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Vehicle routing to multiple warehouses using a memetic algorithm

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Abstract

This paper presents a solution procedure for solving the vehicle routing problem with pick-up and delivery with multiple warehouses (satellites) (Multi warehouse Vehicle Routing Problem with Pickup and Delivery-MDVRPPD) based on a hybrid metaheuristic. The solution strategy includes a memetic algorithm consisting of an evolutionary metaheuristic (genetic algorithm) and a heuristic local search (2-opt algorithm).

1. Introduction

In recent years, the Vehicle Routing Problems (VRP) have received special attention from the academic and research community, example of this are the numerous studies and research that can be easily found on search engines and specialized databases. In these studies are outstanding, vehicle routing problem with pick up and delivery (VRPPD), especially for its strong orientation to the urban areas subject to traffic problems and mobility restrictions (Parragh et al., 2008). In these problems a set of routes must be constructed to meet the transport

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requirements, these requirements can be characterized in delivery locations and output locations. As well as the VRP, the VRPPD receive many sorted classifications into two basic classes: the first class applies to VRPPD with return (Backhauls) in which the goods to be delivered are loaded in one or more central warehouses to be sent to one or various delivery points. The second class of VRPPD correspond to problems where goods are transported from a pick up point and delivery point, an example of such problems is the transport of passengers for a bus that runs through the city streets picking up and dropping passengers. For more detail on the different classifications VRPPD, see Parragh et al. (2008), and some models are presented in Kang et al (2007), and Pisinger Ropke (2006), Sifa et al (2011). In this article we will focus on the first class of models with multiple satellites warehouses.

Other problems of VRP family, which have attracted much attention are the models of two levels, in which a central warehouse sends the goods to several satellites (level 1), and from there to serve the customers in their impingement zone (level 2). Such models are very common in supply chain and logistics systems, however there is little literature on the subject (Feliu et al., 2008). For researchers Crainic et al (2004), two-levels models allow to increase the average occupancy per vehicle and reduce the number of freight transport operations in urban areas, through a rationalization of distribution activities which involve consolidation. An analysis of this type of model could see in Feliu et al. (2008) who formulate a 2E-CVRP model (Two-Echelon Capacitated Vehicle Routing Problem) as an extension of Capacitated Vehicle Routing Problem (CVRP), where deliveries sent to customers pass through intermediate satellites. In Perboli et al (2008) different models and heuristics are analyzed for 2E-CVRP.

The model discussed in this article will be based on the 2E-CVRP models formulated in et al (2008) and Feliu et al. (2008), with the additional component of pick up and deliveries.

2. Model MDVRPPD

In this section, pick-up and delivery problem is presented in a scenario of two levels, a $D$ central warehouses and $S$ satellites warehouses (suitable design in city conditions) which serve $N$ customers. There are two types of vehicles, the first type ($V$) are used at the first level formed by central warehouses and satellites. Vehicles type two ($W$) serve the second level formed by satellites and customers. The $V$ and $W$ vehicles have a capacity $\beta_v$ and $\beta_w$ respectively. Regarding customer service, variable $q_i$ will represent the amount of product associated with customer $i$ that can be a delivery ($-q_i$) or pick up ($+q_i$). The complete vehicles depart from the central store and warehouse, at a cost of route between $i$ and $j$ $C_{ij}^v$, $C_{ij}^w$, while $R_s$ is the cost per load unit on the $s$ satellite. Figure 1 shows a representation of the problem described.

Other variables in the model are:

- $R_s$: Cost per load unit on the $s$ satellite warehouse
- $x_{v_{ij}}$: 1 if $v$ vehicle $\in V$ does the route $(i, j)$ in the first level.
The problem to be solve is expressed as:

$$\min \sum_{v \in V} \sum_{(i,j) \in E} C_{ij} x_{ij} + \sum_{s \in S} \sum_{(i,j) \in E} C_{ij}^{ws} y_{ij}^{ws} + \sum_{j \in V} R^2(\sum_{i \in N} q_i) z_{sj}$$ (1)

The objective function seeks to minimize the operating costs of the routes in the level one and two, also the costs associated with the operation in the central warehouse. Table 1 contains the model constrains.

