\begin{aligned} \min \quad & f(\mathbf{x}) \\ \text{s.t.} \quad & \max_{\mathbf{a}_i} [\overline{g}_i(\mathbf{a}_i, \mathbf{x}) | \textstyle\sum_{j=1}^{n} \delta_{ij} \le \Gamma_i] \le b_i \qquad \forall i \\ & \mathbf{l} \le \mathbf{x} \le \mathbf{u} , \end{aligned} \tag{7}$$

where $\mathbf{a}_i$ is the vector of uncertain coefficients in the i th constraint with each $a_{ij} \in [\overline{a}_{ij} - \hat{a}_{ij}, \overline{a}_{ij} + \hat{a}_{ij}]$ and $\overline{g}_i(\mathbf{a}_i, \mathbf{x}) = \sum_{j=1}^{n} a_{ij} x_j$ is the counterpart of $g_i(\mathbf{x})$ in (4) but with each a_{ij} treated as a variable. Bertsimas and Sim [1] proved that the nonlinear robust formulation in (7) has the following robust linear counterpart

$$
\begin{array}{lll}
\min & \sum_{j=1}^{n} c_j x_j & \\
\text{s.t.} & \sum_{j=1}^{n} \overline{a}_{ij} x_j + \Gamma_i q_i + \sum_{j=1}^{n} r_{ij} \le b_i & \forall i \\
& q_i + r_{ij} \ge \hat{a}_{ij} y_j & \forall i, j \\
& -y_j \le x_j \le y_j ,\ l_j \le x_j \le u_j & \forall j \\
& q_i \ge 0 & \forall i \\
& y_j \ge 0 & \forall j \\
& r_{ij} \ge 0 & \forall i, j .
\end{array}
\tag{8}
$$

A highly attractive feature of this formulation is that this linear counterpart is of the same class as the nominal model in (4) which can be easily solved with standard optimization packages. Moreover, Bertsimas and Sim [1] showed that even if the budget of uncertainty constraints are not satisfied, the robust solution will remain feasible with a very high probability.

4. Application to multiobjective problems

Under uncertainty, the problem coefficients in (1) are uncertain and, hence, the solution must be robust, *i.e.*, the solution should remain feasible (constraint robust), efficient and most preferred by the DM (objective function robust) under almost all possible realizations of uncertain coefficients. Both a_{ij} and c_{kj} are considered uncertain and their uncertainty is captured using the interval uncertainty discussed earlier. The nominal value and the half-interval width of c_{kj} are represented by $\overline{c}_{kj}$ and $\hat{c}_{kj}$, respectively. The k th uncertain objective function is expressed as $\overline{f}_k(\mathbf{c}_k, \mathbf{x}) = \sum_{j=1}^{n} c_{kj} x_j$ where $\mathbf{c}_k$ is the vector of uncertain coefficients in the k th objective function with each $c_{kj} \in [\overline{c}_{kj} - \hat{c}_{kj}, \overline{c}_{kj} + \hat{c}_{kj}]$. While being the counterpart of $f_k(\mathbf{x})$ in (1), $\overline{f}_k(\mathbf{c}_k, \mathbf{x})$ is a function of both $\mathbf{c}_k$ and $\mathbf{x}$ since each c_{kj} is treated as a variable. Similar to δ_{ij} defined for a_{ij} in (5), the absolute value of scaled deviation δ'_{kj} of c_{kj} from its nominal value $\overline{c}_{kj}$ is defined as

$$\delta'_{kj} = |(c_{kj} - \overline{c}_{kj}) / \hat{c}_{kj}| \qquad \forall i, j . \tag{9}$$

Similar to (6), a budget of uncertainty Γ'_k is imposed to the k th objective function such that

$$\sum_{j=1}^{n} \delta'_{kj} \le \Gamma'_k \qquad 0 \le \Gamma'_k \le n , \tag{10}$$

where $\Gamma'_k = 0$ and $\Gamma'_k = n$ correspond to the nominal and worst cases, respectively. Note that while Γ_i controls the robustness of the i th constraint, Γ'_k controls the robustness of the k th objective function against the level of conservatism. For notational convenience, let $\mathbf{\Gamma} \in \Re^m$ and $\mathbf{\Gamma}' \in \Re^K$ be the vectors of budgets of uncertainty for the constraints and for the objective functions, respectively. Imposing the budgets of uncertainty on the constraints and the objective functions will ensure that the solution will remain both constraint robust and objective function robust. The nonlinear robust formulation of the nominal multiobjective programming model in (1) is stated as

$$\begin{aligned} \min \quad & z_k = \max_{\mathbf{c}_k}\left[\overline{f}_k(\mathbf{c}_k,\mathbf{x}) \mid \sum_{j=1}^{n}\delta'_{kj} \le \Gamma'_k\right] && \forall k \\ \text{s.t.} \quad & \max_{\mathbf{a}_i}\left[\overline{g}_i(\mathbf{a}_i,\mathbf{x}) \mid \sum_{j=1}^{n}\delta_{ij} \le \Gamma_i\right] \le b_i && \forall i \\ & x_j \ge 0 && \forall j\,. \end{aligned} \tag{11}$$

