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Long distance chiral corrections in B meson amplitudes

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Abstract

We discuss the chiral corrections to f_B and B_B with particular emphasis on determining the portion of the correction that arises from long distance physics. For very small pion and kaon masses all of the usual corrections are truly long distance, while for larger masses the long distance portion decreases. These chiral corrections have been used to extrapolate lattice calculations towards the physical region of lighter masses. We show in particular that the chiral extrapolation is better behaved if only the long distance portion of the correction is used. We also display the long distance portions of the infrared enhanced chiral logarithms that appear in partially quenched chiral perturbation theory.

1. Introduction

Lattice calculations of B meson properties are presently done with parameters such that the light quark masses are larger than their physical values. In order to make predictions that are relevant for phenomenology, these calculations are extrapolated down to lower quark masses. One of the extrapolation methods uses some results from chiral perturbation theory, and this appears to produce rather large effects due to the chiral corrections. A recent summary of the field [1] noted that this chiral extrapolation is the largest uncertainty (17%) at present in the calculation of the B meson decay constant f_B.

Chiral perturbation theory is an effective field theory involving pions, kaons and η mesons. These mesons are the lightest excitations in QCD and the effective field theory is designed to describe the effects of long range propagation of these light degrees of freedom. Even in loop diagrams there are long distance effects which are described well by the effective field theory. However, chiral perturbation theory is not a good model of physics at short distances and is not valid for large meson masses. If we consider mesons of variable mass, as the masses become heavier, less and less of the loop corrections are truly long distance.

The chiral corrections are sometimes used in ways that hide the separation of long distance and short distance physics. Consider, for example, the chiral correction to the B meson decay constant in dimensional regularization [2,3,5]

$$f_B = f_0\left[1 - \left(\frac{1+3g^2}{16\pi^2 F_\pi^2}\right)\frac{3}{8}m_\pi^2 \ln\frac{m_\pi^2}{\mu^2} + \cdots\right], \tag{1}$$

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where g is the heavy meson coupling to pions. The ellipses denote the kaon and eta contributions as well as analytic terms in the masses that carry unknown coefficients which must be fit. We see that the corrections vanish for massless mesons and grow continuously with large meson masses.[1] This is the opposite of the behavior that one might expect, which would be to have larger chiral corrections when the pions are nearly massless. For very large masses of the "pions", physically we expect that the loop effects must decouple from the observables. The expression of Eq. (1) does not illustrate this decoupling. The key point is that as the mesons become heavier, most of the correction given in Eq. (1) comes from short distance physics, which is not a reliable part of the effective field theory. We will show this in more detail below. This behavior is not a problem in principle. The free coefficients in the chiral Lagrangian allow one to compensate for the unwanted behavior and correctly match the short distance physics of QCD. However the reliance on Eq. (1) at large masses can have a deleterious effect on phenomenology in some applications.

The way that present lattice extrapolations of f_B are performed apply the chiral predictions outside their region of validity. An example is given in Fig. 1, describing the results of the JLQCD Collaboration [4].

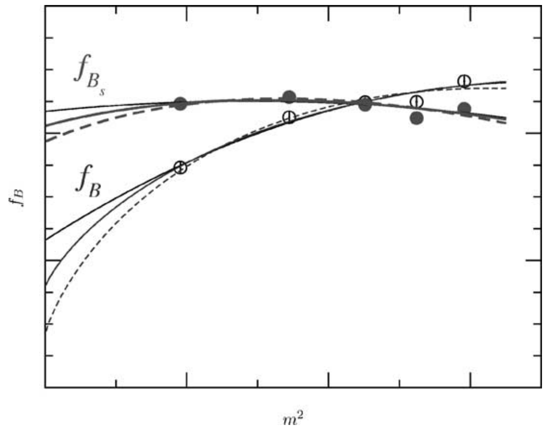

Fig. 1. Lattice data points for f_B and f_{B_s} and fitted curves with quadratic fit (upper solid curve) and with chiral logs for $g = 0.27$ and $g = 0.59$ (dashed).

In order to address the issue of the chiral exptrapolation, the lattice data was fit with the function of Eq. (1) at large mass and the form is used to extrapolate the results to small values of the mass. The fact that there appears to be a large effect at $m = 0$ does not imply that the chiral correction is large here. Indeed, inspection of Eq. (1) shows that the chiral log correction vanishes at zero mass, so the chiral logarithm is not large at the physical masses. Rather, the big effect seen comes from using Eq. (1) at large masses. Since the chiral logs grow at large mass, and appear in this formula with a fixed coefficient, normalizing the function at large mass produces a sizable difference when compared to smaller masses. Since chiral perturbation theory is not applicable at such large masses, this shift is not a valid consequence of chiral perturbation theory.

This presents a problem for lattice calculations. The need to include chiral logarithms in extracting physical results has been persuasively presented by Ryan and Kronfeld [8–10]. However, the analysis that we present below indicates that the lattice has not yet reached the region where the chiral formulas apply and that the current extrapolation is being driven by "nonsense" physics that comes from the chiral loops at short distance, which chiral perturbation theory is not able to describe. The application of Eq. (1) at large masses then amounts to a bad model of the short distance physics. We will argue for the solution where the short distance physics is removed, yet keeping the long distance physics in the region of validity of the chiral theory. At small quark masses, our method is just a different regularization of chiral perturbation theory, and reproduces the usual chiral corrections. When applied at large quark masses, our formulas must also be considered as a model. However, it is a relatively innocuous model in that it makes no assumptions about short distance physics and it produces a small correction since the loop effect decouples at large mass.

When used to extrapolate the lattice results to the physical masses, our results lead to more reasonable estimates of the chiral corrections. Our methods are similar to some work on long distance regularization in baryon chiral perturbation theory [12] and on chi-

[1] Note that we keep the B meson mass unchanged, so that when we refer to large and small meson masses, we are always referring to the masses of the chiral particles—pions, kaons and etas—that occur in the loop diagrams.

ral extrapolations in other processes [13]. In particular, the JLQCD group has explored the use of the Adelaide-MIT approach [13] in the extrapolation of the pion decay constant [4]. There is some controversy concerning these methods—see [11] for an example. We attempt to contribute to this important topic by a fuller discussion of the need for a modified analysis and of the rationale behind the solution of keeping only the long distance corrections.

