Role of thermal diffusion on double-diffusive natural convection in a vertical annular porous medium

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Abstract This paper derives analytical solutions of fully developed natural convection heat and mass transfer in a vertical annular non-Darcy porous medium. This investigation extends the work of Cheng (2006) to a situation where Soret effect is present and the flow is both aided and opposed. The influence of the controlling parameters on the flow characteristics has been seen to be higher for double diffusion in the absence of Soret effect than in the presence of Soret effect. It is found that the presence of Soret effect reduces the dependence of the volume flow rate, the total heat rate and the total species rate on the inner radius-gap ratio and on the modified Darcy number.

KEYWORDS Modified Darcy number; Natural convection; Annular duct; Porous medium; Double diffusion; Soret effect

1. Introduction

The natural convection of binary fluids flow in porous media has attracted great research interest during the past few decades. While a good number of works have made significant contributions for the development of the theory, an equally good number of works have been devoted to the numerous industrial, natural and geophysical applications. Double-diffusive convective flows in a differentially heated vertical annulus have been intensively studied in relation to applications such as oxidation of surface materials, cleaning and dying operations, fluid storage components and energy storage in solar ponds [1]. Consideration of two kinds of problems concerning the convection of a binary mixture filling a porous layer is in the literature. The first kind of problem considers flows induced by the buoyancy forces resulting from the imposition of both thermal and solute boundary conditions on the layer. The second kind of problem considers thermal convection in a binary fluid driven by Soret-effects. For this situation, the species gradients are not due to the imposition of solute boundary conditions. Rather, they result from the imposition of a temperature gradient in an otherwise uniform-concentration mixture. This phenomenon has many applications in geophysics, oil reservoirs, and ground water. Bahoul et al. [2] gave the reviews of previous works done in this direction. A very recent comprehensive overview of double-diffusive convection in saturated porous media, its relevance in the understanding of many
natural systems and its wide variety of engineering applications is well documented in the literature [3–7].

Boutana et al. [8] investigated the Soret effect and double-diffusive natural convection in a rectangular porous medium filled with a binary fluid both analytically and numerically. They reported that the range of buoyancy ratios for the existence of multiple solutions depends on the type of convection induced by the solute gradients, i.e. on the constant $a$. In particular, it was demonstrated that for opposing flows, near the buoyancy ratio greater than $-1$, multiple solutions are possible for a range of $R_t$ which depends on the other governing parameters. Nithyadevi and Yang [9] considered the effect of double-diffusive natural convection of water in a partially heated enclosure with Soret and Dufour coefficients numerically. They concluded that the fluid particle moves with lesser velocity and high mass transfer rate in the presence of Soret coefficient when the partially heated active vertical left side wall has lower concentration while the opposite behaviour is observed when the partially heated active vertical left side wall has higher concentration. Allou and Vasseur [10] studied analytically and numerically the double-diffusive and Soret-induced natural convection in a shallow rectangular cavity filled with a micropolar fluid. Nikbakhti and Rahimi [11] carried out a study of the double-diffusive natural convection in a rectangular cavity with partially thermally active side walls filled with air numerically and found that the average Nusselt number increases by increasing the buoyancy ratio, thus increasing the heat transfer rate in the cavity. Basu and Layek [12] gave a theoretical analysis to investigate the onset of double-diffusive convection at the marginal state in presence of cross-diffusive terms, viz. Soret and Dufour effects in a horizontal fluid layer and concluded that the Soret parameter destabilizes the oscillatory convection while the Dufour does the opposite. Sankar et al. [13] reported a numerical study of double-diffusive convection in a fluid-saturated vertical porous annulus subjected to discrete heat and mass fluxes from a portion of the inner wall. They discovered that the average Nusselt number increases with radius ratio; however, the average Sherwood number increases with radius ratio only up to when the radius ratio is equal to five, and for the radius ratio greater than five, the average Sherwood number does not increase significantly. Nik-Ghazali et al. [14] conducted a study on heat and mass transfer behaviour on porous medium embedded in a square annulus where the inner surface wall is considered to have a cool temperature while the outer surface is exposed to a hot temperature. They discovered that Soret effect tends to make contribution that is more significant to the concentration profile than Dufour effect. Cheng [15] studied the Soret and Dufour effects on the boundary layer flow due to free convection heat and mass transfer over a vertical cylinder in a porous medium saturated with Newtonian fluids with constant wall temperature and concentration. Their results showed that an increase in the Soret number leads to a decrease in the local Sherwood number and an increase in the local Nusselt number. The same author [16], in his study of free convection boundary layer flow over an arbitrarily inclined heated plate in a porous medium with Soret and Dufour effects, pointed out that an increase in the Dufour parameter tends to decrease the local heat transfer rate and an increase in the Soret parameter tends to decrease the local mass transfer rate. Ouriemi et al. [17] presented an analytical and numerical study of natural convection of a binary fluid confined in a tall enclosure, slightly inclined about the gravity field where they showed that, for a given Raleigh number, both “natural” flow, circulating clockwise and “anti-natural” flow, circulating anticlockwise are possible provided the angle of inclination is small enough. Lakshmi Narayana et al. [18] investigated the stability of Soret-driven thermosolutal convection in a shallow horizontal layer of a porous medium subjected to inclined thermal and solutal gradients of finite magnitude theoretically and observed
that the Soret parameter has a significant effect on convective instability. Mahdy [19] presented a non-similar boundary layer analysis to study the flow, heat and mass transfer characteristics of non-Darcian mixed convection of a non-Newtonian fluid from a vertical isothermal plate imbedded in a homogeneous porous medium with the effect of Soret and Dufour and in the presence of either surface injection or suction. They reported that as the Soret parameter increases and the Dufour parameter decreases, both velocity and concentration decrease, whereas the temperature distribution increases. In their study of double-diffusive convection in a horizontal sparsely packed porous layer in presence of Soret effect, Gaikwad and Kamble [20] revealed that as the Soret parameter increases, the critical thermal Raleigh number decreases implying that the increase of Soret effect is to destabilise the system. Chen et al. [21] investigated numerically the double-diffusive (natural) convection in vertical annuluses with opposing temperature and concentration gradients. Prasad et al. [22] considered the thermo-diffusion and diffusion-thermo effects on MHD free convection flow past a vertical porous plate embedded in a non-Darcian porous medium.