Unlike the single objective model (7), each c_{kj} in the objective functions is also considered uncertain in (11).

Any feasible solution to the above model is called a robust feasible solution. The set of all robust feasible solutions, *i.e.*, $X^{\Gamma} = \{\mathbf{x} \in \Re^n \mid \max_{\mathbf{a}_i}[\overline{g}_i(\mathbf{a}_i,\mathbf{x}) \mid \sum_{j=1}^{n}\delta_{ij} \le \Gamma_i] \le b_i \ \forall i, x_j \ge 0\}$, is called the robust feasible region in decision space for a given vector $\mathbf{\Gamma}$. A $\mathbf{x} \in X^{\Gamma}$ is called a robust feasible solution in decision space. The set $Z^{\Gamma,\Gamma'} = \{\mathbf{z} \in \Re^K \mid z_k = \max_{\mathbf{c}_k}[\overline{f}_k(\mathbf{c}_k,\mathbf{x}) \mid \sum_{j=1}^{n}\delta'_{kj} \le \Gamma'_k], \mathbf{x} \in X^{\Gamma}\}$ is the robust feasible region in criterion space for the given vectors $\mathbf{\Gamma}'$ and $\mathbf{\Gamma}$. A $\mathbf{z} \in Z^{\Gamma,\Gamma'}$ is called a robust feasible solution in criterion space or a robust criterion vector. A nondominated robust criterion vector $\overline{\mathbf{z}} \in Z^{\Gamma,\Gamma'}$, an efficient robust solution $\overline{\mathbf{x}} \in X^{\Gamma}$, and an optimal robust solution $\hat{\mathbf{z}} \in Z^{\Gamma,\Gamma'}$ can be defined similarly to their counterparts for the nominal model (1). The robust ideal point $\mathbf{z}^* \in \Re^K$ is defined as $z_k^* = \min_{\mathbf{x}}\{\max_{\mathbf{c}_k}[\overline{f}_k(\mathbf{c}_k,\mathbf{x}) \mid \sum_{j=1}^{n}\delta'_{kj} \le \Gamma'_k], \mathbf{x} \in X^{\Gamma}\}$. A robust utopian point is also defined as $\mathbf{z}^{**} \in \Re^K$ such that $z_k^{**} = z_k^* - \varepsilon_k$ with $\varepsilon_k > 0$ and small.

For a given weighting vector $\mathbf{w} \in \mathbf{W}$, a robust augmented weighted Tchebycheff program for the nonlinear programming model in (11) is formulated from (3) as the following

$$\begin{aligned} \min \quad & \alpha + \rho\sum_{k=1}^{K}(z_k - z_k^{**}) \\ \text{s.t.} \quad & \alpha \ge w_k(z_k - z_k^{**}) && \forall k \\ & z_k = \max_{\mathbf{c}_k}[\overline{f}_k(\mathbf{c}_k,\mathbf{x}) \mid \sum_{j=1}^{n}\delta'_{kj} \le \Gamma'_k] && \forall k \\ & \max_{\mathbf{a}_i}[\overline{g}_i(\mathbf{a}_i,\mathbf{x}) \mid \sum_{j=1}^{n}\delta_{ij} \le \Gamma_i] \le b_i && \forall i \\ & x_j \ge 0 && \forall j \\ & z_k \ \text{unrestricted} && \forall k \\ & \alpha \ge 0\,. \end{aligned} \tag{12}$$

Similar to the coefficients in the objective function of model (7), the coefficients in the objective function of model (12) are exactly known.