2. The separation of long and short distance physics

Effective field theory is a technique for extracting the low energy predictions of a theory without explicitly involving the high energy degrees of freedom. One imagines integrating out all the high energy physics, including the quantum corrections, and keeping the full field theoretic apparatus for the low energy degrees of freedom. In the present application, one is interested in matching the low energy theory, described by chiral perturbation theory, to the high energy theory, which is QCD solved via lattice simulations. Inherent in this procedure must be a separation of the long distance and short distance scales of the theory, since the two regimes are treated by different methods. Let us call this procedure Wilsonian effective field theory because it was Wilson whose methods emphasized the integrating out of degrees of freedom beyond a given high energy scale [14].

The basic problem addressed in this Letter arises because we do not do Wilsonian effective field theory in practice. In a relativistic theory is it inconvenient to separate low energy and high energy because one must specify in which frame to define the separation scale. Instead, dimensional regularization is regularly used. The problem is that dimensional regularization has no intrinsic scale—it knows nothing about the separation scale appropriate for an effective field theory of QCD. So there is a dichotomy in this application of effective field theory. The scale of QCD is contained only in the low energy constants in the chiral Lagrangian, while the loop effects are sensitive to all scales.[2] We will demonstrate that the large chiral logarithm corrections which occur at large mass are effects that come from the short distance portion of loops. With a Wilsonian separation of scale, such effects should not be included in the low energy effective field theory.

Let us examine another calculational framework in order to get a sense of what is experimentally known about this problem. There are a few chiral calculations that can equally well be formulated as dispersion relations, and this gives a direct insight into the transition from long distance to short distance in connection with chiral logarithms. Useful in this regard are the Weinberg and DMO sum rules for the pion decay constant and for the chiral parameter L_{10} [15],

$$F_\pi^2 = \int_{4m_\pi^2}^{\infty} ds\,(\rho_V - \rho_A),$$

$$-4\overline{L}_{10} = \int_{4m_\pi^2}^{\infty} \frac{ds}{s}(\rho_V - \rho_A) \tag{2}$$

with

$$\overline{L}_{10} = L_{10}^{\text{ren}}(\mu) + \frac{1}{144\pi^2}\left(\ln\frac{m_\pi^2}{\mu^2} + 1\right). \tag{3}$$

Here $\rho_V(s) - \rho_A(s)$ is the difference of the vector and axial vector spectral functions, which are measured in e^+e^- annihilation and in tau decay. Since these sum rules are rigorous consequences of QCD, the chiral logarithms can also be found in dispersive evaluations of the sum rules. Let us see how this can occur. At lowest order in chiral symmetry, one predicts the low energy behavior of the spectral functions

$$\rho_V(s) = \frac{1}{48\pi^2}\left[1 - \frac{4m_\pi^2}{s}\right]^{3/2},$$

$$\rho_A(s) = 0. \tag{4}$$

The threshold behavior of the sum rule integration will then yield chiral logarithm behavior. Momentarily halting the upper limit of the integration at some scale

[2] We should emphasize that this is not a fundamental problem for chiral perturbation theory in isolation, as any incorrect short distance physics in loops can be corrected by adjustment of the unknown low energy constants. However it *does* cause a problem when trying to match to full solution to QCD such as the lattice which already includes a solution to the short distance physics.

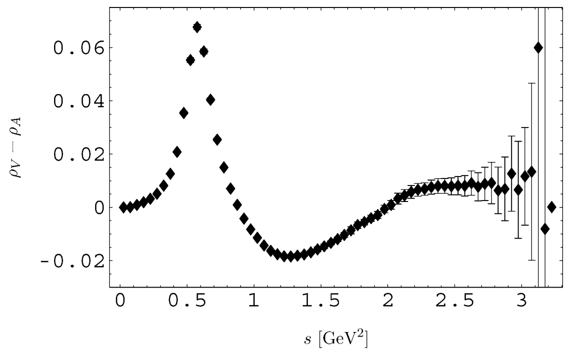

Fig. 2. The ALEPH data on the $V-A$ spectral function.

$s = \Lambda^2$, one finds

$$\int_{4m_\pi^2}^{\Lambda^2} ds \, \frac{1}{48\pi^2}\left(1 - \frac{4m_\pi^2}{s}\right)^{1/2} = \frac{m_\pi^2}{8\pi^2} \ln m_\pi^2 + \cdots,$$

$$\int_{4m_\pi^2}^{\Lambda^2} \frac{ds}{s} \frac{1}{48\pi^2}\left(1 - \frac{4m_\pi^2}{s}\right)^{1/2} = -\frac{1}{48\pi^2} \ln \frac{m_\pi^2}{\Lambda^2} + \cdots. \tag{5}$$

This reproduces the chiral logarithm in $\overline{L}_{10}$ and a portion of the chiral log corrections to F_π, with the remainder coming from tadpole diagrams. We see that the threshold behavior of the spectral function is the source of these chiral logs. However, since we know the full spectral function we can use the data to study the limits to validity of this approximation.

Now let us look at the full experimental results for the spectral functions. Using ALEPH data [16] in our normalization convention, one finds the spectral function of Fig. 2. An expanded view of the low energy end is given in Fig. 3, along with the leading chiral approximation to the spectral function. One sees that the leading chiral approximation of Eq. (4) is appropriate right at threshold, although it is modified relatively quickly.

The corrections to Eq. (4) can be accounted for at higher orders in the chiral expansion and with enough terms one would converge to agree with the low energy end of the spectral function. However, for our purposes the key feature that can be seen in the data is the transition from long distance physics, to be treated in chiral perturbation theory, to short distance physics, which in general must be solved by other means. An inspection of the spectral function

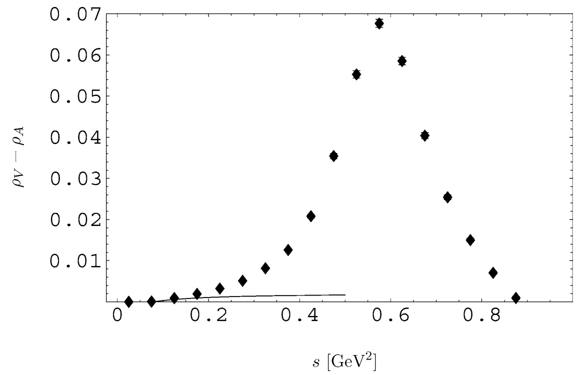

Fig. 3. The low energy end of the spectral function. The solid line is the leading chiral approximation to the spectral function, given in Eq. (4).

