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On the underlying gauge group structure of $D=11$ supergravity

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Abstract

The underlying gauge group structure of $D=11$ supergravity is revisited. It may be described by a one-parametric family of Lie supergroups $\tilde{\Sigma}(s) \otimes\!\!\!\!\times SO(1,10)$, $s \neq 0$. The family of superalgebras $\tilde{\mathfrak{E}}(s)$ associated to $\tilde{\Sigma}(s)$ is given by a family of extensions of the M-algebra $\{P_a, Q_\alpha, Z_{ab}, Z_{a_1\cdots a_5}\}$ by an additional fermionic central charge Q'_α. The Chevalley–Eilenberg four-cocycle $\omega_4 \sim \Pi^\alpha \wedge \Pi^\beta \wedge \Pi^a \wedge \Pi^b \Gamma_{ab\alpha\beta}$ on the standard $D=11$ supersymmetry algebra may be trivialized on $\tilde{\mathfrak{E}}(s)$, and this implies that the three-form field A_3 of $D=11$ supergravity may be expressed as a composite of the $\tilde{\Sigma}(s)$ one-form gauge fields e^a, ψ^α, B^{ab}, $B^{a_1\cdots a_5}$ and η^α. Two superalgebras of $\tilde{\mathfrak{E}}(s)$ recover the two earlier D'Auria and Fré decompositions of A_3. Another member of $\tilde{\mathfrak{E}}(s)$ allows for a simpler composite structure for A_3 that does not involve the $B^{a_1\cdots a_5}$ field. $\tilde{\Sigma}(s)$ is a deformation of $\tilde{\Sigma}(0)$, which is singularized by having an enhanced $Sp(32)$ (rather than just $SO(1,10)$) automorphism symmetry and by being an expansion of $OSp(1|32)$.

1. Introduction

M-theory (see [1]) emerged at the time of the second superstring revolution in the mid nineties. In contrast with other theories like the standard model, QCD or general relativity, M-theory is at present not based on a definite Lagrangian or on an S-matrix description; rather, it is characterized by its different perturbative and low energy limits (string models and supergravities) and by dualities [2] among them. Such dualities, including those

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relating apparently different models, are believed to be symmetries of M-theory; the full set of M-theory symmetries[1] should include these dualities as well as the symmetries of the different superstring and supergravity limits.

In this Letter we are interested in the underlying gauge symmetry of $D = 11$ supergravity as a way of understanding the symmetry structure of M-theory. The problem of the hidden or underlying geometry of $D = 11$ supergravity was raised already in the pioneering paper by Cremmer, Julia and Scherk (CJS) [16] (see also [17,18]), where the possible relevance of $OSp(1|32)$ was suggested. It was specially considered by D'Auria and Fré [19], where the search for the local supergroup of $D = 11$ supergravity was formulated as a search for a composite structure of its three-form A_3. Indeed, while the graviton and gravitino are given by one-form fields $e^a = dx^\mu e_\mu^a(x)$, $\psi^\alpha = dx^\mu \psi_\mu^\alpha(x)$ and can be considered, together with the spin connection $\omega^{ab} = dx^\mu \omega_\mu^{ab}(x)$, as gauge fields for the standard super-Poincaré group [20], the $A_{\mu_1\mu_2\mu_3}(x)$ Abelian gauge field is not associated with a symmetry generator and it rather corresponds to a three-form A_3. However, one may ask whether it is possible to introduce a set of additional one-form fields such that they, together with e^a and ψ^α, can be used to express A_3 in terms of products of one-forms. If so, the 'old' and 'new' one-form fields may be considered as gauge fields of a larger supergroup, and all the CJS supergravity fields can then be treated as gauge fields, with A_3 expressed in terms of them. This is what is meant by the underlying gauge group structure of $D = 11$ supergravity: it is hidden when the standard $D = 11$ supergravity multiplet is considered, and manifest when A_3 becomes a composite of the one-form gauge fields associated with the extended group. The solution to this problem is equivalent (see Section. 2) to trivializing a standard $D = 11$ supersymmetry algebra four-cocycle (related to dA_3) on an enlarged superalgebra.

Two superalgebras with a set of 528 bosonic and $32 + 32 = 64$ fermionic generators

$$P_a, \quad Q_\alpha, \quad Z_{a_1a_2}, \quad Z_{a_1\cdots a_5}, \quad Q'_\alpha, \tag{1}$$

including the M-algebra [21] ones plus a central fermionic generator Q'_α, were found in [19] to allow for a decomposition of A_3. Both superalgebras are clearly larger than $osp(1|32)$, but an analysis [22] of its possible relation with $osp(1|64)$ and $su(1|32)$ (by an İnönü–Wigner contraction) gave a negative answer. The two D'Auria–Fré superalgebras are particular elements (namely, $\tilde{\mathfrak{E}}(3/2)$ and $\tilde{\mathfrak{E}}(-1)$) of a one-parametric family of superalgebras $\tilde{\mathfrak{E}}(s)$ characterized by specific structure constants, the meaning of which has been unclear until present.

In fact, the first message of this Letter is that the underlying gauge supergroup structure of the $D = 11$ supergravity can be described by any representative of a *one-parametric family of supergroups* $\tilde{\Sigma}(s) \rtimes SO(1,10)$ *for* $s \neq 0$, and that these are non-trivial ($s \neq 0$) deformations of $\tilde{\Sigma}(0) \rtimes SO(1,10) \subset \tilde{\Sigma}(0) \rtimes Sp(32)$, where $\rtimes$ means semidirect product. The second point is the relation of the underlying gauge supergroups with $OSp(1|32)$. Recently, a new method for obtaining Lie algebras from a given one has been proposed in [23] and developed in [24]. The relevant feature of this procedure, the *expansion method* [24] is that, although it includes the İnönü–Wigner contraction as a particular case, it is not a dimension preserving process in general, and leads to (super)algebras of higher dimension than the (super)algebras that are expanded. We show that $\tilde{\Sigma}(0) \rtimes SO(1,10)$ may be obtained from $OSp(1|32)$ by an expansion: $\tilde{\Sigma}(0) \rtimes SO(1,10) \approx OSp(1|32)(2,3,2)$ (see Appendix A). The $SO(1,10)$ automorphism group of $\tilde{\Sigma}(s)$ is enhanced to $Sp(32)$ for $\tilde{\Sigma}(0)$. It is also seen that $\tilde{\Sigma}(0) \rtimes Sp(32)$ is the expansion $OSp(1|32)(2,3)$.