An optimal solution to (12) minimizes the augmented weighted Tchebycheff metric between $\mathbf{z}^{**}$ and any $\mathbf{z} \in Z^{\Gamma,\Gamma'}$ while respecting the budget of uncertainty constraints. The solution to this formulation has some interesting properties. First, it is a nondominated solution for the selected $\mathbf{\Gamma}$ and $\mathbf{\Gamma}'$. Second, unlike its nominal counterpart, it is robust, *i.e.*, insensitive to uncertainties in the coefficients of both the objective functions and constraints. This means that given all possible realizations of a_{ij} and c_{kj}, the solution of (12) not only will have a much higher probability of feasibility than the nominal solution of (3) but also will have a corresponding criterion vector which performs comparable to the nondominated nominal criterion vector. These properties are significant because model (12) can assist the DM as a tool in finding nondominated robust solutions by properly balancing performance versus robustness. Using this formulation, the nominal solution closest to the nominal $\mathbf{z}^{**}$, measured by the augmented weighted Tchebycheff metric, is slightly sacrificed but in return, this sacrifice is compensated by the robustness of the solution.

Proposition. Model (12) has the following linear programming counterpart

$$
\begin{array}{lll}
\min & d & \\
\text{s.t.} & \alpha + \rho \sum_{k=1}^{K} \alpha_k - d \le 0 & \\
& w_k \alpha_k - \alpha \le 0 & \forall k \\
& \sum_{j=1}^{n} \bar{c}_{kj} x_j + \Gamma'_k q'_k + \sum_{j=1}^{n} r'_{kj} - \alpha_k \le z_k^{**} & \forall k \\
& \sum_{j=1}^{n} \bar{a}_{ij} x_j + \Gamma_i q_i + \sum_{j=1}^{n} r_{ij} \le b_i & \forall i \\
& q'_k + r'_{kj} \ge \hat{c}_{kj} y_j & \forall k, j \\
& q_i + r_{ij} \ge \hat{a}_{ij} y_j & \forall i, j \\
& x_j \le y_j ,\ y_j \ge 0 ,\ x_j \ge 0 & \forall j \\
& q_i \ge 0 & \forall i \\
& \alpha_k , q'_k \ge 0 & \forall k \\
& r'_{kj} \ge 0 & \forall k, j \\
& r_{ij} \ge 0 & \forall i, j \\
& d, \alpha \ge 0 . &
\end{array}
\tag{13}
$$

Proof. Model (12) is first reformulated as

$$
\begin{array}{lll}
\min & d & \\
\text{s.t.} & d \ge \alpha + \rho \sum_k \alpha_k & \forall k \\
& \alpha \ge w_k \alpha_k & \forall k \\
& \max_{\mathbf{c}_k} [\, \bar{f}_k(\mathbf{c}_k, \mathbf{x}) \,|\, \sum_j \delta'_{kj} \le \Gamma'_k \,] - \alpha_k \le z_k^{**} & \forall k \\
& \max_{\mathbf{a}_i} [\bar{g}_i(\mathbf{a}_i, \mathbf{x}) \,|\, \sum_j \delta_{ij} \le \Gamma_i \,] \le b_i & \forall i \\
& x_j \ge 0 & \forall j .
\end{array}
$$

Using the derivation of (8) from (7), (13) follows.■

The linear programming counterpart (13) of the robust augmented weighted Tchebycheff program (12) can be directly used to generate nondominated robust solutions. By using dispersed weighting vectors, dispersed nondominated robust solutions are obtained that can be used within an interactive procedure, such as the interactive weighted Tchebycheff procedure [2, 9, 10]. Each iteration of the interactive weighted Tchebycheff procedure has two phases. In the solution generation phase, a set of weighting vectors is generated and then filtered to a widely dispersed subset, a robust augmented weighted Tchebycheff program is solved to generate a nondominated robust criterion vector for each weighting vector in the subset, and the resulting nondominated robust criterion vectors are filtered to obtain a smaller subset of dispersed ones. In the solution evaluation phase, this subset of nondominated robust criterion vectors is presented to the DM who articulates the preference information by selecting the most preferred one. In the next iteration, the weighting vector space is reduced around the weighting vector corresponding to the current most preferred solution selected by the DM. New dispersed weighting vectors are then generated in this reduced weighting vector space, new nondominated robust solutions are generated, and so on. The procedure terminates after a predetermined number of iterations have been performed or when the DM is satisfied with a nondominated robust solution that has already been identified.

5. Conclusions

Multiobjective linear programming problems with imprecise coefficients in the objective functions and constraints are considered. A robust augmented weighted Tchebycheff program is formulated and its linear counterpart is developed that can be employed to generate nondominated robust solutions within an interactive procedure. An extraordinary strength of this approach is that robustness is achieved without bothering the DM in supplying unknown distribution details for the imprecise coefficients. This approach can be applied to a wide variety of multiobjective linear programming problems with imprecise data.

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