

A comment on the measurement of neutrino masses in β -decay experiments

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Abstract

We discuss the physics potential of future tritium β -decay experiments having a sensitivity to a neutrino mass $\sim \sqrt{|\Delta m_{23}^2|} \sim 5 \times 10^{-2}$ eV. The case of three-neutrino mixing is analyzed. A negative result of such an experiment would imply that the neutrino mass spectrum is of normal hierarchical type. The interpretation of a positive result would depend on the value of the lightest neutrino mass; if the lightest neutrino mass satisfies the inequality $\min(m_j) \ll \sqrt{|\Delta m_{23}^2|}$, it would imply that the neutrino mass spectrum is of the inverted hierarchical type.

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1. Introduction

The discovery of neutrino oscillations in the experiments with solar, atmospheric and reactor neutrinos [1–8] is the first particle physics evidence for existence of a new beyond the Standard Model physics. The solar, atmospheric, reactor and K2K neutrino data imply the presence of 3ν mixing in the weak charged lepton current:

$$\nu_{lL}(x) = \sum_{i=1}^3 U_{li} \nu_{iL}(x), \quad l = e, \mu, \tau. \quad (1)$$

Here $\nu_{lL}(x)$ is the mixed flavor neutrino field, U is the 3×3 unitary PMNS [9,10] mixing matrix and $\nu_{iL}(x)$ is the field of neutrino with mass m_i . All currently existing ν -oscillation data, except the data of the LSND experiment² [11], can be described perfectly well assuming 3ν mixing in vacuum and

we will consider this possibility in what follows. The minimal 4ν mixing scheme which could incorporate the LSND indications for ν -oscillations is strongly disfavored by the data. The ν -oscillation explanation of the LSND results is possible assuming 5ν mixing [13].

The experimental study of neutrino oscillations allowed to determine the values of the two neutrino mass-squared differences Δm_{12}^2 and $|\Delta m_{23}^2|$ ($\Delta m_{ik}^2 = m_k^2 - m_i^2$) and to obtain information on the three neutrino mixing angles θ_{12} , θ_{23} and θ_{13} , which characterize the oscillations in the case of 3 -neutrino mixing. From analysis of the Super-Kamiokande atmospheric neutrino data, the following best fit values and 95% C.L. allowed ranges of values of the parameters $|\Delta m_{23}^2|$ and $\sin^2 2\theta_{23}$ were obtained [5,14]:

$$|\Delta m_{23}^2| \cong 2.4 \times 10^{-3} \text{ eV}^2, \quad \sin^2 2\theta_{23} = 1.0, \quad (2)$$

$$1.7 \times 10^{-3} \leq |\Delta m_{23}^2| \leq 2.9 \times 10^{-3} \text{ eV}^2, \quad \sin^2 2\theta_{23} > 0.90. \quad (3)$$

The combined analysis of the solar neutrino and KamLAND data allowed to determine Δm_{12}^2 and $\sin^2 \theta_{12}$. For the best fit values and 95% allowed ranges it was found [6,7,15]

$$\Delta m_{12}^2 = 8.1 \times 10^{-5} \text{ eV}^2, \quad \sin^2 \theta_{12} = 0.31, \quad (4)$$

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² In the LSND experiment indications for oscillations $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ with $(\Delta m^2)_{\text{LSND}} \simeq 1 \text{ eV}^2$ were obtained. The LSND results are being tested in the MiniBooNE experiment [12].

$$7.3 \times 10^{-5} \leq \Delta m_{12}^2 \leq 9.0 \times 10^{-5} \text{ eV}^2, \\ 0.26 \leq \sin^2 \theta_{12} \leq 0.37. \quad (5)$$

Only an upper bound on the mixing angle θ_{13} has been obtained so far. A combined analysis of the data of the CHOOZ experiment with reactor neutrinos [16] and the data from solar neutrino and KamLAND experiments leads to the following limit [15]

$$\sin^2 \theta_{13} < 0.027 \text{ (0.044)}, \quad 95\% \text{ (99.73\%)} \text{ C.L.} \quad (6)$$

The existing atmospheric neutrino data does not allow to determine the $\text{sgn}(\Delta m_{23}^2)$. As a consequence, two types of neutrino mass spectrum in the case of three-neutrino mixing are possible (see, e.g., [17]):