[1] Several groups may play a role, as the rank 11 Kac–Moody E_{11} group [3] or $OSp(1|64)$ [4,5] and its subgroup $GL(32)$ [6,7]. This group is the automorphism group of the M-algebra $\{Q_\alpha, Q_\beta\} = P_{\alpha\beta}$; it is also a manifest symmetry of the actions [8,9] for BPS preons [10], the hypothetical constituents of M-theory. Clearly, in $D = 11$ supergravity one might see only a fraction of the M-theory symmetries. As it was noticed recently [11,12] (see also [9]), a suggestive analysis of partially supersymmetric $D = 11$ supergravity solutions can be carried out in terms of generalized connections with holonomy group $SL(32)$. The case for a $OSp(1|32) \otimes OSp(1|32)$ gauge symmetry in a Chern–Simons context was presented in [13–15].

2. Trivialization of a Chevalley–Eilenberg four-cocycle and composite nature of the A_3 field

Supergravity is a theory of local supersymmetry. The graviton $e^a_\mu(x)$ and the gravitino $\psi^\alpha_\mu(x)$ can be considered as gauge fields associated with the standard supertranslations algebra $\mathfrak{E}$ ($\equiv \mathfrak{E}^{(D|n)}$ in general, $\mathfrak{E}^{(11|32)}$ for $D=11$),

$$\{Q_\alpha, Q_\beta\} = \Gamma^a_{\alpha\beta} P_a, \qquad [P_a, Q_\alpha] = 0, \qquad [P_a, P_b] = 0. \tag{2}$$

The supergravity one-forms e^a, ψ^α and ω^{ab} (spin connection) generate a free differential algebra (FDA)[2] defined by the expressions for the FDA curvatures

$$\mathbf{R}^a := de^a - e^b \wedge \omega_b{}^a + i\psi^\alpha \wedge \psi^\beta \Gamma^a_{\alpha\beta} = T^a + i\psi^\alpha \wedge \psi^\beta \Gamma^a_{\alpha\beta}, \tag{3}$$

$$\mathbf{R}^\alpha := d\psi^\alpha - \psi^\beta \wedge \omega_\beta{}^\alpha \quad \left(\omega_\alpha{}^\beta = \frac{1}{4}\omega^{ab}\Gamma_{ab\,\alpha}{}^\beta\right), \tag{4}$$

$$\mathbf{R}^{ab} := d\omega^{ab} - \omega^{ac} \wedge \omega_c{}^b, \tag{5}$$

where $T^a := De^a = de^a - e^b \wedge \omega_b{}^a$ is the torsion and $\mathbf{R}^{ab}$ coincides with the Riemann curvature, and by the requirement that they satisfy the Bianchi identities that constitute the selfconsistency or integrability conditions for Eqs. (3)–(5). When all curvatures are set to zero, $\mathbf{R}^a = 0$, $\mathbf{R}^\alpha = 0$, $\mathbf{R}^{ab} = 0$, Eqs. (3) and (4) reduce, if we remove the Lorentz ω^{ab} part, to the Maurer–Cartan (MC) equations for $\mathfrak{E}$,

$$de^a = -i\psi^\alpha \wedge \psi^\beta \Gamma^a_{\alpha\beta}, \qquad d\psi^\alpha = 0. \tag{6}$$

One easily solves (6) by

$$e^a = \Pi^a := dx^a - i\,d\theta^\alpha \Gamma^a_{\alpha\beta}\theta^\beta, \qquad \psi^\alpha = \Pi^\alpha := d\theta^\alpha, \tag{7}$$

where Π^a, Π^α are the MC forms for the supertranslation algebra. Considered as forms on rigid superspace ($\Sigma^{(D|n)}$ in general), one identifies x^a and θ^α with the coordinates $Z^M = (x^a, \theta^\alpha)$ of this superspace.[3] When e^a and ψ^α are forms on spacetime, x^a are still spacetime coordinates while θ^α are Grassmann functions, $\theta^\alpha = \theta^\alpha(x)$, the Volkov–Akulov Goldstone fermions [27]. For one-forms defined on curved standard superspace, $e^a = dZ^M E^a_M(Z)$, $\psi^\alpha = dZ^M E^\alpha_M(Z)$, $\omega^{ab}(Z) = dZ^M \omega^{ab}_M(Z)$ the FDA (3), (4), (5) with non-vanishing $\mathbf{R}^\alpha$ and $\mathbf{R}^{ab} = R^{ab}$ but vanishing $\mathbf{R}^a = 0$ gives a set of superspace supergravity constraints (which are kinematical or *off-shell* for $D = 4$, $N = 1$ and *on-shell*, i.e., containing equations of motion among their consequences, for higher D including $D = 11$ [28]). However, the FDA makes also sense for forms on spacetime, where $e^a = dx^\mu e^a_\mu(x)$ and $\psi^\alpha = dx^\mu \psi^\alpha_\mu(x)$ are the gauge fields for the supertranslations group.

For $D = 11$ supergravity, however, the above FDA description is incomplete since the CJS supergravity supermultiplet includes, in addition to $e^a_\mu(x)$ and $\psi^\alpha(x)$, the antisymmetric tensor field $A_{\mu\nu\rho}(x)$ associated with the three-form A_3. The FDA (3), (4), (5) has to be completed by the definition of the four-form field strength [19]

$$\mathbf{R}_4 := dA_3 + \frac{1}{4}\psi^\alpha \wedge \psi^\beta \wedge e^a \wedge e^b \Gamma_{ab\alpha\beta}. \tag{8}$$

Note that, considering the FDA (3), (4), (5), (8) on the $D = 11$ superspace and setting $\mathbf{R}^a = 0$ and $\mathbf{R}_4 = F_4 := 1/4! e^{a_4} \wedge \cdots \wedge e^{a_1} F_{a_1\cdots a_4}$ one arrives at the original on-shell $D = 11$ superspace supergravity constraints [29,30]. But, and in contrast with the $D = 4$ case, the above FDA for vanishing curvatures cannot be associated with the MC equations of a *Lie* superalgebra due to the presence of the *three*-form A_3. However, on rigid superspace $\Sigma^{(11|32)}$

[2] In essence, a FDA (introduced in this context in [19] as a *Cartan integrable system*) is an exterior algebra of forms, with constant coefficients, that is closed under the exterior derivative d; see [25,19,26].

[3] *Rigid* superspace is the group manifold of the supertranslations group $\Sigma^{(D|n)}$. We shall use the same symbol $\Sigma^{(D|n)}$, $\tilde{\Sigma}$, to denote both the supergroups and their manifolds.