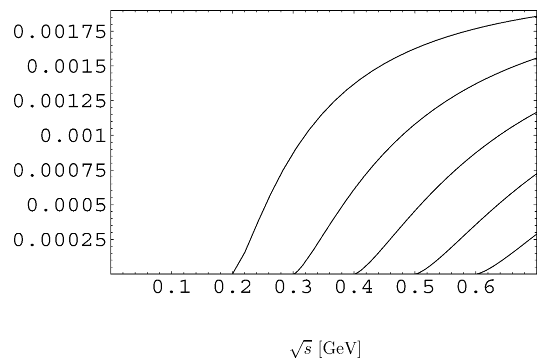

Fig. 4. The leading chiral approximation to the long distance part of the integrand for the pion decay constant sum rule, calculated with pion masses having values of $m = 100$, 150, 200, 250, 300 MeV.

reveals that this transition cannot be taken to be higher in energy than $s = (700\ \text{MeV})^2$. Beyond this point, chiral perturbation theory will be useless as a description of the spectral function and the data reveals the resonances of QCD as the appropriate short distance physics. One can then perform a calculation of L_{10} or F_π by using a chiral approximation for the low energy end of the spectral function, but then use the data for the short distance physics. This is a visible manifestation of the Wilsonian separation of scales.

Given this separation scale, let us look at what happens to the chiral logs as the meson mass gets larger. Let us define the long distance contribution to the integral of the chiral spectral function up to $\Lambda \sim 700$ MeV. The lowest order approximation to the spectral function is shown in Fig. 4 for a series of meson masses. For small values of the mass there is a well-defined threshold behavior for the low energy contribution. As the mass increases, the threshold for the dispersive integral of course also increases. Moreover, one sees that at larger masses there is only a small portion of the threshold region that contributes before one enters the region of short distance physics. The chiral approximation is not a useful one beyond a mass of 300 MeV.

Let us show this more completely by looking at the chiral approximation to the long distance contribution. For the Weinberg sum rule one has

$$F_\pi^2 = \int_{4m_\pi^2}^{\Lambda^2} ds\,\frac{1}{48\pi^2}\left(1-\frac{4m_\pi^2}{s}\right)^{1/2} + \cdots$$
$$= \frac{1}{48\pi^2\Lambda^2}\left[\left(\Lambda^2+8m_\pi^2\right)\sqrt{1-\frac{4m_\pi^2}{\Lambda^2}} + 6\ln\frac{2m_\pi^2}{\Lambda^2\left(1+\sqrt{1-\frac{4m_\pi^2}{\Lambda^2}}\right)-2m_\pi^2}\right] + \cdots$$
$$= \frac{\Lambda^2}{48\pi^2} + \frac{m_\pi^2}{48\pi^2}\left(\ln\frac{m_\pi^2}{\Lambda^2}+1\right) + \cdots. \qquad (6)$$

Here the second line is the complete long distance contribution using Eq. (4). In chiral perturbation theory at one loop, this result would be approximated by a constant, a chiral log and a slope term. This chiral approximation is given in the last line. The Λ^2 term combines up with the rest of the spectral integral to give an overall value of the pion decay constant, leaving the chiral log and the slope term to express the dependence of the result on the pion mass. How far are we allowed to trust this dependence? This question is answerable in the present framework because we have calculated the full long distance contribution. In Fig. 5 we display the full long distance contribution and the chiral approximation as a function of mass. Fig. 6 displays the ratio of the long distance integral to its chiral approximation. These have been matched to agree exactly in magnitude and slope at $m = 0$. One sees that the agreement is fine at small masses but that the chiral approximation develops a large variation in the region where there is no longer any residual true long distance effect. Both of these figures show that the chiral approximation starts out being a good approximation

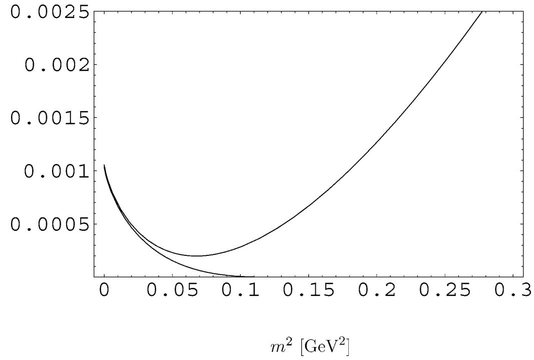

Fig. 5. The chiral approximation (upper curve) of the form constant $+\,am^2 + bm^2 \ln m^2$ to the full long distance spectral integral, Eq. (6) (lower curve).

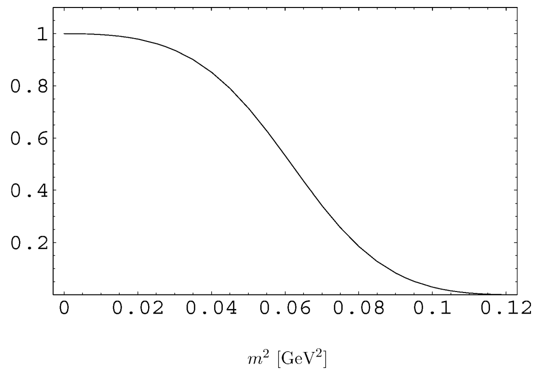

Fig. 6. The quality of the chiral approximation to the long distance integral, Eq. (6), as a function of mass. The ordinate displays the ratio of the real integral to the chiral approximation defined by keeping terms up to and including the chiral logs $m^2 \ln m^2$. The chiral approximation is seen to be excellent at small masses, including the physical pion mass, but to fail at larger masses.

for the mass dependence at low mass, but it deviates drastically beyond $m \sim 300$ MeV. The chiral approximation continues to grow and to have a rapid variation with mass at higher values of the mass. However the long distance component of the sum rule disappears. The reason for this is clear—the mesons are heavy enough that even their threshold effects falls outside of the long distance regime. An identical conclusion follows if one studies the chiral logarithm in the L_{10} sum rule.