<table>
<thead>
<tr>
<th>Table 1. Model constrains.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sum_{i \in S} x_{sj}^{wl} \leq v$</td>
</tr>
<tr>
<td>$\sum_{j \in S} x_{ij}^{wl} = \sum_{i \in S} x_{ij}^{wl}$</td>
</tr>
<tr>
<td>$\sum_{s \in S} \sum_{i \in E} y_{ij}^{ws} \leq w$</td>
</tr>
<tr>
<td>$\sum_{i \in E} \sum_{j \in S} y_{ij}^{ws} = \sum_{i \in E} \sum_{j \in S} y_{ij}^{ws}$</td>
</tr>
<tr>
<td>$y_{ij}^{ws} = 1 \to Q_{ij}^w = Q_{ij}^w + q_i^j - q_j^i$</td>
</tr>
<tr>
<td>$0 \leq Q_{ij}^w \leq \beta_w$</td>
</tr>
<tr>
<td>$\sum_{v} Q_{ij}^w = - \sum_{j} q_i$</td>
</tr>
<tr>
<td>$x_{ij}^{wl} = 1 \to Q_{ij}^w = Q_{ij}^w - \sum_{i} q_j z_{im}$</td>
</tr>
<tr>
<td>$\sum_{w} Q_{ij}^{ws} = \sum_{j} q_j^s z_{sj}$</td>
</tr>
<tr>
<td>$y_{ij}^{ws} z_{sj} + y_{ij}^{ws} \leq z_{sj}$</td>
</tr>
<tr>
<td>$\sum_{i \in S} y_{ij} = z_{sj} \geq 1$</td>
</tr>
<tr>
<td>$\sum_{i \in S} z_{sj} = 1$</td>
</tr>
</tbody>
</table>

3. Solution strategy

The VRP problems and by extension MDVRPPD are combinatorial models that fall into the NP-complete category, these can be solved for small instances with exact methods, but the computational cost increases exponentially when the number of nodes increases (Tam and Ma, 2008). For this reason, heuristic and metaheuristic methods have become popular tools to find good quality solutions in a short time for such problems. For the analysis of studies on the state of the art in the VRP research the study performed by El-Sherbeny (2010) describing exact, heuristic and metaheuristic methods is recommended.
The method that will be developed in the next sections considers an evolutionary metaheuristic (also called evolutionary algorithm EA), which is based on the iterative transformation of the population of individuals representing the set of possible solutions to the given problem (Drezewski et al. 2006). Evolution consists in the formation of successive generations, using so-called genetic operators (or variation operators) and the selection process. Process of evolution should tend to generate better individuals and finally to find the solution of the problem (usually approximate). Although, EA often suffer from the loss of population diversity, which often results in a premature convergence, which means the location of the basin of attraction of local optimum instead of a global one. This is especially important considering transportation problems as VRP or VRPPD due to introduced constraints, which virtually eliminate many new individuals in the population (Drezewski et al., 2006). Several techniques have been proposed to allow the formation of species through modification of mechanisms for selecting individuals for the next generation (group), or the selection mechanism of the parents, including memetic algorithms (MA).

3.1. Memetic algorithms

Genetic algorithms (GA), are best known EA; in these, the individual solutions do not evolve during their lifetimes: they are created, are evaluated, may be selected as parents to new solutions, mutate and are destroyed. However, research into memetic and genetic local search algorithms has shown that performance may be improved if solutions evolve during their own life (Marinakis and Marinaki 2010). The MA belong to EA class to apply an independent local search process to refine solutions. These methods are inspired by models of adaptation in natural systems that combine the evolutionary adaptation of populations of people with individual learning within a lifetime. Furthermore, the MA are inspired by the concept of a meme from Dawkins (1976), which represents a unit of cultural evolution that can exhibit local refinement.