(the group manifold of the $D=11$ supertranslations group), where one also sets $\mathbf{R}_4=0$ by consistency, the bosonic four-form

$$a_4 = -\frac{1}{4}\psi^\alpha \wedge \psi^\beta \wedge e^a \wedge e^b \Gamma_{ab\alpha\beta} \tag{9}$$

becomes a Chevalley–Eilenberg (CE) [31,32] Lie algebra cohomology four-cocycle on $\mathfrak{E}$,

$$\omega_4\big(x^a,\theta^\alpha\big) = -\frac{1}{4}\Pi^\alpha \wedge \Pi^\beta \wedge \Pi^a \wedge \Pi^b \Gamma_{ab\alpha\beta} = d\omega_3\big(x^a,\theta^\alpha\big) \tag{10}$$

since ω_4 is invariant and closed. The $\mathfrak{E}^{(11|32)}$ four-cocycle ω_4 is, furthermore, a non-trivial CE one, since the above three-form $\omega_3=\omega_3(x^a,\theta^\alpha)$ cannot be expressed in terms of the invariant MC forms of $\mathfrak{E}^{(11|32)}$. Now, we may ask whether there exists an *extended* Lie superalgebra, generically denoted $\tilde{\mathfrak{E}}$, with MC forms on its associated extended superspace $\tilde{\Sigma}$, on which the CE four-cocycle ω_4 becomes trivial. In this way, the problem of writing the original A_3 field in terms of one-form fields becomes purely geometrical: it is equivalent to looking, in the spirit of the fields/superspace variables democracy of [33], for an *enlarged* supergroup manifold $\tilde{\Sigma}$ on which one can find a new three-form $\tilde{\omega}_3$ (corresponding to A_3) written in terms of products of $\tilde{\mathfrak{E}}$ MC forms on $\tilde{\Sigma}$ (corresponding to one-form gauge fields) that depend on the coordinates $\tilde{Z}$ of $\tilde{\Sigma}$. That such a form $\tilde{\omega}_3(\tilde{Z})$ should exist here is also not surprising if we recall that the $(p+2)$-CE cocycles on $\mathfrak{E}$ that characterize [34] the Wess–Zumino terms of the super-p-brane actions and their associated FDA's, can also be trivialized on larger superalgebras $\tilde{\mathfrak{E}}$ [35,33] associated to extended superspaces $\tilde{\Sigma}$, and that the pull-back of $\tilde{\omega}_3(\tilde{Z})$ to the supermembrane worldvolume defines an invariant WZ term.

The MC equations of the larger Lie superalgebra $\tilde{\mathfrak{E}}^{(11|32)}$ trivializing ω_4 can be 'softened' by adding the appropriate curvatures. Considering the resulting FDA for the 'soft' forms over eleven-dimensional spacetime, one arrives at a theory of $D=11$ supergravity in which A_3 is a *composite*, not elementary, field. Its FDA curvature, R_4 in Eq. (8), is then expressed through the curvatures of the old and new one-form gauge fields.

3. A family of extended superalgebras $\tilde{\mathfrak{E}}(s)$ allowing for a trivialization of the CE four-cocycle ω_4

It was found in [19] that it was possible to write the three-form A_3 of the $D=11$ supergravity FDA (3), (4), (5), (8) in terms of one-forms, at the prize of introducing two new bosonic one-forms, $B^{a_1a_2}$, $B^{a_1\cdots a_5}$, and one new fermionic one-form η^α, obeying the FDA equations

$$\mathcal{B}_2^{a_1a_2} = DB^{a_1a_2} + \psi^\alpha \wedge \psi^\beta \Gamma_{\alpha\beta}^{a_1a_2}, \tag{11}$$

$$\mathcal{B}_2^{a_1\cdots a_5} = DB^{a_1\cdots a_5} + i\psi^\alpha \wedge \psi^\beta \Gamma_{\alpha\beta}^{a_1\cdots a_5}, \tag{12}$$

$$\mathcal{B}_2^{\alpha} = D\eta^\alpha - i\delta e^a \wedge \psi^\beta \Gamma_{a\,\beta}{}^{\alpha} - \gamma_1 B^{ab} \wedge \psi^\beta \Gamma_{ab\,\beta}{}^{\alpha} - i\gamma_2 B^{a_1\cdots a_5} \wedge \psi^\beta \Gamma_{a_1\cdots a_5\beta}{}^{\alpha}, \tag{13}$$

for two sets of specific values of the parameters, namely

$$\begin{aligned} &\delta = 5\gamma_1, \quad \gamma_2 = \frac{\gamma_1}{2\cdot 4!} \quad (\gamma_1 \neq 0) \quad \text{and} \\ &\delta = 0, \quad \gamma_2 = \frac{\gamma_1}{3\cdot 4!} \quad (\gamma_1 \neq 0). \end{aligned} \tag{14}$$

For vanishing curvatures and spin connection, $\omega^{ab}=0$, Eqs. (11)–(13) read

$$dB^{a_1a_2} = -\psi^\alpha \wedge \psi^\beta \Gamma_{\alpha\beta}^{a_1a_2}, \tag{15}$$

$$dB^{a_1\cdots a_5} = -i\psi^\alpha \wedge \psi^\beta \Gamma_{\alpha\beta}^{a_1\cdots a_5}, \tag{16}$$

$$d\eta^\alpha = \psi^\beta \wedge \big(-i\,\delta e^a \Gamma_{a\,\beta}{}^{\alpha} - \gamma_1 B^{ab}\Gamma_{ab\,\beta}{}^{\alpha} - i\gamma_2 B^{a_1\cdots a_5}\Gamma_{a_1\cdots a_5\beta}{}^{\alpha}\big). \tag{17}$$

Eqs. (6) together with Eqs. (15)–(17) provide the MC equations for the superalgebra

$$\{Q_\alpha, Q_\beta\} = \Gamma^a_{\alpha\beta} P_a + i\Gamma^{a_1a_2}_{\alpha\beta} Z_{a_1a_2} + \Gamma^{a_1\cdots a_5}_{\alpha\beta} Z_{a_1\cdots a_5}, \tag{18}$$

$$[P_a, Q_\alpha] = \delta \Gamma_{a\,\alpha}{}^\beta Q'_\beta,$$
$$[Z_{a_1a_2}, Q_\alpha] = i\gamma_1 \Gamma_{a_1a_2\,\alpha}{}^\beta Q'_\beta, \qquad [Z_{a_1\cdots a_5}, Q_\alpha] = \gamma_2 \Gamma_{a_1\cdots a_5\,\alpha}{}^\beta Q'_\beta. \tag{19}$$

Actually, Eqs. (15)–(17) and (18), (19) are not restricted to the cases of Eq. (14); it is sufficient that

$$\delta + 10\gamma_1 - 6!\gamma_2 = 0, \tag{20}$$

as required by the Jacobi identities [19].