Cheng [23] studied the fully developed heat and mass transfer by natural convection of a non-Darcian fluid flowing through a vertical annular porous medium with asymmetric wall temperatures and concentrations. The study reports that an increase in the modified Darcy number, the buoyancy ratio, and the inner radius-gap ratio increases the volume flow rate, the total heat rate added to the fluid and the total species rate added to the fluid.

The present study deals not only with the double-diffusive natural convection in the absence of Soret effect in an annulus as considered by Cheng [23] but also extends it to the case when Soret-induced natural convection is present and when the flow is both aiding and opposing. Future investigation on this problem can be done on a spherical geometry.

2. Analysis

Consider a steadily fully developed laminar natural convection flow in an annular region of infinite length embedded in a homogeneous fluid-saturated porous medium (see Fig 1). The convection current is induced by both the temperature and concentration gradients. The fluid is assumed to satisfy the Boussinesq approximation, with constant properties except for the density variations in the buoyancy force term. The density variation with temperature and concentration is described by the equation of state $\rho = \rho_0(1 - \beta_1(T - T_0) - \beta_2(C - C_0))$ where $\rho_0$ is the fluid mixture density at temperature $T = T_0$ and mass fraction $C = C_0$. Due to fully developed flow assumptions, the fluid enters the part of the annular passage under consideration with an axial velocity profile which remains invariant in the entire channel (i.e. $\partial u'/\partial z' = 0$). Because the flow is fully developed, the flow depends only on the radial coordinate $r'$. In the present investigation, the Dufour effect is neglected since it is well known that the modification of heat flow due to the concentration gradient is of importance in gases but negligible in liquids. Under these assumptions, the equations governing the steady-state conservation of momentum, energy, and constituent for non-Darcy flow through a homogeneous porous medium inside the annular duct can be written as [23–25]

\[
\frac{v}{\varepsilon} \frac{1}{r'} \frac{d}{dr'} \left( r' \frac{du'}{dr'} \right) - \frac{v}{K} u' = -g\beta_1(T - T_0) - g\beta_2(C - C_0),
\]

\[
\frac{d^2 T}{dr'^2} + \frac{1}{r'} \frac{dT}{dr'} = 0,
\]

\[
\frac{d^2 C}{dr'^2} + \frac{1}{r'} \frac{dC}{dr'} = 0.
\]