The lessons of the previous exercise are that (1) the data exhibit a transition from the long distance description to short distance that occurs at or before a scale $\Lambda \sim 700$ MeV, and (2) the approximation consisting of a chiral logarithm and a slope term fails to describe the long distance regime for meson masses beyond $m \sim 300$ MeV. The value of the mass for which this transition occurs is smaller than many people would expect, but is readily understood in this case because the physical threshold starts at $2m$, i.e., $s = 4m^2$.

Why does one not see this behavior in the usual application of chiral perturbation theory? In practice we do not do a Wilsonian separation of scales inside loop diagrams. With dimensional regularization of loop integrals all momentum scales are probed and the dominant contribution (after renormalization) come from momentum close to the meson mass. As the meson mass grows, the resulting chiral logarithm appears to grow without bound. While this is not a problem for chiral perturbation theory in isolation, it is a problem if one tries to match on to a calculation done in lattice gauge theory. Lattice calculations will completely calculate the short distance physics. At large mass, the short distance behavior of the chiral loops is large and incorrect (i.e., in disagreement with the data or the lattice calculation). Therefore, in trying to match chiral calculations to lattice work, it is better to exclude the short distance portions of the chiral loops and keep only the long distance effects.

The dispersive analysis has been convenient for identifying an appropriate separation scale. However, not all field theory calculations have dispersive analogs so we cannot always use this technique to implement the long distance corrections. In particular, we do not know how to formulate the chiral calculations of f_B into a useful dispersive framework. However, the results above can be mimicked by use of field theory techniques with a momentum space cutoff. The method of using a cutoff to extract the long distance predictions has already been developed and applied in SU(3) baryon chiral perturbation theory [12], where it was useful for understanding the kaon loop effects. In the next section, we will explore the same issue of separation of long and short distance in a field theoretic context. This will lead us to the use of field theoretic cutoff techniques as a regularization scheme in chiral perturbation theory. Such a regularization reproduces the usual results for small values of the pion mass. However, with an appropriate choice of the cutoff, one can also use this technique to implement the desired separation of scales, keeping only the long distance portions of the loop integrals.

3. A study of the chiral corrections to f_B

The chiral corrections were initially calculated by Grinstein et al. [2] (see also [3,5–7]). The methods are standard and we will not reproduce the details. However we note that, although there are various Feynman diagrams in the calculation, in the end the loop calculations involve only one loop integral,

$$\mathbf{I}(m) = i \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 - m^2 + i\epsilon)}. \tag{7}$$

The chiral expansion involves unknown parameters for the reduced decay constant at zero mass ($\bar{f}_0$) and for the slopes (α_1, α_2) parameterizing linear dependence in the masses. The results are [2,3,5,6]

$$\begin{aligned} f_{B_{u,d}} = {} & \frac{1}{\sqrt{m_B}} \bar{f}_0 \\ & \times \Bigg[1 + \alpha_1 m_\pi^2 + \alpha_2 \big(2m_K^2 + m_\pi^2\big) \\ & \quad - \frac{1+3g^2}{4F_\phi^2} \\ & \quad \times \left(\frac{3}{2}\mathbf{I}(m_\pi) + \mathbf{I}(m_K) + \frac{1}{6}\mathbf{I}(m_\eta) \right) \Bigg] \end{aligned} \tag{8}$$

and

$$\begin{aligned} f_{B_s} = {} & \frac{1}{\sqrt{m_{B_s}}} \bar{f}_0 \\ & \times \Bigg[1 + \alpha_1 \big(2m_K^2 - m_\pi^2\big) + \alpha_2 \big(2m_K^2 + m_\pi^2\big) \\ & \quad - \frac{1+3g^2}{4F_\phi^2} \left(2\mathbf{I}(m_K) + \frac{2}{3}\mathbf{I}(m_\eta) \right) \Bigg], \end{aligned} \tag{9}$$

where g is the coupling of heavy mesons to pions[3] and F_ϕ is the pseudo-Goldstone meson decay constant in the chiral limit.[4] Of course, the integral still needs

[3] In our numerical work, we will use $g = 0.59$.

[4] We use the normalization such that $F_\pi = 0.0924$ GeV.

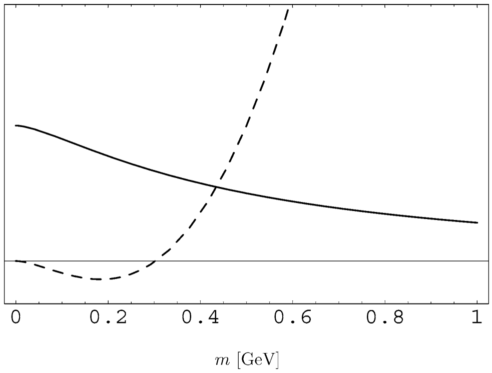

Fig. 7. Integrals $\mathbf{I}(m, \Lambda)$ with $\Lambda = 500$ MeV and $\mathbf{I}^{\text{d.r.}}(m)$ with $\mu = 500$ MeV (dashed).

to be regularized. In dimensional regularization, one absorbs the $1/(d-4)$ divergences into the slopes and finds the residual integral

$$\mathbf{I}^{\text{d.r.}}(m) = \frac{1}{16\pi^2}\left[m^2 + m^2 \ln\frac{m^2}{\mu^2}\right], \tag{10}$$

where μ is the arbitrary mass parameter that enters in dimensional regularization. The physical results do not depend on μ, as it can be absorbed into a shift in the unknown slope coefficients.