One parameter (γ_1 if non-vanishing, δ otherwise) can be removed by rescaling the new fermionic generator Q'_α and it is thus inessential. Hence Eqs. (18)–(20) describe, effectively, a one-parameter family of Lie superalgebras that may be denoted $\tilde{\mathfrak{E}}(s)$ by using a parameter s given by[4]

$$s := \frac{\delta}{2\gamma_1} - 1, \quad \gamma_1 \neq 0 \quad \Rightarrow \quad \begin{cases} \delta = 2\gamma_1(s+1), \\ \gamma_2 = 2\gamma_1(s/6! + 1/5!). \end{cases} \tag{21}$$

In terms of s, Eq. (19) reads:

$$[P_a, Q_\alpha] = 2\gamma_1(s+1)\Gamma_{a\,\alpha}{}^\beta Q'_\beta,$$
$$[Z_{a_1a_2}, Q_\alpha] = i\gamma_1 \Gamma_{a_1a_2\,\alpha}{}^\beta Q'_\beta, \qquad [Z_{a_1\cdots a_5}, Q_\alpha] = 2\gamma_1\left(\frac{s}{6!} + \frac{1}{5!}\right)\Gamma_{a_1\cdots a_5\,\alpha}{}^\beta Q'_\beta, \tag{22}$$

and the MC equations for $\tilde{\mathfrak{E}}(s)$ are given by Eqs. (6), (15), (16) and

$$d\eta^\alpha = -2\gamma_1 \psi^\beta \wedge \left(i(s+1)\, e^a \Gamma_{a\,\beta}{}^\alpha + \frac{1}{2} B^{ab} \Gamma_{ab\,\beta}{}^\alpha + i\left(\frac{s}{6!} + \frac{1}{5!}\right) B^{a_1\cdots a_5} \Gamma_{a_1\cdots a_5\beta}{}^\alpha \right). \tag{23}$$

The $\tilde{\mathfrak{E}}(s)$ family includes the two superalgebras [19] of Eq. (14); they correspond to $\tilde{\mathfrak{E}}(3/2)$ and $\tilde{\mathfrak{E}}(-1)$. We show below, however, that the CE trivialization of ω_4 is possible for all the $\tilde{\mathfrak{E}}(s)$ algebras but for $\tilde{\mathfrak{E}}(0)$, i.e., for all but one values of the constants δ/γ_1, γ_2/γ_1 obeying Eq. (20). For these, there exists a $\tilde{\omega}_3$, $d\tilde{\omega}_3 = \omega_4$, that may be written in terms of the $\tilde{\mathfrak{E}}(s)$ MC one-forms defined on the enlarged superspace group manifold $\tilde{\Sigma}(s)$, $s \neq 0$. Such a trivialization will lead to a composite structure of the 3-form field A_3 in terms of one-form gauge fields of $\tilde{\Sigma}(s)$.

The $\tilde{\mathfrak{E}}(0)$ superalgebra constitutes a special case. It can be written as

$$\{Q_\alpha, Q_\beta\} = P_{\alpha\beta}, \qquad [P_{\alpha\beta}, Q_\gamma] = 64\gamma_1 C_{\gamma(\alpha} Q'_{\beta)}, \tag{24}$$

which follows indeed from Eqs. (22), (23) (cf. (18)) because for $s = 0$ one can use the Fierz identity

$$\delta_{(\alpha}{}^\gamma \delta_{\beta)}{}^\delta = \frac{1}{32} \Gamma^a_{\alpha\beta} \Gamma^{\gamma\delta}_a - \frac{1}{64} \Gamma^{a_1a_2}{}_{\alpha\beta} \Gamma_{a_1a_2}{}^{\gamma\delta} + \frac{1}{32\cdot 5!} \Gamma^{a_1\cdots a_5}{}_{\alpha\beta} \Gamma_{a_1\cdots a_5}{}^{\gamma\delta}. \tag{25}$$

Similarly, it is possible to collect the bosonic one-forms e^a, $B^{a_1a_2}$, $B^{a_1\cdots a_5}$ in Eqs. (6), (15), (16) and (23) with $s = 0$ in a symmetric spin-tensor one-form $\mathcal{E}^{\alpha\beta}$,

$$\mathcal{E}^{\alpha\beta} = \frac{1}{32}\left(e^a \Gamma_a^{\alpha\beta} - \frac{i}{2} B^{a_1a_2} \Gamma_{a_1a_2}{}^{\alpha\beta} + \frac{1}{5!} B^{a_1\cdots a_5} \Gamma_{a_1\cdots a_5}{}^{\alpha\beta} \right), \tag{26}$$

that allows us to write the MC equations of $\tilde{\mathfrak{E}}(0)$ in compact form as

$$d\mathcal{E}^{\alpha\beta} = -i\psi^\alpha \wedge \psi^\beta, \qquad d\psi^\alpha = 0, \qquad d\eta^\alpha = -64i\gamma_1 \psi^\beta \wedge \mathcal{E}_\beta{}^\alpha; \tag{27}$$

[4] The case $\gamma_1 \to 0$, $s \to \infty$, may be included with $\gamma_1 s \to \delta/2 \neq 0$. The corresponding algebra can be denoted $\tilde{\mathfrak{E}}(\infty)$.

Eqs. (24) or (27) exhibit the $Sp(32)$ automorphism symmetry of $\tilde{\mathfrak{E}}(0)$.

All the $\tilde{\mathfrak{E}}(s)$ superalgebras, $s \neq 0$, can be considered as deformations of $\tilde{\mathfrak{E}}(0)$. Furthermore, the $\tilde{\mathfrak{E}}(0)$ superalgebra is singled out because its full automorphism group is $Sp(32)$ while, $\forall s \neq 0$, $\tilde{\mathfrak{E}}(s)$ has the smaller $SO(1,10)$ group of automorphisms. Hence, the generalizations of the super-Poincaré group for the $s \neq 0$ and $s = 0$ cases are the semidirect products $\tilde{\Sigma}(s) \otimes SO(1,10)$ and $\tilde{\Sigma}(0) \otimes Sp(32)$, respectively. It is shown in Appendix A that, precisely for $s = 0$, both $\tilde{\Sigma}(0) \otimes SO(1,10)$ and $\tilde{\Sigma}(0) \otimes Sp(32)$ can be obtained from $OSp(1|32)$ by the expansion method [24]; they are given, respectively, by the expansions $Osp(1|32)(2,3,2)$ and $Osp(1|32)(2,3)$.

To trivialize the cocycle (10) over the $\tilde{\mathfrak{E}}(s)$ enlarged superalgebra one considers the most general ansatz[5] for the three-form A_3 expressed in terms of wedge products of e^a, ψ^α; $B^{a_1a_2}$, $B^{a_1\cdots a_5}$, η^α,

$$\begin{aligned}
4A_3 = {} & \lambda B^{ab} \wedge e_a \wedge e_b - \alpha_1 B_{ab} \wedge B^b{}_c \wedge B^{ca} - \alpha_2 B_{b_1a_1\cdots a_4} \wedge B^{b_1}{}_{b_2} \wedge B^{b_2a_1\cdots a_4} \\
& - \alpha_3 \epsilon_{a_1\cdots a_5 b_1\cdots b_5 c} B^{a_1\cdots a_5} \wedge B^{b_1\cdots b_5} \wedge e^c - \alpha_4 \epsilon_{a_1\cdots a_6 b_1\cdots b_5} B^{a_1a_2a_3}{}_{c_1c_2} \wedge B^{a_4a_5a_6c_1c_2} \wedge B^{b_1\cdots b_5} \\
& - 2i\psi^\beta \wedge \eta^\alpha \wedge \left(\beta_1 e^a \Gamma_{a\alpha\beta} - i\beta_2 B^{ab}\Gamma_{ab\,\alpha\beta} + \beta_3 B^{a_1\cdots a_5}\Gamma_{a_1\cdots a_5\,\alpha\beta}\right),
\end{aligned} \tag{28}$$