3.2. MA Methodology for MDVRPPD

Surekha and Sumathi (2011) proposed a three-step process to solve the MDVRP using GA and consisting of: 1 - group, 2 - routing and 3 - programming and optimization. Meanwhile Ombuki-Berman and Hanshar (2009) proposed a more complete process to solve the MDVRP through GA, this process will be adapted in this research to solve MDVRPPD with a MA instead of a GA.

3.2.1. Initial allocation of customers to central warehouses.

In this early stage, clients are allocated to central warehouses based on the Euclidean distance between them (see Figure 3). Surekha and Sumathi (2011) describe this first step as follows:

![Fig. 2. (a) Pseudocode GA, (b) Pseudocode MA](image-url)
- If distance \( (i, S_1) < \) distance \( (i, S_2) \), \( i \) is allocated to \( S_1 \).
- If distance \( (i, S_2) < \) distance \( (i, S_1) \), \( i \) is allocated to \( S_2 \).
- If distance \( (i, S_1) = \) distance \( (i, S_2) \), the allocation may be to \( S_1 \) or \( S_2 \).
- The Euclidean distance between two nodes \( i \) and \( j \) is given by:

\[
distance(i, j) = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}
\]  

(2)

3.2.2 Structuring and initiation of the population

The representation of the individual will be made from two types of chromosomes to level one and three to level two (see Figure 4). At level one the first chromosome is based on the route between the satellites and the second to vehicles. For the second level, the first chromosome will define satellites and their relationship with the nodes that will serve, the second chromosome will contemplate routes within each satellite and the third is based on the routing of vehicles.

![Fig. 3. Allocation zones of coverage for central warehouses](image)

![Fig. 4. a) Chromosome solution for level 1, b) solution chromosome for level 2](image)

For the first level, the first chromosome shows the sequence in the route between the satellites, while the second allocates the route that the vehicle does indicating the field in initiating the established sequence in the first chromosome. For example, the first route starts in field 1 and the second in the field three, over the first chromosome the established routes are: Route 1: 2 and 1 satellites; Route 2: satellite 3. For the second level is
similar. The first chromosome set for each satellite the start field of the represented customer allocation and sequenced on chromosome 2 while the third chromosome does the same but separating vehicles route. In Figure 3, the 3rd satellite would have allocated nodes 10, 11, 14, 1, 6, 9, and two routes would settle: Route 2: 10 - 11 - 14 and Route 3: 1 - 6-9.

Once defined the form of representation of the solutions (individual), the next is to generate a first solution using a greedy algorithm which shows a pseudocode in Table 2.

Table 2. Pseudocode greedy algorithm

```
Start Greedy (s ∈ S)
C: = {1, 2, 3, ..., n};
Repeat
Find k, l with g_{kl} = \min_{C} \in C
s_k; = l;
C: = C \setminus \{k\};
Until C = Ø;
Return s;
End
```

In this solution, the vehicle routes are not set, but the sequence of routes is set to the first and second level, based on the objective function and constraints of the model.

3.2.3 Scheduling route

The routes for each vehicle are scheduled in two steps: 1 - the division of routes in each satellite is performed according to the set sequences in the previous step and the availability of the vehicle. 2 - The last customer of each vi route is reassigned to become the first customer in the vi +1 route, if the change allows vi +1 is feasible and that the cost of the route vi + vi +1 in the second step is less than that amount in the first, the change is retained, but the routes are left under step 1.

3.2.4 Stock assessment

Once each route has been established as a feasible route, proceeds to their assessment according of the objective function that seeks to minimize transportation costs and the load costs and unloading on the satellite.

3.2.5 Selection

The selection method used is tournament selection, which takes a random subset of individuals in the population, the best individual in the subgroup is selected as a parent and the process is repeated to complete the population.