Let us explore the loop integral and study the long-distance part. In order to do this, we use a cutoff defined in the rest frame of the B meson in order to remove the short distance component. Specifically, we use a dipole cutoff yielding

$$\mathbf{I}(m, \Lambda) = i\Lambda^4 \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 - m^2 + i\epsilon)(k^2 - \Lambda^2 + i\epsilon)^2}. \tag{11}$$

In related contexts, other forms of cutoffs have been studied [12,13]—qualitatively similar results are found with other forms, although the parameter Λ will have different meanings in each case. We employ a finite value for the cutoff of order the size of the B meson. The integral may be calculated and has the form

$$\mathbf{I}(m, \Lambda) = \frac{\Lambda^4}{16\pi^2}\left[-\frac{1}{m^2 - \Lambda^2} + \frac{m^2}{(m^2 - \Lambda^2)^2} \ln\frac{m^2}{\Lambda^2}\right]. \tag{12}$$

More illuminatingly, this result is shown in Fig. 7. In this figure we compare the dimensionally regularized result to the long distance portion, defined by Eq. (12).

The long distance component is seen to have several reassuring features in the cutoff regularization. It is largest when the meson is massless, as one would expect. It is small when the mass is big and exhibits decoupling, vanishing as the mass goes to infinity. It smoothly interpolates between these limits. When comparing it to the dimensionally regularized result, one sees a shift in the intercept at zero mass—this is not surprising because the regularization corresponds to removing the value when $m = 0$. One also notices that, aside from this shift, both forms have the same logarithmic behavior near $m = 0$. The small curvature noted at the smallest mass values is the nonlinear behavior due to the chiral log factor $m^2 \ln m^2$. Without this term the result would be able to be Taylor expanded about $m = 0$, with the first term being a linear slope in m^2—the nonlinear behavior is the result of the logarithm.

We also see that the chiral log by itself grows large quickly and has a large curvature at large masses in dimensional regularization. This effect is not mirrored in the long distance component, so that it is clear that this behavior comes from the short distance portion of the integral. This is not surprising. In dimensional regularization, there is no scale within the integration aside from the particle's mass, so that the whole integral scales with $k \sim m$. These short distance effects are ones which are not reliably calculated by the effective field theory.

The above calculation has been a diagnosis of the problem. We are then faced with the question of what to do in order to better perform the chiral extrapolation. It is clear that the only perfect solution is that the lattice effort should continue until they can deal with quark masses as small as observed in nature. However, this is a long way off in the future and we are interested in the best possible estimate of B meson properties at the present time. To extrapolate with an analytic polynomial is to ignore the known existence of chiral logs. To use the formula of Eq. (1) at large mass is to use a very bad model of the short distance physics. A better solution is to use a model extrapolation that includes all of the chiral logs at long distance, but which makes no assumption about short distance physics.[5] We discuss how such a model can be exactly equivalent to chiral perturbation theory at small mass, yet decouple at large mass.

4. Long distance regularization of the chiral calculation

At small quark masses, the cutoff treatment of the integral can be promoted to a regularization of chiral perturbation theory. This has been studied in the context of baryon chiral perturbation theory in Ref. [12], where it was called long distance regularization. The use of a cutoff is clearly more painful calculationally than the usual dimensional regularization, but when the masses are small it reproduces the usual one-loop chiral expansion for matrix elements such as we are studying.

In order to regularize the calculation using the cutoff, the divergent pieces are separated in the Feynman integral. The result is

$$\mathbf{I}(m,\Lambda)=\frac{1}{16\pi^2}\left[\Lambda^2-m^2\ln\frac{\Lambda^2}{\mu^2}\right]+\mathbf{I}^{\text{ren}}(m,\Lambda), \tag{13}$$

where $\mathbf{I}^{\text{ren}}(m,\Lambda)$ is finite in the limit $\Lambda\to\infty$. This residual integral has the form

$$\mathbf{I}^{\text{ren}}(m,\Lambda)=\mathbf{I}^{\text{d.r.}}(m)+\frac{1}{16\pi^2}\left[-\frac{m^4}{m^2-\Lambda^2}-\frac{m^4(m^2-2\Lambda^2)}{(m^2-\Lambda^2)^2}\ln\frac{m^2}{\Lambda^2}\right]. \tag{14}$$

We see that there are potentially divergent contributions proportional to Λ^2 and $\ln\Lambda^2$. However, since the cutoff regularization scheme is consistent with chiral symmetry, these have exactly the right structure to be absorbed into the chiral parameters. In particular, the renormalization is

$$\bar{f}_0^{\text{ren}}=\bar{f}_0-\frac{8}{3}\bar{f}_0\frac{1+3g^2}{64\pi^2F_\phi^2}\Lambda^2,$$

$$\alpha_1^{\text{ren}}=\alpha_1+\frac{5}{6}\frac{1+3g^2}{64\pi^2F_\phi^2}\ln\frac{\Lambda^2}{\mu^2},$$

$$\alpha_2^{\text{ren}}=\alpha_2+\frac{11}{18}\frac{1+3g^2}{64\pi^2F_\phi^2}\ln\frac{\Lambda^2}{\mu^2}. \tag{15}$$

After renormalization, we can express the chiral amplitudes in terms of these parameters plus the logarithmic contribution in the residual integral $\mathbf{I}^{\text{ren}}t(m,\Lambda)$, providing the renormalized observables

$$f_{B_{u,d}}=\frac{1}{\sqrt{m_B}}\bar{f}_0^{\text{ren}}\times\Big[1+\alpha_1^{\text{ren}}m_\pi^2+\alpha_2^{\text{ren}}\big(2m_K^2+m_\pi^2\big)-\frac{1+3g^2}{4F_\phi^2}\Big(\frac{3}{2}\mathbf{I}^{\text{ren}}(m_\pi,\Lambda)+\mathbf{I}^{\text{ren}}(m_K,\Lambda)+\frac{1}{6}\mathbf{I}^{\text{ren}}(m_\eta,\Lambda)\Big)\Big] \tag{16}$$

and

$$f_{B_s}=\frac{1}{\sqrt{m_{B_s}}}\bar{f}_0^{\text{ren}}\times\Big[1+\alpha_1^{\text{ren}}\big(2m_K^2-m_\pi^2\big)+\alpha_2^{\text{ren}}\big(2m_K^2+m_\pi^2\big)-\frac{1+3g^2}{4F_\phi^2}\times\Big(2\mathbf{I}^{\text{ren}}(m_K,\Lambda)+\frac{2}{3}\mathbf{I}^{\text{ren}}(m_\eta,\Lambda)\Big)\Big]. \tag{17}$$

Since at small mass, the residual integral $\mathbf{I}^{\text{ren}}(m,\Lambda)$ tends to $\mathbf{I}^{\text{d.r.}}(m)$, the usual chiral expansion is recovered at $m^2\ll\Lambda^2$. At small mass, the cutoff is just another way to regularize the calculation.