and looks for the values of the constants $\alpha_1,\ldots,\alpha_4$, $\beta_1,\ldots,\beta_3$ *and* λ such that $dA_3 = a_4$ in Eq. (9) provided e^a, ψ^α, $B^{a_1a_2}$, $B^{a_1\cdots a_5}$ and η^α are MC forms obeying (6), (15)–(17) (we do not distinguish notationally in Eq. (28) and below between the MC one-forms and the one-form gauge fields, nor between A_3 and $\tilde{\omega}_3$). If a solution exists, then Eq. (28) for the appropriate values of the constants $\alpha_1,\ldots,\beta_3$ and λ also provides an expression for a composite A_3 satisfying (8) in terms of the one-forms obeying the FDA Eqs. (3), (4), (5), (11)–(13). This is so because given a Lie algebra through its MC equations, the Jacobi identities also guarantee that the algebra obtained by adding non-zero curvatures is a gauge FDA.

The condition that (28) satisfies (9) produces a set of equations for the constants $\alpha_1,\ldots,\beta_3$ and λ including δ, γ_1 and γ_2 as parameters.[6] This system has a non-trivial solution for

$$\Delta = (2\gamma_1 - \delta)^2 = 4s^2\gamma_1^2 \neq 0. \tag{29}$$

The general solution has the form

$$\begin{aligned}
&\lambda = \frac{1}{5}\frac{s^2+2s+6}{s^2}, \qquad \beta_1 = -\frac{1}{10\gamma_1}\frac{2s-3}{s^2}, \qquad \beta_2 = \frac{1}{20\gamma_1}\frac{s+3}{s^2}, \qquad \beta_3 = \frac{3}{10\cdot 6!\gamma_1}\frac{s+6}{s^2}, \\
&\alpha_1 = -\frac{1}{15}\frac{2s+6}{s^2}, \qquad \alpha_2 = \frac{1}{6!}\frac{(s+6)^2}{s^2}, \qquad \alpha_3 = \frac{1}{5\cdot 6!5!}\frac{(s+6)^2}{s^2}, \qquad \alpha_4 = -\frac{1}{9\cdot 6!5!}\frac{(s+6)^2}{s^2},
\end{aligned} \tag{30}$$

and exists $\forall s \neq 0$, i.e., for any δ, γ_1, γ_2 obeying (20) except, as mentioned above, for $\delta = 2\gamma_1$, $\gamma_2 = 2\gamma_1/5!$ ($\Delta = 0$) which corresponds to $s = 0$ in (21). Thus, the ω_4 cocycle (10) can be trivialized ($\omega_4 = d\tilde{\omega}_3$) over all the $\tilde{\mathfrak{E}}(s)$ superalgebras when $s \neq 0$; the impossibility of doing it over $\tilde{\mathfrak{E}}(0)$ may be related with the fact that just $\tilde{\mathfrak{E}}(0)$ has an enhanced automorphism symmetry, $Sp(32)$. As a result, the three-form field[7] A_3 of the standard CJS $D = 11$ supergravity can be considered as a composite of the gauge fields of the $\tilde{\Sigma}(s)$ supergroups, $s \neq 0$. In this case, taking the exterior derivatives of (28) with the constants in (30) one also finds the expression for $\mathbf{R}_4$ in terms of the two-form FDA curvatures.

[5] This was the starting point of [19], although for $\lambda = 1$. Since more general possibilities—all including an additional fermionic generator—exist (cf. [35,33]), one can motivate Eq. (28) as follows. As the $D = 11$ super-Poincaré algebra is not sufficient to account for the gauge group structure of $D = 11$ supergravity, the next possibility would be to include the tensor charges [36,37] of the M-algebra. The ansatz would then be Eq. (28) for $\beta_1 = \beta_2 = \beta_3 = 0$ (no η^α), where only the first term may reproduce, under the action of d, the bifermionic four form a_4, Eq. (9). This would fix λ to be one. However, such an ansatz still does not allow to obtain an A_3 obeying the FDA with (8). A new fermionic one-form η^α is thus unavoidable and its inclusion provides a new contribution $\propto \omega_4$, thus allowing for $\lambda \neq 1$.

[6] This system of eight equations $\beta_1 + 10\beta_2 - 6!\beta_3 = 0$, $\lambda - 2\delta\beta_1 = 1$, $\lambda - 2\gamma_1\beta_1 - 2\delta\beta_2 = 0$, $3\alpha_1 + 8\gamma_1\beta_2 = 0$, $\alpha_2 - 10\gamma_1\beta_3 - 10\gamma_2\beta_2 = 0$, $\alpha_3 - \delta\beta_3 - \gamma_2\beta_1 = 0$, $\alpha_2 - 5!10\gamma_2\beta_3 = 0$, $\alpha_3 - 2\gamma_2\beta_3 = 0$, $3\alpha_4 + 10\gamma_2\beta_3 = 0$, is essentially that of [19] once λ is set equal to one.

[7] One may show that the (Abelian) gauge transformation properties $\delta A_3 = d\alpha_2$ can be reproduced from the gauge transformation properties of the new fields.

The two particular solutions in [19] are recovered by adjusting s (i.e., δ, γ_1 in Eq. (21)) so that $\lambda = 1$ in Eq. (30). This is achieved for $\delta = 5\gamma_1$ (δ non-vanishing but otherwise arbitrary), or for $\delta = 0$ (with γ_1 non-vanishing but otherwise arbitrary). Thus, the two D'Auria and Fré decompositions of A_3 are characterized by

$$\begin{aligned}
&\delta = 5\gamma_1 \neq 0, \quad \gamma_2 = \frac{\gamma_1}{2\cdot 4!} \quad \left(\Leftrightarrow \quad \tilde{\mathfrak{E}}\left(\frac{3}{2}\right)\right),\\
&\lambda = 1, \qquad \beta_1 = 0, \qquad \beta_2 = \frac{1}{10\gamma_1}, \qquad \beta_3 = \frac{1}{6!\gamma_1},\\
&\alpha_1 = -\frac{4}{15}, \qquad \alpha_2 = \frac{25}{6!}, \qquad \alpha_3 = \frac{1}{6!4!}, \qquad \alpha_4 = -\frac{1}{54(4!)^2},
\end{aligned} \tag{31}$$

and

$$\begin{aligned}
&\delta = 0, \quad \gamma_1 \neq 0, \quad \gamma_2 = \frac{\gamma_1}{3\cdot 4!} \quad \left(\Leftrightarrow \quad \tilde{\mathfrak{E}}(-1)\right),\\
&\lambda = 1, \qquad \beta_1 = \frac{1}{2\gamma_1}, \qquad \beta_2 = \frac{1}{10\gamma_1}, \qquad \beta_3 = \frac{1}{4\cdot 5!\gamma_1},\\
&\alpha_1 = -\frac{4}{15}, \qquad \alpha_2 = \frac{25}{6!}, \qquad \alpha_3 = \frac{1}{6!4!}, \qquad \alpha_4 = -\frac{1}{54(4!)^2}.
\end{aligned} \tag{32}$$