The appropriate boundary conditions applied on the outer surface of the inner cylinder and inner surface of the outer cylinder are as follows:

Case I: double-diffusive convection in the absence of Soret effect

\[
u' = 0, \quad T = T_1, \quad C = C_1 \quad \text{at} \quad r' = r_1
\]

\[
u' = 0, \quad T = T_2, \quad C = C_2 \quad \text{at} \quad r' = r_2
\]

Case II: double-diffusive convection in the presence of Soret effect [26]

\[
u' = 0, \quad T = T_1, \quad \frac{dC}{dr'} = - \frac{D'}{D} C_0(1 - C_0) \frac{dT}{dr'} \quad \text{at} \quad r' = r_1
\]

\[
u' = 0, \quad T = T_2, \quad \frac{dC}{dr'} = - \frac{D'}{D} C_0(1 - C_0) \frac{dT}{dr'} \quad \text{at} \quad r' = r_2
\]

A unified solution for both cases can be obtained by combining (4) and (5). By doing so, combined boundary conditions are obtained as

\[
u' = 0, \quad T = T_1, \quad (1 - a)C + a \frac{dC}{dr'} = (1 - a)C_1 - a \frac{D'}{D} C_0(1 - C_0) \frac{dT}{dr'}, \quad \text{at} \quad r' = r_1
\]

\[
u' = 0, \quad T = T_2, \quad (1 - a)C + a \frac{dC}{dr'} = (1 - a)C_2 - a \frac{D'}{D} C_0(1 - C_0) \frac{dT}{dr'}, \quad \text{at} \quad r' = r_2
\]

Figure 1 The geometry of the problem with a porous medium inside the annulus.
in which \(a = 0\) corresponds to double-diffusive convection in the absence of Soret effect and \(a = 1\) corresponds to double-diffusive convection in the presence of Soret effect.

The governing equations are non-dimensionalised by introducing the following non-dimensional variables:

\[
R = (r' - r_1)/(r_2 - r_1), \quad \dot{\lambda} = r_1/(r_2 - r_1), \quad U = u(r_2 - r_1)/Gr_0, \quad \theta = (T - T_0)/(T_1 - T_0), \quad \phi = (C - C_0)/\Delta C, \quad \text{Gr} = \beta g(T - T_0)(r_2 - r_1)^3/\nu^2,
\]

where \(\Delta C = C_1 - C_0\) for double-diffusive convection in the absence of Soret effect and \(\Delta C = -C_0(1 - C_0)(T_1 - T_0)D^*/D\) for Soret-driven convective double diffusion.

The dimensionless equations governing the present problem then read

\[
\frac{d^2 U}{dR^2} + \frac{1}{R + \dot{\lambda}} \frac{dU}{dR} - Da^{-1} U = -\theta - N\phi, \quad (8)
\]

\[
\frac{d\theta}{dR} + \frac{1}{R + \dot{\lambda}} \frac{d\theta}{dR} = 0, \quad (9)
\]

\[
\frac{d\phi}{dR} + \frac{1}{R + \dot{\lambda}} \frac{d\phi}{dR} = 0. \quad (10)
\]

The corresponding boundary conditions in dimensionless form are

\[
U = 0, \quad \theta = 1, \quad (a - 1)\phi + a \frac{d\phi}{dR} = 0 \quad \text{at} \quad R = 0, \quad (11)
\]

\[
U = 0, \quad \theta = R, \quad (a - 1)\phi + a \frac{d\phi}{dR} = (a - 1)R_c + a \frac{d\phi}{dR} \quad \text{at} \quad R = 1 \quad (12)
\]

where \(N = \beta/\Delta C/\beta g(T_1 - T_0), \quad R_c = (T_2 - T_0)/(T_1 - T_0), \quad R_c = (C_2 - C_0)/(C_1 - C_0)\) and \(Da = K/\nu(r_2 - r_1)^2\).