- with normal mass ordering (or hierarchy) corresponding to $\Delta m_{23}^2 > 0$,

$$m_1 < m_2 < m_3, \quad \Delta m_{12}^2 \ll \Delta m_{23}^2; \quad (7)$$

- with inverted mass ordering (or hierarchy), associated with $\Delta m_{23}^2 < 0$,

$$m_3 < m_1 < m_2, \quad \Delta m_{12}^2 \ll |\Delta m_{23}^2| \cong |\Delta m_{13}^2|. \quad (8)$$

The absolute values of neutrino masses m_j are unknown at present. In particular, for each of the two types of the neutrino mass ordering there can be strong hierarchy between the masses, or the splitting between the masses can be much smaller than the absolute values of the masses. Correspondingly, depending on the sign of Δm_{23}^2 and the value of the lightest neutrino mass, $\min(m_j)$, the ν -mass spectrum can be (see, e.g., [17,18]):

- *Normal hierarchical* (NH)

$$m_1 \ll m_2 \ll m_3: \\ m_2 \cong \sqrt{\Delta m_{12}^2} \sim 9 \times 10^{-3} \text{ eV}, \\ m_3 \cong \sqrt{\Delta m_{23}^2} \sim 4.9 \times 10^{-2} \text{ eV}. \quad (9)$$

- *Inverted hierarchical* (IH)

$$m_3 \ll m_1 < m_2: \\ m_{1,2} \cong \sqrt{|\Delta m_{23}^2|} \sim 4.9 \times 10^{-2} \text{ eV}. \quad (10)$$

- *Quasi-degenerate* (QD)

$$m_1 \cong m_2 \cong m_3 \cong m_0, \quad m_j^2 \gg |\Delta m_{23}^2|: \\ m_0 \gtrsim 0.10 \text{ eV}. \quad (11)$$

The measurement of the absolute values of neutrino masses and the determination of the type of neutrino mass spectrum is one of the highest priority and most difficult problems in neutrino physics (see, e.g., [17]). The solution of this problem is of fundamental importance for the progress in understanding the origin of neutrino masses and mixing.

Information about the absolute values of neutrino masses can be obtained from:

(a) precision measurements of the β -spectrum in the end-point region [19,20];

(b) investigation of neutrinoless double β -decay, if the neutrinos with definite mass are Majorana particles [21,22];

(c) measurement of power spectrum of the large scale distribution of galaxies (see, e.g., [23]).

From the data of the Mainz [24] and Troitsk [25] tritium β -decay experiments the following upper bound on the measurable neutrino mass was obtained (95% C.L.):

$$m_\beta \cong m_0 < 2.3 \text{ eV}. \quad (12)$$

In the future KATRIN experiment [26] a sensitivity to

$$m_\beta \cong m_0 \cong 0.2 \text{ eV}, \quad (13)$$

is planned to be achieved.³

Using the Cosmic Microwave Background (CMB) data of the WMAP experiment, combined with data from large scale structure surveys (2dFGRS, SDSS), for the sum of neutrino masses upper bounds in the range

$$\sum_i m_i \leq (0.4\text{--}1.7) \text{ eV}, \quad (14)$$

were found (see, e.g., [28]). Data on weak lensing of galaxies by large scale structure, combined with data from the WMAP and PLANCK experiments, may allow $\sum m_i$ to be determined with an uncertainty of $\delta \sim 0.04 \text{ eV}$ [29].

The sign of Δm_{23}^2 , which drives the dominant atmospheric neutrino oscillations, can be determined by studying oscillations of neutrinos and antineutrinos, say, $\nu_\mu \rightarrow \nu_e$ and $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$, in which matter effects are sufficiently large. This can be done, e.g., in long-baseline ν -oscillation experiments (see, e.g., [30]). Information about $\text{sgn}(\Delta m_{23}^2)$ can be obtained also in atmospheric neutrino experiments by studying the oscillations of the atmospheric ν_μ and $\bar{\nu}_\mu$ which traverse the Earth [31].

In the present Letter we investigate the physics potential of β -decay experiments having a sensitivity which permits to probe the neutrino mass range corresponding to $\sqrt{|\Delta m_{23}^2|} \cong 5 \times 10^{-2} \text{ eV}^2$. At present such a sensitivity does not seem reachable in any realistic experiment. However, this situation may change in the future. In our analysis we take into account the existing and prospective neutrino oscillation data on the neutrino mass squared differences and neutrino mixing angles. We consider different possible types of neutrino mass spectrum as well.⁴

2. On the measurement of neutrino mass in β -decay experiments

The measurement of β -spectrum in the end-point region in tritium β -decay,



³ For an alternative method of direct neutrino mass measurement, based on a calorimetric study of β -decay of ${}^{187}\text{Re}$, see Ref. [27].