3.2.6 Crossing

Hanshar and Ombuki-Berman (2009) describe the following method of crossing, which will be used in the model: Given a population P = (p1, ... p\text{n}) of viable individuals for MDVRPPD where each pi = (dij, .. dim), m is the number of satellites, and d_{ij} = (r_k, ... r_v) is the part of the chromosome corresponding to satellites with r_i allocated routes. Each r_k = c_1, ... c_l, c_i are customers served on k route.

a) Randomly selected p_1 and p_2 \in P

b) From each selected individual chooses a random satellite.

c) Each selected satellite in b, a route (r) is chosen randomly
d) All customers are eliminated in c $\in r_1$ from p$_2$ and the c $\in r_2$ from p$_1$

e) For each c $\in r_1$: the cost of insertion or feasibility of c $\in r_1$ is calculated for each location in p$_2$. c$_1$ a feasible route is assigned

f) Repeat for c$_2$.

3.2.7 Mutation

The mutation occurs within the satellite routes and mutation operator based on the change (SM) proposed by Syswerda (1991) is used, randomly selects a subtour in the solution and give them a new random order (see Figure 5)

![Initial route](image)

**Fig. 5.** (a) First picture; (b) Second picture.

3.2.8 Local Search

One of the most important operators in the MA is the local search operator in which the improvement of the offspring is concentrated in GA. This operator simulates the development of individuals who may be in a generation and that are not subject to evolutionary conditions, such as crossover or mutation. Usually k-optimal heuristics or metaheuristics are used as tabu search, simulated annealing, among others. In this analysis, the 2-opt heuristic proposed by Croes (1958) will be used. The 2-opt method adapted by Liu et al. (2012) for multiple satellite models, involves taking two satellites and to make a move on each route divides into two sub-routes and connect them together (see Figure 6). This process replaces the mutation operator between satellites proposed by Berman and Ombuki-Hanshar (2009).

![2-opt heuristic](image)

**Fig. 6.** 2-opt heuristic.
4. Computational results

The algorithm has been programmed using the Java language, running on a machine with Athlon X2 Dual Core processor with 2000 MB Ram and one instance for MDVRP was used, available http://neo.lcc.uma.es/vrp/vrp-instances/multiple-depot-vrp-instances/, where 249 customers are served from 3 satellites (demand in the instance has been modified to pick up and deliveries).

When an AE with a GRASP algorithm is started, takes the risk of limiting the search space to a region of good solutions but far from optimal, but it is also valid to mention that this algorithm debugs the population allowing to explore through the evolutionary algorithm, potentially better solutions, and precisely this dynamic is reflected in figure 7, where the fast convergence characteristic in the MA is seen. The runtime was 2 minutes.

![Fig. 7. MA Solution with N=249](image)

Programmed routes are 16, as showed in table 3, which run through between 68 and 219 and remote units. This solution differs from the found solution in the source instance gave this is considered as both delivery and pick up points, also handling costs in satellites are present.

<table>
<thead>
<tr>
<th>Satellite</th>
<th>Route</th>
<th>Cost</th>
<th>Load</th>
<th>Route</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>450</td>
<td>351</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>446</td>
<td>360</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>460</td>
<td>403</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>380</td>
<td>412</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>362</td>
<td>425</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
<td>452</td>
<td>493</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
<td>400</td>
<td>459</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
<td>455</td>
<td>423</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
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<td>502</td>
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<td>0</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>462</td>
<td>423</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>11</td>
<td>398</td>
<td>444</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>12</td>
<td>421</td>
<td>407</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>13</td>
<td>405</td>
<td>415</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>14</td>
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</tr>
</tbody>
</table>
5. Conclusions

This article developed an effective solution process for the MDVRP, based on MA which includes a Greedy algorithm and another 2-opt. To use it, you need to remember that the solution provided by the method is not optimal, but the computational time is very short compared to the time that an exact method would spend for solving combinatorial problems such as those addressed in this study.

Another aspect that stands in MA, is the way that the search spaces are exhaustively explored that they are set from the AE, which allows to get quickly solutions very close to the optimal. But for some researchers, the speed can result in an early convergence to search spaces that deviates from the optimal solution, it is suggested to add in the process population management mechanisms that allow to explore populations with more diverse individuals. In future studies are expected to add this feature to the process considered here.

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