In the present formulation, the particular case \(a = 0\) corresponds to double-diffusive convection in the absence of Soret effect for which the solute buoyancy forces are induced by the imposition of a constant concentration such that \(\phi = 1\) at \(R = 0\) and \(\phi = R\) at \(R = 1\). On the other hand, \(a = 1\) corresponds to the case of a binary fluid subject to the Soret-effect. For this situation, setting \(a = 1\) in Eqs. (11) and (12) yields \(d\phi/\text{d}R = d\theta/\text{d}R\) at both annular boundaries, which shows that the solutal flux is dependent on thermal flux at the boundaries.

Solving Eqs. (9) and (10) with their corresponding boundary conditions (11) and (12), the dimensionless temperature and concentration are

\[
\theta(R) = a_1 \ln(R + \dot{\lambda}) + a_2, \quad (13)
\]

\[
\phi(R) = a_1 \ln(R + \dot{\lambda}) + a_3, \quad (14)
\]

where

\[
a_1 = (R_c - 1)/\ln(1 + \dot{\lambda}), \quad a_2 = \ln(1 + \dot{\lambda}) - R_0 \ln \dot{\lambda}/\ln(1 + \dot{\lambda}), \quad a_3 = ((1 - a)(1 - R_0) + aa_1/\dot{\lambda}(1 + \dot{\lambda}))/\dot{\lambda}(1 + \dot{\lambda}) - (1 - a)\ln(1 + \dot{\lambda}).
\]

Substituting Eqs. (13) and (14) into Eq. (8) and then solving it with its corresponding boundary conditions in Eqs. (11) and (12), we obtain the following solution:

\[
U = a_5 \ln(R + \dot{\lambda})/\sqrt{Da} + a_6 \ln[(R + \dot{\lambda})/\sqrt{Da}] + a_7 \ln(R + \dot{\lambda}) + a_8 \ln(1 + \dot{\lambda}) \quad \text{at} \quad R = 0, \quad (15)
\]

where

\[
a_5 = [f_1 K_0(f_3) - f_2 K_0(f_2)]/[1/Da I_0(f_3) K_0(f_2) - I_0(f_2) K_0(f_3)],
\]

\[
a_6 = [f_2 I_0(f_3) - f_1 I_0(f_2)]/[1/Da I_0(f_3) K_0(f_2) - I_0(f_2) K_0(f_3)],
\]

\[
a_7 = Da(a_1 + a_3 N), \quad a_8 = Da(a_2 + a_3 N), \quad f_1 = (a_1 + a_3 N) \ln(1 + \dot{\lambda}) + a_2 + a_3 N, \quad f_2 = (a_1 + a_3 N) \ln(\dot{\lambda}) + a_2 + a_3 N, \quad f_3 = \dot{\lambda} \sqrt{Da}, \quad f_4 = (1 + \dot{\lambda}) \sqrt{Da}.
\]

We present expression for the rate of heat transfer which is expressed as a Nusselt number by the use of Eq. (13). Denoting the Nusselt number by \(Nu\), we obtain

\[
Nu = a_5 = \frac{R_c - 1}{\dot{\lambda} \ln(1 + 1/\dot{\lambda})}. \quad (16)
\]

From where we have

\[
Nu_0 = \frac{R_c - 1}{\dot{\lambda} \ln(1 + 1/\dot{\lambda})}, \quad (17)
\]

\[
Nu_1 = \frac{R_c - 1}{(1 + \dot{\lambda}) \ln(1 + 1/\dot{\lambda})}. \quad (18)
\]

Eqs. (17) and (18) indicate that the Nusselt numbers at the outer surface of inner cylinder (\(Nu_0\)) and at the inner surface of outer cylinder (\(Nu_1\)) are zero for symmetric heating (\(R_c = 1\)). However, for asymmetric wall temperatures, heat is transferred from the hot wall to the cold wall.

The dimensionless volume flow rate is given by

\[
Q = 2\pi \int_0^1 (R + \dot{\lambda}) U dR. \quad (19)
\]

where \(Q = Q'/(Gr_0(r_2 - r_1))\) and \(Q'\) is the flux of fluid flowing through the annular region.