5. Partially quenched chiral logarithms

The results of the previous section can be simply extended to the case of partially quenched chiral perturbation theory (PQChPTh) [18]. Sharpe and Zhang [3] have calculated the chiral logs in that theory and we will give the modification that occurs when using long distance regularization.

In the partially quenched theory, one differentiates between valence quarks (V) and sea quarks (S). The valence quarks live in the external hadrons and one adds a set of commuting pseudo-quarks with the same mass as the valence quarks to cancel off

[5] The "smooth matching" procedure of Ref. [5] is another attempt to apply the chiral results only in their region of validity.

the fermion determinant of the valence quarks. The sea quarks then provide the fermion determinant, and in general they may have different masses from the valence quarks. Real QCD is obtained when $m_V = m_S$. The propagators for flavor nonsinglet mesons are the same as in full QCD. However, in the flavor diagonal channel the propagators are modified by sea effects which involve the mixing with the heavy singlet meson, the "η'". In this case, one has for a flavor diagonal meson propagator (in the notation of Ref. [19])

$$G(p) = \frac{1}{p^2 + M_{VV}^2} - \frac{m_0^2 + \alpha_\Phi p^2}{(p^2 + M_{VV}^2)^2} \times \frac{1}{1 + (N_f/3)(m_0^2 + \alpha_\Phi p^2)/(p^2 + M_{SS}^2)}, \tag{18}$$

where m_0 is related to the η' mass and α_Φ to its propagator. However, in the limit that the sea meson mass M_{SS} is small compared to the η' mass, the propagator simplifies to

$$G(p) = \left(1 - \frac{1}{N_f}\right)\frac{1}{p^2 + M_{VV}^2} - \frac{M_{SS}^2 - M_{VV}^2}{(p^2 + M_{VV}^2)^2}. \tag{19}$$

The first propagator has been modified by the removal of the flavor singlet meson. The double propagator vanishes in the QCD limit—it is the source of the enhanced chiral logarithms that occur in PQChPTh.

The chiral loop correction now includes a new Feymann integral, corresponding to the double pole. When using our regularization this becomes

$$\begin{aligned} \mathbf{J}\big(m^2, \Lambda\big) &= i\Lambda^4 \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 - m^2 + i\epsilon)^2 (k^2 - \Lambda^2 + i\epsilon)^2} \\ &= \frac{\partial}{\partial m^2} I\big(m^2, \Lambda\big) \\ &= \frac{\Lambda^4}{16\pi^2}\left(\frac{2}{(m^2 - \Lambda^2)^2} - \frac{m^2 + \Lambda^2}{(m^2 - \Lambda^2)^3} \ln \frac{m^2}{\Lambda^2}\right). \end{aligned} \tag{20}$$

As $\Lambda \to \infty$ or small meson mass we recover the dimensional regularization result for this integral

$$\begin{aligned} \mathbf{J}\big(m^2, \Lambda \to \infty\big) &= \mathbf{J}\big(m^2 \to 0, \Lambda\big) \to \frac{1}{16\pi^2}\left(2 + \ln \frac{m^2}{\Lambda^2}\right) \end{aligned} \tag{21}$$

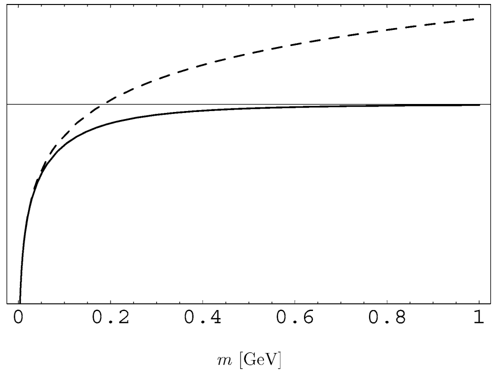

Fig. 8. Integrals $\mathbf{J}(m, \Lambda)$ with $\Lambda = 500$ MeV and $\mathbf{J}^{\text{d.r.}}(m)$ with $\mu = 500$ MeV (dashed).

while for large mass this rapidly vanishes

$$\mathbf{J}\big(m^2 \to \infty, \Lambda\big) \to -\frac{1}{16\pi^2}\frac{\Lambda^4}{m^4}\left(\ln \frac{m^2}{\Lambda^2} - 2\right). \tag{22}$$

Because of the double pole, this integral is more infrared sensitive than the usual chiral loop integral. However, correspondingly the integral is less sensitive to UV effects as the mass becomes large. In Fig. 8 we show the integral **J** using a cutoff at $\Lambda = 500$ MeV compared to the dimensionally regularized form with $\mu = \Lambda$. As expected, the two forms agree exactly at small mass, and disagree at larger masses, although the disagreement is not as large as was seen for the previous integral **I**. We also see that the dimensionally regularized form does not have the same rapid variation at large mass that was seen in the integral **I**.

Let us carry out the renormalization in the same way as in the last section. One defines a renormalized integral $\mathbf{J}^{\text{ren}}$ by subtracting a constant term which goes into the renormalization of the slope parameters. Specifically,

$$\begin{aligned} \mathbf{J}^{\text{ren}}(m, \Lambda) &= \mathbf{J}(m, \Lambda) - \frac{1}{16\pi^2}\left(2 + \ln \frac{\mu^2}{\Lambda^2}\right) \\ &\to \frac{1}{16\pi^2} \ln \frac{m^2}{\mu^2} + \cdots. \end{aligned} \tag{23}$$