It is worth noting that there is a specially simple trivialization of ω_4. It is achieved for the family element $\tilde{\mathfrak{E}}(-6)$, characterized by $\gamma_2 = 0$,

$$\tilde{\mathfrak{E}}(-6): \quad \delta \neq 0, \quad \delta = -10\gamma_1, \quad \gamma_2 = 0. \tag{33}$$

In $\tilde{\mathfrak{E}}(-6)$ the generator $Z_{a_1\cdots a_5}$ is central (see Eq. (19)) and does not play any rôle in the trivialization of the ω_4 cocycle. Indeed, for these values of the parameters, Eqs. (18)–(20) allow us to consider the $\tilde{\mathfrak{E}}_{\rm min}$ superalgebra whose extension by the central charge $Z_{a_1\cdots a_5}$ gives $\tilde{\mathfrak{E}}(-6)$ in Eq. (33). It is the $(66+64)$-dimensional superalgebra $\tilde{\mathfrak{E}}_{\rm min}$,

$$\{Q_\alpha, Q_\beta\} = \Gamma^a_{\alpha\beta} P_a + i\Gamma^{a_1a_2}_{\alpha\beta} Z_{a_1a_2}, \tag{34}$$

$$[P_a, Q_\alpha] = -10\gamma_1 \Gamma_{a\alpha}{}^{\beta} Q'_\beta, \qquad [Z_{a_1a_2}, Q_\alpha] = i\gamma_1 \Gamma_{a_1a_2\alpha}{}^{\beta} Q'_\beta, \tag{35}$$

associated with the most economic $\tilde{\Sigma}_{\rm min} = \Sigma^{(66|32+32)}$ extension of the standard supertranslation group (rigid superspace) on which ω_4 becomes trivial. The values of Eq. (33) in Eq. (30) give

$$\begin{aligned}
&\lambda = \frac{1}{6}, \qquad \beta_1 = \frac{1}{4!\gamma_1}, \qquad \beta_2 = -\frac{1}{2\cdot 5!\gamma_1}, \qquad \beta_3 = 0,\\
&\alpha_1 = \frac{1}{90}, \qquad \alpha_2 = 0, \qquad \alpha_3 = 0, \qquad \alpha_4 = 0,
\end{aligned} \tag{36}$$

and one notices in Eq. (28) that all the terms containing $B^{a_1\cdots a_5}$ are zero. This makes the expression for A_3 simpler,

$$A_3 = \frac{1}{4!} B^{ab}\wedge e_a \wedge e_b - \frac{1}{3\cdot 5!} B_{ab}\wedge B^b{}_c \wedge B^{ca} - \frac{i}{4\cdot 5!\gamma_1}\psi^\beta \wedge \eta^\alpha \wedge \left(10e^a \Gamma_{a\alpha\beta} + iB^{ab}\Gamma_{ab\,\alpha\beta}\right) \tag{37}$$

and thus $\Sigma^{(66|32+32)}$ can be regarded as a minimal underlying gauge supergroup of $D = 11$ supergravity.

The other $s \neq 0$ representatives of the $\tilde{\mathfrak{E}}(s)$ family are similar, although not isomorphic. For instance, the momentum generator is central for $\tilde{\mathfrak{E}}(-1)$ while Z_{ab} is central for $\tilde{\mathfrak{E}}(\infty)$ ($\gamma_1 = 0$). They all trivialize the ω_4 CE cocycle and, hence, provide a composite expression of A_3 in terms of one-form gauge fields of the enlarged supergroup $\tilde{\Sigma}(s)$.

4. Concluding remarks

We have shown that the cocycle ω_4 (Eq. (10)) on the standard $D = 11$ supersymmetry algebra $\mathfrak{E}^{(11|32)}$ may be trivialized on the one-parametric family of superalgebras $\tilde{\mathfrak{E}}(s)$, for $s \neq 0$, defined by Eqs. (18)–(20) or (22). These superalgebras are central extensions of the M-algebra (of generators P_a, Q_α, Z_{ab}, $Z_{a_1\cdots a_5}$) by a fermionic charge Q'_α. Trivializing the supertranslation algebra cohomology four-cocycle ω_4 on the larger superalgebra $\tilde{\mathfrak{E}}(s)$, so that $\omega_4 = d\tilde{\omega}_3$, is tantamount to finding a composite structure for the three-form field A_3 of the standard Cremmer–Julia–Scherk supergravity [16] in terms of one-form gauge fields of $\tilde{\Sigma}(s)$, $A_3 = A_3(e^a, \psi^\alpha; B^{a_1a_2}, B^{a_1\cdots a_5}, \eta^\alpha)$, Eq. (28) with (30). Such an expression is given by the same equation (28) that describes the $\tilde{\omega}_3$ trivialization of the ω_4 cocycle, in which the Maurer–Cartan forms of $\tilde{\mathfrak{E}}(s)$ are replaced by one-forms obeying a free differential algebra with curvatures, Eqs. (3)–(5), (11)–(13). Thus one may treat the standard CJS $D = 11$ supergravity as a gauge theory of the $\tilde{\Sigma}(s) \otimes SO(1,10)$ supergroup for any $s \neq 0$.

This fact was known before for two superalgebras [19] that correspond to $\tilde{\Sigma}(3/2)$, Eq. (31), and $\tilde{\Sigma}(-1)$, Eq. (32) (although the whole family $\tilde{\mathfrak{E}}(s)$ that results from Eq. (20) was defined in [19]). In this respect the novelty of our results is that, for $s \neq 0$, any of the $\tilde{\Sigma}(s)$ supergroups may be equally treated as an underlying gauge supergroup of the $D = 11$ supergravity. A special representative of the family of trivializations is given by $\tilde{\mathfrak{E}}(-6)$ for which the $Z_{a_1\cdots a_5}$ generator is central. The expression for A_3 trivializing the cocycle ω_4 over $\tilde{\mathfrak{E}}(-6)$ is particularly simple: it does not involve the one-form $B^{a_1\cdots a_5}$. Thus, the smaller $\tilde{\Sigma}_{\min} = \tilde{\Sigma}^{(66|32+32)}$ may be considered as the minimal underlying gauge supergroup of $D = 11$ CJS supergravity.