Substituting Eq. (15) into Eq. (19) and integrating, we obtain the dimensionless volume flow rate as

\[
Q = a_9 I_0 \left[\frac{(1 + \dot{\lambda})}{\sqrt{Da}}\right] + a_{10} I_1 \left[\frac{(1 + \dot{\lambda})}{\sqrt{Da}}\right] + a_{11} K_1 \left[\frac{(1 + \dot{\lambda})}{\sqrt{Da}}\right] + a_{12} K_1 \left[\frac{(\dot{\lambda})}{\sqrt{Da}}\right] + a_{13} \ln(1 + \dot{\lambda}) + a_{14} \ln(\dot{\lambda}) + a_{15}, \quad (20)
\]

where

\[
a_0 = 2\pi \sqrt{Da} a_0 (1 + \dot{\lambda}), \quad a_9 = -2\pi \sqrt{Da} a_0 \dot{\lambda}, \quad a_{11} = -2\pi \sqrt{Da} a_0 \dot{\lambda}, \quad a_{12} = 2\pi \sqrt{Da} a_0 \dot{\lambda}, \quad a_{13} = \pi a_3 (1 + \dot{\lambda})^2, \quad a_{14} = -\pi a_3 \dot{\lambda}^2, \quad a_{15} = \pi (1 + 2\dot{\lambda})(2a_8 - a_7)/2.
\]
The dimensionless total heat rate added to the fluid is
\[ H = 2\pi \int_{0}^{1} (R + \lambda) U \psi dR, \]  
(21)
where \( H = H'/[Gr\alpha(T_1 - T_0)(r_2 - r_1)] \) and \( H' \) is the total heat rate added to the fluid.

Substituting Eqs. (13) and (15) into Eq. (21) and integrating give the dimensionless total heat rate added to the fluid as
\[ H = a_5 \lambda + a_6 \lambda I_1 \left( \frac{1 + \lambda}{\sqrt{Da}} \right) + a_7 \lambda I_1 \left( \frac{\lambda}{\sqrt{Da}} \right) \]
\[ + a_8 \lambda I_0 \left( \frac{1 + \lambda}{\sqrt{Da}} \right) + a_9 \lambda I_0 \left( \frac{\lambda}{\sqrt{Da}} \right) \]
\[ + a_{10} K_1 \left( \frac{1 + \lambda}{\sqrt{Da}} \right) + a_{11} K_1 \left( \frac{\lambda}{\sqrt{Da}} \right) \]
\[ + a_{12} K_0 \left( \frac{1 + \lambda}{\sqrt{Da}} \right) + a_{13} K_0 \left( \frac{\lambda}{\sqrt{Da}} \right) \]
\[ + a_{14} \ln(\lambda + 1) + a_{15} \ln(\lambda + 1) \]
\[ + a_{16} \ln \lambda + a_{17} \lambda. \]
(22)
where
\[ a_{16} = 2\pi \sqrt{Da} a_5 \lambda \ln(1 + \lambda), \]
\[ a_{17} = -2\pi \sqrt{Da} a_5 \lambda \ln \lambda, \]
\[ a_{18} = 2\pi \sqrt{Da} a_6 \lambda \ln(1 + \lambda) \]
\[ a_{19} = 2\pi \sqrt{Da} a_7 \lambda \ln \lambda, \]
\[ a_{20} = 2\pi \sqrt{Da} a_8 \lambda \ln(1 + \lambda), \]
\[ a_{21} = 2\pi \sqrt{Da} a_9 \lambda \ln \lambda, \]
\[ a_{22} = -2\pi a_5 a_6 D_4 a_7 \lambda(1 + \lambda), \]
\[ a_{23} = 2\pi a_5 a_7 D_4 a_8 \lambda, \]
\[ a_{24} = \pi a_5 a_7 \lambda^2, \]
\[ a_{25} = \pi a_5 a_7 \lambda^2, \]
\[ a_{26} = -\pi a_5 a_7 \lambda^2, \]
\[ a_{27} = (\pi a_5 (1 + 2\lambda))(\lambda - a_4)/2. \]

The dimensionless total species rate added to the fluid is
\[ \psi = 2\pi \int_{0}^{1} (R + \lambda) V \phi dR, \]  
(23)
where \( \psi = \psi'/[Gr\alpha(C_1 - C_0)(r_2 - r_1)] \) and \( \psi' \) is the total species rate added to the fluid.