One then finds that the result of Sharpe and Zhang [3] is reproduced for the limit $m^2 \ll \Lambda^2$. For larger masses, the removal of the short distance component leads to the modification using the integrals $\mathbf{I}^{\text{ren}}(m, \Lambda)$

and $\mathbf{J}^{\rm ren}(m, \Lambda)$. For N_f degenerate flavors

$$f_{B_V} = \frac{1}{\sqrt{m_B}} f_0 \times \left[1 + c_1^{PQ} m_{VV}^2 + c_2^{PQ} m_{SS} + \frac{1+3g^2}{4F_\phi^2} \times \left(\frac{N_f}{2}\mathbf{I}^{\rm ren}(m_{VS}) - \frac{1}{2N_f}\mathbf{I}^{\rm ren}(m_{VV}) + \frac{1}{2N_f}\left(m_{VV}^2 - m_{SS}^2\right) \times \mathbf{J}^{\rm ren}(m_{VV}, \Lambda)\right)\right]. \tag{24}$$

The result has some interesting features. One sees that the enhanced chiral logs persist in this regularization even when m_{SS} is large. This is because the factor $m_{SS}^2 J(m_{VV})$ blows up in the limit that $m_{VV} \to 0$ at fixed m_{SS}. The infrared sensitive double pole persists in this regularization since the propagating particle is a valence meson. It appears that the large m_{VV} and m_{VS} effects decouple, but the large m_{SS} effects do not unless m_{VV} is also large. However, this is a consequence of the approximation that m_{SS} is small compared to the η' mass. As can be seen from the propagator in Eq. (18), the sea effects obey a form of decoupling at large mass. If m_{SS} is larger than the η' mass then the sea quark masses become irrelevant and the propagator becomes that of fully quenched chiral perturbation theory. It is only in the region where the sea masses are small compared to the η' mass that the PQChPTh results are applicable.

The partially quenched results provide an additional method for exploring the properties of the chiral logarithmic corrections.

6. The chiral extrapolation of f_B

If we are going to use any meson loop calculation at larger masses in order to match to the lattice, then all treatments are model dependent. We have argued above that the use of chiral logs at these scales amounts to a bad model because it builds in very large and spurious short distance effects. Our calculation above removes the short distance effects in the one-loop diagrams. This is then a reasonable formalism to apply to the lattice calculation. The lattice calculation supplies the correct short distance physics, described there through terms analytic in m^2 (linear behavior, quadratic…). In addition, at smaller masses, our formulas naturally include the chiral logarithms in the regions where they should be valid. This motivates us to use the long distance loop calculation in the chiral extrapolation for B meson properties.

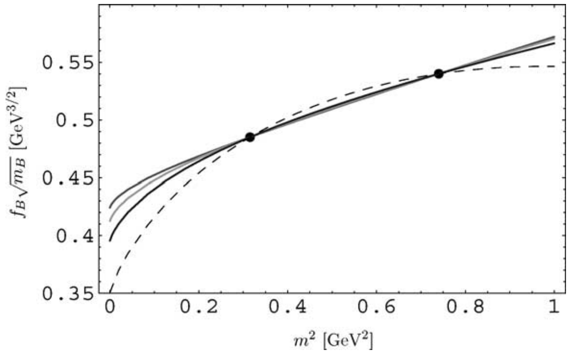

Fig. 9. $f_B\sqrt{m_B}$ as a function of m^2 fitted to the lattice data points for $\Lambda = 400$, 600, 1000 MeV and for the result from dimensional regularization (dashed).

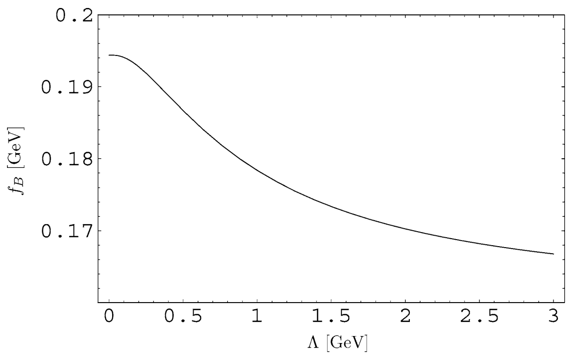

Fig. 10. f_B at the physical pion mass as a function of Λ.

Let us first fit our expression to a caricature of the lattice data by matching the data at two points. Such a linear extrapolation is appropriate for one-loop since we have only the constants and linear counterterms in the one-loop expression. This fit is demonstrated in Fig. 9, for various values of Λ. We see that the extrapolation is smoother and that there is no large curvature induced at large mass.

There remains dependence of the extrapolated value on the parameter Λ. This is shown in Fig. 10. In the range $\Lambda = 400$ MeV $\to$ 1000 MeV, this amounts to a 5% uncertainty in the extrapolated value. The formula used in previous extrapolations corresponds to $\Lambda \to \infty$. It is clear that the loop contributions that arise beyond the scale of $\Lambda = 1000$ MeV are of too short distance to be physically relevant for the effective field theory—there is no reliable chiral physics beyond this scale.

This extrapolation can be systematically improved. Most favorably would be the situation in which the lattice data can be calculated at smaller mass squared—eventually no extrapolation would be needed. Even if the improved data goes only part of the distance to the physical masses, it would remove some of the model dependence of the result. The extrapolation needed would be smaller and the residual Λ dependence would be smaller. Another way that improvement possibly may be made is with increased precision even at larger masses. As shown by Eq. (14) above, the extrapolations for different Λ values differ only at order m^4/Λ^2. If one includes an extra $\mathcal{O}(m^4)$ in the one-loop chiral calculation, fitting to a quadratic expression, then the extrapolations will be in closer agreement at this chiral order. Note however that the low mass region is still being extrapolated by a one-loop chiral formula—this procedure is not equivalent to a two-loop result in chiral perturbation theory.

As the lattice data reaches higher precision and/or smaller quark masses, it may be that the range of Λ for which a good fit is obtained may shrink. While we are treating Λ as a regularization parameter, it is meant as a rough parameterization of a physical effect—the transition from long distance to short distance in the loop calculation. Therefore when using a fit to a given order in the chiral expansion, the lattice data may only be describable with Λ within some range near the scale of this physical effect. Indeed, already the present data is a poor fit for $\Lambda \to \infty$. Of course if one allows arbitrary orders in the chiral expansion, with free parameters at each order, it is always possible to correct the loop effect for any incorrect short distance behavior by adjusting the parameters. However, when using the one-loop integral with precise data it may not be possible to obtain good fits for large values of Λ without introducing *several* new parameters at higher orders in the masses. In contrast, simpler fits with fewer parameters may be obtained with Λ within some optimal range.