All other representatives of the family $\tilde{\mathfrak{E}}(s)$ are equivalent, although they are not isomorphic. Their significance might be related to the fact that the field $B^{a_1\cdots a_5}$ is needed [9] for a coupling to BPS preons, the hypothetical basic constituents of M-theory [10]. In a more conventional perspective, one can notice that the charges Z_{ab} and $Z_{a_1\cdots a_5}$ can be treated as topological charges [37] of M2 and M5 branes. In the standard CJS supergravity the M2-brane solution carries a charge of the three-form gauge field A_3 thus it should have a relation with the charge Z_{ab}; that is reflected by Eq. (37) for a composite A_3 field and especially by its first term $B_{ab} \wedge e^a \wedge e^b$ given by the natural three-form constructed from the Z_{ab} gauge field B^{ab}. Similarly, the $Z_{a_1\cdots a_5}$ gauge field $B^{a_1\cdots a_5}$ should be related to the six-form gauge field A_6 which is dual to the A_3 field and is necessary to consider the action for the coupling of supergravity to the M5 brane [38]. One might expect that this A_6 field could also be a composite of one-forms with basic term (the counterpart of the first one in Eq. (37)) of the form $B^{a_1\cdots a_5} \wedge e_{a_1} \wedge \cdots \wedge e_{a_5}$. The rôle of the fermionic central charge Q'_α and its gauge field η^α in this perspective also requires further study. Notice that such a fermionic central charge is also present in the Green algebra [39] (see also [40,35,33]).

Although the presence of a full family of superalgebras $\tilde{\mathfrak{E}}(s)$—rather than a unique one—trivializing the standard $\mathfrak{E}^{(11|32)}$ algebra four-cocycle ω_4, suggests that the obtained underlying gauge symmetries of $D = 11$ supergravity may be incomplete (this is almost certainly the case if one considers the symmetries of M-theory), the singularity of the $\tilde{\mathfrak{E}}(0)$ case looks a reasonable one. The $\tilde{\Sigma}(0)$ supergroup is special because it possesses an enhanced automorphism symmetry $Sp(32)$ and the full $\tilde{\Sigma}(0) \otimes Sp(32)$, that replaces the $D = 11$ super-Poincaré group, is the expansion $OSp(1|32)(2,3)$ of $OSp(1|32)$ (Appendix A). The other members of the $\tilde{\Sigma}(s)$ family only have a $SO(1,10)$ automorphism symmetry and are deformations of the $s = 0$ element. Thus our conclusion is that the underlying gauge group structure of $D = 11$ supergravity is determined by a one-parametric non-trivial deformation of $\tilde{\Sigma}(0) \otimes SO(1,10) \subset \tilde{\Sigma}(0) \otimes Sp(32)$.

We would like to conclude with two remarks. The first is that we did not consider in the expression of the A_3 field (see Eq. (28)) Chern–Simons-like contributions as $B_{a_1a_2} \wedge \mathcal{B}_2^{a_1a_2}$, $B_{a_1\cdots a_5} \wedge \mathcal{B}_2^{a_1\cdots a_5}$, etc. These clearly would not affect our cocycle trivialization arguments; their presence would modify the expression of the composite $\mathbf{R}_4$ by topological densities (see [41] and, e.g., [42]). The second is that, unlike the lower dimensional versions, $D = 11$ supergravity forbids a cosmological term extension. The reason may be traced [43] to a cohomological obstruction due to the presence of the three-form field A_3. It would be interesting to analyze the implications of its composite structure for this problem. The application of the results of the present Letter, and in particular the consequences of a composite structure of A_3 for $D = 11$ supergravity and M-theory, will be considered elsewhere.

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Appendix A

A.1. $\tilde{\Sigma}(0) \otimes SO(1,10)$ *as the expansion* $OSp(1|32)(2,3,2)$

To apply the expansion method [23,24], it will be sufficient here to consider the case in which the superalgebra $\mathcal{G}$ admits the splitting $\mathcal{G} = V_0 \oplus V_1 \oplus V_2$, where V_0, V_2 (V_1), are even (odd) subspaces of dimension $\dim V_p$, $p = 0,1,2$, and V_0 is a subalgebra of $\mathcal{G}$. Then, a rescaling of the group parameters $g^{i_p} \to \lambda^p g^{i_p}$, $i_p = 1,\dots,\dim V_p$, makes the MC forms $\omega^{i_p}(\lambda)$ corresponding to the pth subspace V_p, with the natural grading $\omega^{i_p}(-\lambda) = (-1)^p \omega^{i_p}(\lambda)$, to expand as a series in λ as

$$\omega^{i_p}(\lambda) = \lambda^p \omega^{i_p,p} + \lambda^{p+2}\omega^{i_p,p+2} + \lambda^{p+4}\omega^{i_p,p+4} + \cdots \quad (p = 0,1,2). \tag{A.1}$$

The insertion of these series into the MC equations of $\mathcal{G}$,

$$d\omega^{i_p} = -\frac{1}{2} c^{i_p}_{j_q k_s} \omega^{j_q} \wedge \omega^{k_s} \quad (p,q,s = 0,1,2;\ i_{p,q,s} = 1,2,\dots,\dim V_{p,q,s}), \tag{A.2}$$

produces a set of equations identifying equal powers in λ. The equations involving only the ω^{i_p,α_p} up to certain orders $\alpha_p = N_p$, $p = 0,1,2$ ($\alpha_p = p, p+2, \dots, N_p$) will determine the MC equations of a Lie algebra provided that the highest ω^{i_p,N_p} orders retained satisfy

$$N_0 = N_1 + 1 = N_2 \quad \text{or} \quad N_0 = N_1 - 1 = N_2 \quad \text{or} \quad N_0 = N_1 - 1 = N_2 - 2. \tag{A.3}$$

The dimension of this new Lie algebra, the *expansion* $\mathcal{G}(N_0,N_1,N_2)$ of $\mathcal{G}$, is [24]

$$\dim \mathcal{G}(N_0,N_1,N_2) = \left[\frac{N_0+2}{2}\right] \dim V_0 + \left[\frac{N_1+1}{2}\right] \dim V_1 + \left[\frac{N_2}{2}\right] \dim V_2. \tag{A.4}$$