Substituting Eqs. (14) and (15) into Eq. (23) and integrating give the dimensionless total species rate added to the fluid as
\[ \psi = a_{28} \lambda + a_{29} \lambda I_1 \left( \frac{1 + \lambda}{\sqrt{Da}} \right) + a_{30} \lambda I_1 \left( \frac{\lambda}{\sqrt{Da}} \right) \]
\[ + a_{31} \lambda I_0 \left( \frac{1 + \lambda}{\sqrt{Da}} \right) + a_{32} \lambda I_0 \left( \frac{\lambda}{\sqrt{Da}} \right) \]
\[ + a_{33} K_1 \left( \frac{1 + \lambda}{\sqrt{Da}} \right) + a_{34} K_1 \left( \frac{\lambda}{\sqrt{Da}} \right) \]
\[ + a_{35} K_0 \left( \frac{1 + \lambda}{\sqrt{Da}} \right) + a_{36} K_0 \left( \frac{\lambda}{\sqrt{Da}} \right) \]
\[ + a_{37} \ln(\lambda + 1) + a_{38} \ln(\lambda + 1) \]
\[ + a_{39} \ln \lambda + a_{40}, \]  
(24)
where
\[ a_{39} = \pi a_5 a_6 (1 + \lambda)^2, \]
\[ a_{40} = -\pi a_5 a_6 \lambda^2 \]
\[ a_{41} = (\pi a_5 (1 + 2\lambda))(\lambda - a_4)/2. \]

By the use of Eq. (14), the local Sherwood number for both walls is
\[ Sh = \frac{d\phi}{dR} \bigg|_{R=0.1} = \frac{\lambda(1 + \lambda)(1 - a)(1 - R_c) + a_{41}}{(\lambda + R)(\lambda - a)(1 + \lambda)(1 - a)(1 + \ln(1 + 1/\lambda))}. \]  
(25)
For \( a = 0, \)
\[ Sh_0 = -\frac{1 - R_c}{\lambda \ln(1 + 1/\lambda)}. \]  
(26)
\[ Sh_1 = -\frac{1 - R_c}{(1 + \lambda) \ln(1 + 1/\lambda)}. \]  
(27)
For \( a = 1, \)
\[ Sh_0 = \frac{R_c - 1}{(1 + \lambda) \ln(1 + 1/\lambda)}. \]  
(28)
\[ Sh_1 = -\frac{R_c - 1}{(1 + \lambda) \ln(1 + 1/\lambda)}. \]  
(29)
Eqs. (26)–(29) show that the local Sherwood number depends on the solute concentrate for \( a = 0 \) and on the thermal condition for \( a = 1. \) When \( a = 0, \) Eqs. (26) and (27) show that the local Sherwood number is zero for symmetric annular wall concentration \( (R_c = 1) \) and for asymmetric wall concentration, species are transferred from the higher concentration wall to the lower concentration wall. When \( a = 1, \) Eqs. (28) and (29) show that the local Sherwood number is zero for symmetric annular wall heating \( (R_c = 1) \) and for asymmetric wall heating, species are transferred from the hotter wall to the cold wall.

3. Results and discussion

The fixed values selected for all cases are \( R_c = 0.6 \) and \( R_c = 0.2 \) [23]. The effects of the governing physical parameters such as the buoyancy ratio \( (N), \) the modified Darcy number \( (Da) \) and the inner radius-gap ratio \( (\lambda) \) on the heat and mass transfer are displayed graphically in Figs. 2–9. Results are discussed for convection induced by double diffusion both in the absence of Soret effect \( (a = 0) \) represented by solid lines and in the presence of Soret effect \( (a = 1) \) represented by dot lines.

Fig. 2 illustrates the influence of \( N \) on the velocity for \( \lambda = 0.5 \) and \( Da = 0.1 \). The buoyancy ratio is varied from \( 4 \) to \( 4 \) to cover the spectrum from opposite but solute-dominated flow \( (N = -4) \) through purely thermal-dominated flow \( (N = 0) \) to aided solute-dominated flow \( (N = 4) \). First, we consider the case \( N = 0 \) that corresponds to a pure thermal situation for which the flow is induced by the imposed temperature gradients only. The graph shows that for this situation, the velocity is independent of the convective mode \( (a = 0) \) or \( a = 1 \). Next, we consider a double-diffusive system without taking into account the Soret effect \( (a = 0) \). When \( N < 0, \) the thermal and solute buoyancy forces act in opposite directions. It is clearly seen in the graph that the solute buoyancy forces are dominating the flow such that the velocity decreases with an increase in \( N. \) On the other hand, when \( N > 0, \) it is clear that the velocity increases as \( N \) increases. This is because the thermal and solute buoyancy forces act in the same direc-
Figure 2  Effect of buoyancy ratio on velocity profiles 
\( (Da = 0.1, R_t = 0.6, R_c = 0.2, \lambda = 0.5) \).