Our procedure might be criticized as being a model, due to the choice of a separation function and a separation scale. However, at large masses, the dimensional regularization result is really more of a model as it introduces large and unphysical short distance physics. Our procedure is the "anti-model" because it removes most of that physics. The residual dependence on Λ comes from the ambiguity concerning how much of the short distance physics to remove. The value of Λ from the lattice results, introduced through the dipole cutoff, parameterizes the amount of short distance physics included in the loop. However, this dependence can itself be adjusted by using the coefficients of the chiral Lagrangian. Despite the decoupling of the loop at large mass, we retain all of the correct chiral behavior in the limit of small quark mass.

7. Application to B_B

All of the preceding formalism can also be applied to the chiral extrapolation of the B_B parameter for B–$\overline{B}$ mixing. We have reproduced the calculations of Refs. [2,3] using throughout the method of long distance regularization. As above, only the integral $\mathbf{I}^{\text{ren}}$ is needed in the final answer. The chiral formulas after renormalization of the parameters are

$$B_{B_d} = B_0^{\text{ren}}\Big[1 + \beta_1^{\text{ren}} m_\pi^2 + \beta_2^{\text{ren}}\big(2m_k^2 + m_\pi^2\big) - \frac{1-3g^2}{4F_\phi^2} \times \Big(\mathbf{I}^{\text{ren}}(m_\pi, \Lambda) + \frac{1}{3}\mathbf{I}^{\text{ren}}(m_\eta, \Lambda)\Big)\Big], \tag{25}$$

$$B_{B_s} = B_0^{\text{ren}}\Big[1 + \beta_1^{\text{ren}}\big(2m_K^2 + m_\pi^2\big) + \beta_2^{\text{ren}}\big(2m_k^2 + m_\pi^2\big) - \frac{1-3g^2}{3F_\phi^2}\mathbf{I}^{\text{ren}}(m_\eta, \Lambda)\Big], \tag{26}$$

in the same notation as before. Here the new chiral constants B_0, β_1, β_2 describe the intercept and slope of the chiral expansion. At small masses the usual dimensional regularization results of Refs. [2,3] are recovered in the limit of small m/Λ, as is seen using Eq. (14).

The chiral corrections for B_B are proportional to $1 - 3g^2$, while in the case of f_B the corrections contain the factor $1 + 3g^2$. This modification makes an important change in the result. For the coupling $g = 0.59$ that is favored by recent measurements [17] and supported by recent lattice calculations and theoretical predictions [17], the factor $1 - 3g^2$ almost vanishes. In this case, the one-loop chiral corrections are tiny whether one employs the standard scheme or our long distance regularization methods. (See also [20] for a discussion of this effect.) For this reason, we do not display the numerical effect of the chiral extrapolation of B_B. Use of a significantly smaller value of the coupling g would lead to measurable effect in the B_B extrapolation. Similarly, if the coupling was larger, the chiral logarithm effects could lead to an increase in the value of B_B, rather than a decrease such as we saw for f_B.

8. Conclusions

The chiral extrapolation of lattice calculations is a tricky subject because the regions of validity of chiral loops and of present lattice simulations do not overlap significantly. In Section 2 we have provided a data-based exploration of the limits of validity of the chiral formulation of loop diagrams. For meson masses that are larger than 300 MeV, the loops start to enter the short distance region and are no longer well represented by the effective field theory. Lattice simulations get most of their signal for larger masses than this. In the long run, the only satisfactory treatment requires the lattice to be applied at the physical quark masses. In the meantime one must attempt to provide the best possible treatment for the extrapolation. All such treatments are model dependent since they must be applied outside the range of validity of chiral loops.

Our method to connect them is to use just the long distance components of a one-loop calculation. This includes the chiral logarithm in the region where it is valid. It has the advantage that it removes the large and unphysical short distance effects that caused problems in previous extrapolations.

The use of long distance regularization has been applied to baryon properties by Donoghue, Holstein and Borasoy [12]. Related regularization schemes have been applied to other chiral extrapolations by the Adelaide group [13]. The regularization schemes can differ in details such as the form of the cutoff function. However, experience indicates that this form is not of great importance in the physical applications. What is important is that all such schemes exclude the short distance portions of loop diagrams. Alternatively, if the lattice calculation can be extended into the region where the chiral formulas are valid, then the smooth matching procedure of [5] also has the feature of not depending on the short distance physics in chiral loops.

There is still some model dependence that is visible in the variation of the results on Λ. This is presently inevitable because the matching between long and short distances cannot be achieved to great accuracy. This variation, and also the difference between the cutoff schemes and dimensional regularization, are perhaps disconcerting. Ultimately, physics does not depend on the regularization scheme. One might be tempted to assign an uncertainty to the calculation that is given by the spread in the scheme dependence, ranging from $\Lambda = 0$ for no chiral logarithms up to $\Lambda = \infty$ (which corresponds to using the dimensionally regularized formulas at large meson masses). However, this is too extreme. It is certain that the physics that comes from chiral loop diagrams beyond 1 GeV is incorrect. There is no reason to consider this spurious short distance physics as a measure of the uncertainty in the chiral extrapolation. Similarly there is no reason to doubt the existence of the chiral corrections below a scale of 400 MeV. Therefore at the least the range of uncertainty can be reduced to the spread in values for $\Lambda = 400 \to 1000$ MeV. The uncertainty in an extrapolation for f_B is about 5% when the cutoff is constrained to this range. For B_B the uncertainty in the chiral extrapolation is negligible for $g = 0.59$. We would recommend that our method only be applied for values in this range. As lattice simulations are applied to smaller masses, this range in the cutoff may need to be narrowed in order to agree with the lattice data. This corresponds to a more accurate matching of the long and short distance portions of the calculation.

The chiral corrections have the effect of producing a slight decrease in the extrapolated values of f_B when compared to an extrapolation which does not include chiral effects. This is the effect of the nonanalytic behavior of the chiral logarithm at long distance. Our estimates suggest that the decrease due to the chiral log puts the chirally corrected result at 0.945 ± 0.025 of the uncorrected extrapolation for f_B. We hope that our method will be applied in future extrapolations of lattice data.

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