Consider now the MC equations of $\tilde{\mathfrak{E}}(0)$, Eqs. (6), (15), (16) and (23) for $s = 0$,

$$d\eta^\alpha = -2\gamma_1 \psi^\beta \wedge \left(i e^a \Gamma_{a\beta}{}^\alpha + \frac{1}{2} B^{ab} \Gamma_{ab\beta}{}^\alpha + \frac{i}{5!} B^{a_1\cdots a_5} \Gamma_{a_1\cdots a_5\beta}{}^\alpha \right), \tag{A.5}$$

to which we might add the ω^{ab} terms that implement the $SO(1,10)$ automorphisms. The superalgebra $osp(1|32)$ is defined by the MC equations

$$d\rho^{\alpha\beta} = -i\rho^{\alpha\gamma} \wedge \rho_\gamma{}^\beta - i\nu^\alpha \wedge \nu^\beta, \quad d\nu^\alpha = -i\nu^\beta \wedge \rho_\beta{}^\alpha, \quad \alpha,\beta = 1,\dots,32, \tag{A.6}$$

where $\rho^{\alpha\beta}$ are the $sp(32)$ bosonic one-forms ($\rho_\gamma{}^\beta = C_{\gamma\alpha}\rho^{\alpha\beta}$, where $C_{\alpha\beta}$ is identified with the $D = 11$ imaginary charge conjugation matrix) and ν^α are the fermionic ones. The decomposition

$$\rho^{\alpha\beta} = \frac{1}{32}\left(\rho^a \Gamma_a - \frac{i}{2}\rho^{ab}\Gamma_{ab} + \frac{1}{5!}\rho^{a_1\cdots a_5}\Gamma_{a_1\cdots a_5} \right)^{\alpha\beta}, \quad a,b = 0,1,\dots,10, \tag{A.7}$$

is adapted to the splitting [24] $osp(1|32) = V_0 \oplus V_1 \oplus V_2$, where V_0 is generated by ρ^{ab}, V_1 by ν^α and V_2 by ρ^a and $\rho^{a_1\cdots a_5}$. The series (A.1) take here the form

$$\begin{aligned}&\nu^\alpha = \lambda\nu^{\alpha,1} + \lambda^3\nu^{\alpha,3} + \cdots, \qquad \rho^{ab} = \rho^{ab,0} + \lambda^2\rho^{ab,2} + \cdots, \qquad \rho^a = \lambda^2\rho^{a,2} + \cdots,\\ &\rho^{a_1\cdots a_5} = \lambda^2\rho^{a_1\cdots a_5,2} + \cdots.\end{aligned} \tag{A.8}$$

Choosing $N_0 = 2$, $N_1 = 3$, $N_2 = 2$ (in agreement with conditions (A.3)) one obtains the MC equations of the expansion $osp(1|32)(2,3,2)$:

$$\begin{aligned}&d\rho^{ab,0} = -\frac{1}{16}\rho^{ac,0}\wedge\rho_c{}^{b,0}, \qquad d\rho^{a,2} = -\frac{1}{16}\rho^{b,2}\wedge\rho_b{}^{a,0} - i\nu^{\alpha,1}\wedge\nu^{\beta,1}\Gamma^a_{\alpha\beta},\\ &d\rho^{ab,2} = -\frac{1}{16}\left(\rho^{ac,0}\wedge\rho_c{}^{b,2} + \rho^{ac,2}\wedge\rho_c{}^{b,0}\right) - \nu^{\alpha,1}\wedge\nu^{\beta,1}\Gamma^{ab}_{\alpha\beta},\\ &d\rho^{a_1\cdots a_5,2} = \frac{5}{16}\rho^{b[a_1\cdots a_4|,2}\wedge\rho_b{}^{|a_5],0} - i\nu^{\alpha,1}\wedge\nu^{\beta,1}\Gamma^{a_1\cdots a_5}_{\alpha\beta},\\ &d\nu^{\alpha,1} = -\frac{1}{64}\nu^{\beta,1}\wedge\rho^{ab,0}\Gamma_{ab\,\beta}{}^\alpha,\\ &d\nu^{\alpha,3} = -\frac{1}{64}\nu^{\beta,3}\wedge\rho^{ab,0}\Gamma_{ab\,\beta}{}^\alpha - \frac{1}{32}\nu^{\beta,1}\wedge\left(i\rho^{a,2}\Gamma_a + \frac{1}{2}\rho^{ab,2}\Gamma_{ab} + \frac{i}{5!}\rho^{a_1\cdots a_5,2}\Gamma_{a_1\cdots a_5}\right)_\beta{}^\alpha.\end{aligned} \tag{A.9}$$

Setting $\rho^{ab,0} = -16\omega^{ab}$, Eqs. (A.9) coincide with those of $\tilde{\mathfrak{E}}(0) \supsetplus so(1,10)$ (see Eqs. (6), (15), (16) and (A.5)), with the further identifications $\rho^{a,2} = e^a$, $\rho^{ab,2} = B^{ab}$, $\rho^{a_1\cdots a_5,2} = B^{a_1\cdots a_5}$, $\nu^{\alpha,1} = \psi^\alpha$ and $\nu^{\alpha,3} = \eta^\alpha/64\gamma_1$ (notice that $\gamma_1 \neq 0$ just defines the scale of Q'_α). Thus, we conclude that $\tilde{\Sigma}(0)\otimes SO(1,10) \approx OSp(1|32)(2,3,2)$ of dimension $2\cdot 55 + 2\cdot 32 + 473 = 647$ by Eq. (A.4).

A.2. $\tilde{\Sigma}(0)\otimes Sp(32)$ *as the expansion* $OSp(1|32)(2,3)$

Let $osp(1|32) = V_0 \oplus V_1$ where V_0 (V_1) is generated by $\rho^{\alpha\beta}$ (ν^α). Choosing $N_0 = 2$ and $N_1 = 3$ we obtain the expansion $osp(1|32)(2,3)$ defined by the MC equations:

$$\begin{aligned}&d\rho^{\alpha\beta,0} = -i\rho^{\alpha\gamma,0}\wedge\rho_\gamma{}^{\beta,0}, \qquad d\rho^{\alpha\beta,2} = -i\left(\rho^{\alpha\gamma,0}\wedge\rho_\gamma{}^{\beta,2} + \rho^{\alpha\gamma,2}\wedge\rho_\gamma{}^{\beta,0}\right) - i\nu^{\alpha,1}\wedge\nu^{\beta,1},\\ &d\nu^{\alpha,1} = -i\nu^{\beta,1}\wedge\rho_\beta{}^{\alpha,0}, \qquad d\nu^{\alpha,3} = -i\nu^{\beta,3}\wedge\rho_\beta{}^{\alpha,0} - i\nu^{\beta,1}\wedge\rho_\beta{}^{\alpha,2}.\end{aligned} \tag{A.10}$$

Identifying $\rho^{\alpha\beta,0}$ in (A.10) with the $sp(32)$ connection $\Omega^{\alpha\beta}$, Eqs. (A.10) are those of $\tilde{\mathfrak{E}}(0) \supsetplus sp(32)$ (see Eqs. (27)) with $\rho^{\alpha\beta,2} = \mathcal{E}^{\alpha\beta}$, $\nu^{\alpha,1} = \psi^\alpha$ and $\nu^{\alpha,3} = \eta^\alpha/64\gamma_1$. Further, $\dim(\tilde{\mathfrak{E}}(0) \supsetplus sp(32)) = 528 + 64 + 528 = \dim osp(1|32)(2,3)$ by Eq. (A.4).

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