Figure 3  Effect of modified Darcy number on velocity profiles 
\( (N = 2.0, R_t = 0.6, R_c = 0.2, \lambda = 0.5) \).

Figure 4  Effects of buoyancy ratio and modified Darcy number on volume flow rate 
\( (R_t = 0.6, R_c = 0.2, \lambda = 0.5) \).

Figure 5  Effects of buoyancy ratio and modified Darcy number on total heat rate added to fluid 
\( (R_t = 0.6, R_c = 0.2, \lambda = 0.5) \).

Figure 6  Effects of buoyancy ratio and modified Darcy number on total species rate added to fluid 
\( (R_t = 0.6, R_c = 0.2, \lambda = 0.5) \).

Figure 7  Effects of inner radius-gap ratio and modified Darcy number on volume flow rate 
\( (R_t = 0.6, R_c = 0.2, N = 2.0) \).
It is shown in the figure that increasing the buoyancy number (\(N\)) for such behaviour is that the porous medium with higher \(N\) increases the velocity is observed to increase for \(a = 0\) and to decrease for \(a = 1\) thus respectively increasing and decreasing the volume of fluid flowing through the annular duct. In addition, the figure reveals that as the modified Darcy number (\(Da\)) increases, the volume flow rate increases in the case of \(a = 0\) in the range \(N > -1.5\) and decreases in the range \(N < -1.5\) while it is independent of the modified Darcy number at \(N = -1.5\). On the other hand, by increasing the modified Darcy number in the case of \(a = 1\) the volume flow rate increases in the range \(N < 2.5\), decreases in the range \(N > 2.5\) and independent of the Darcy number at \(N = 2.5\).

Fig. 5 displays the total heat rate (\(H\)) added to the fluid as a function of the buoyancy ratio (\(N\)) for different modified Darcy numbers (\(Da\)) when \(\lambda = 0.5\). It is observed that with an increase in \(N\), the total heat rate added to the fluid increases for the case of \(a = 0\) and decreases for the case of \(a = 1\). Due to the increase in the velocity with an increase in the buoyancy ratio, for \(a = 0\), the heat transfer rate between the wall and the fluid increases leading to the increase in the total heat rate added to the fluid. On the other hand, the decrease in velocity with an increase in \(N\) for \(a = 1\) makes the heat transfer rate between the wall and the fluid to decrease leading to the decrease in the total heat rate added to the fluid. In addition, increasing the modified Darcy number increases the velocity of the fluid in the annulus in the range \(-1.5 < N < 2.5\) which leads to an increase in the heat transfer rate between the annular walls and the fluid in both cases of \(a = 0\) and \(a = 1\) thus increasing the total heat rate added to the fluid.

Fig. 6 shows the total species rate added to the fluid for varying values of buoyancy ratio and modified Darcy number. The figure shows that the total species rate added to the fluid increases with increase in the buoyancy ratio for both cases of \(a = 0\) and \(a = 1\). This follows from the increase in the velocity of the fluid, which enhances the mass transfer rate between the fluid and the wall thus raising the total species rate added to the fluid. Again, by increasing the modified Darcy number, the total species rate added to the fluid increases in the range \(-1.5 < N < 2.5\) and decreases in the range \(-1.5 > N > 2.5\) for both cases of \(a = 0\) and \(a = 1\) while it is independent of the modified Darcy number at \(N = -1.5\) when \(a = 0\) and at \(N = 2.5\) when \(a = 1\).

<table>
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<th>Table 1</th>
<th>Comparison of the values of the velocity obtained in the present work with those obtained by Cheng [23].</th>
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<td>Velocity ((N = 2.0, R_i = 0.6, R_o = 0.2, \lambda = 0.5))</td>
<td>Da</td>
</tr>
<tr>
<td>----------</td>
<td>--------------------------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>0.01</td>
<td>0.2</td>
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<tr>
<td></td>
<td>0.4</td>
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<tr>
<td></td>
<td>0.6</td>
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<tr>
<td></td>
<td>0.8</td>
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<tr>
<td>0.05</td>
<td>0.2</td>
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<td></td>
<td>0.4</td>
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<td></td>
<td>0.6</td>
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<td>0.1</td>
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<td></td>
<td>0.4</td>
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<td>0.6</td>
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<tr>
<td></td>
<td>0.8</td>
</tr>
</tbody>
</table>

Discussion and the flow is considered to be aided. Finally, we study the double-diffusive convection in the presence of Soret effect (\(a = 1\)). The figure reveals that the presence of the Soret effect reverses the flow phenomenon with respect to the buoyancy ratio.

Fig. 3 shows the effect of the modified Darcy number (\(Da\)) on the velocity when \(\lambda = 0.5\) and \(N = 2.0\). The figure indicates that increasing \(Da\) of the porous medium leads to an increase in the fluid velocity in both cases of \(a = 0\) and \(a = 1\). The reason for such behaviour is that the porous medium with higher modified Darcy number has lower resistance to the fluid flow.

The effect of buoyancy ratio (\(N\)) and the modified Darcy number (\(Da\)) on the volume flow rate (\(Q\)) are depicted in Fig. 4. It is shown in the figure that increasing the buoyancy ratio raises the volume flow rate in the case of \(a = 0\) and lowers it in the case of \(a = 1\). This follows from the fact that as \(N\)}
Figs. 7–9 reveal the effect of the inner radius-gap ratio (λ) on the volume flow rate (Q), the total heat rate added to the fluid (H) and the total species rate added to the fluid (ψ) respectively. It is worthy to note that increasing the inner radius-gap ratio (λ) leads to an increase in the cross-sectional area of the annular duct by the definition of λ. Therefore, in Fig. 7 the volume flow rate of the fluid increases due to the increase in the cross-sectional area of the duct. This increase in the volume flow rate coupled with the fact that the heat transfer area increases with the increase in the cross-sectional area of the duct results in an increase in the total heat rate added to the fluid as shown in Fig. 8. In addition, Fig. 9 shows that the increase in the volume flow rate with an increase in the mass transfer area of the fluid due to the increase in the cross-sectional area of the duct increases the total species rate added to the fluid.

Table 1 gives a comparison of the numerical values of the velocity obtained in the present work when a = 0 with those obtained by Cheng [23] for N = 2.0, R = 0.6, R_c = 0.2, and λ = 0.5. As can be seen from Table 1, the solutions of the present work perfectly agree with those of Cheng [23].

4. Conclusion

The fully developed heat and mass transfer by natural convection of a non-Darcian fluid flowing through a vertical annular porous medium with asymmetric wall temperatures and concentrations is studied. The exact solution for the problem has been obtained. The effects of the buoyancy ratio, the modified Darcy number, and the inner radius-gap ratio on the volume flow rate, total heat rate, and total species rate added to the fluid are carefully examined. The results presented in this study illustrate the difference between double diffusion in the absence of Soret effect (a = 0) [23] and in the presence of Soret-induced convection (a = 1). For instance, the volume flow rate, the total heat rate added to the fluid and the total species rate added to the fluid depend considerably on the inner radius-gap ratio and the modified Darcy number in the case of a = 0, whereas, their dependence on the inner radius-gap ratio and the modified Darcy number in the case of a = 1 is small. Also, the results indicate that the influence of the controlling parameters on the flow characteristics is higher for a = 0 than for a = 1. It is seen that both the volume flow rate, the total heat rate and the total species rate added to the fluid increase with increase in the modified Darcy number for certain ranges of buoyancy ratio in the case of both a = 0 and a = 1. On the other hand, the volume flow rate, the total heat rate and the total species rate added to the fluid are observed to increase in the case of a = 0 and decrease in the case of a = 1 with increase in the buoyancy ratio. It is equally observed that there are values of the buoyancy ratio (N = 1.5 for a = 0 and N = 2.5 for a = 1) for which the flow characteristics are independent of the modified Darcy number. Finally, the volume flow rate, the total heat rate and the total species rate added to the fluid are observed to increase with increase in the inner radius-gap ratio for both a = 0 and a = 1. It is worthy to note that the results for a = 0 perfectly agree with the results of Cheng [23].

References


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