⁴ For recent related analyses see, e.g., Ref. [32].

is the classical method of direct determination of neutrino mass, which originated from the pioneering articles of Fermi and Perin on β -decay [19,20]. This decay has many advantages (see, e.g., [24]). First of all, it is a super-allowed decay. Thus, the nuclear matrix element is a constant and the electron spectrum is determined by the relevant phase space factor only. Other advantages of this decay are the relatively small energy release ($E_0 \simeq 18.574.3 \pm 1.7$ eV) and convenient half-life (12.3 years).

Taking into account the neutrino mixing, for the effective Hamiltonian of the process (15) we have

$$\mathcal{H}_I^{\text{CC}} = \frac{G_F}{\sqrt{2}} 2 \sum_i U_{ei} \bar{e}_L \gamma_\alpha \nu_{iL} J^\alpha + \text{h.c.}, \quad (16)$$

where J^α is the hadronic charged current. For the state vector of the final neutrinos and electron we obtain from (16)

$$|f\rangle = \sum_i |\bar{\nu}_i\rangle |e^-\rangle \langle \bar{\nu}_i e^-^3\text{He} | S | ^3\text{H}\rangle, \quad (17)$$

where

$$\begin{aligned} & \langle \bar{\nu}_i e^-^3\text{He} | S | ^3\text{H}\rangle \\ &= -i2 \frac{G_F}{\sqrt{2}} N U_{ei} \bar{u}_L(p) \gamma_\alpha \nu_L(p_i) \langle ^3\text{He} | J^\alpha(0) | ^3\text{H}\rangle \\ & \quad \times (2\pi)^4 \delta(P' - P). \end{aligned} \quad (18)$$

Here N is the product of standard normalization factors, p is the momentum of electron, p_i is the momentum of antineutrino (right-handed neutrino in the Majorana case) with mass m_i , P and P' are the total initial and final momenta.

The final state neutrinos are not detected in tritium β -decay experiments. Taking into account the orthogonality of the vectors $|\bar{\nu}_i\rangle$ and neglecting the recoil of the ^3He nucleus, for the electron spectrum we get the incoherent sum

$$\frac{d\Gamma}{dE_e} = \sum_i |U_{ei}|^2 \frac{d\Gamma(m_i)}{dE_e}, \quad (19)$$

where

$$\begin{aligned} \frac{d\Gamma(m_i)}{dE_e} &= C p_e (E_e + m_e) (E_0 - E_e) \sqrt{(E_0 - E_e)^2 - m_i^2} \\ & \quad \times F(E_e) \theta(E_0 - E_e - m_i). \end{aligned} \quad (20)$$

Here $E_e \leq E_0 - m_i$ is the kinetic energy of the electron, E_0 is the energy released in the decay (15), p_e is the electron momentum, m_e is the mass of the electron, $F(E_e)$ is the Fermi function which takes into account the Coulomb interaction of the final state particles, and C is a constant. In Eq. (20) $(E_0 - E_e)$ is the neutrino energy and $p_i = \sqrt{(E_0 - E_e)^2 - m_i^2}$ is the momentum of neutrino with mass m_i .

Neutrino masses enter into the expression for the electron spectrum through neutrino momenta. It is obvious that the maximal distortion of the electron spectrum can be observed in the region $(E_0 - E_e) \sim m_i$, which is less than of the order of few eV. However, for reasons of luminosity, in tritium experiments relatively large end-point intervals of the electron spectrum are measured: in the Mainz experiment [24] $E_0 - E_e \leq 70$ eV;

in the future KATRIN experiment [26] the region $E_0 - E_e \lesssim 20$ eV will be explored.

Usually the quantity m_β defined by

$$m_\beta = \sqrt{\sum_i |U_{ei}|^2 m_i^2}, \quad (21)$$

is considered as the neutrino mass related observable in β -decay experiments. The expression (21) is obtained (see Refs. [33, 34]) by developing the neutrino momentum over $\frac{m_i^2}{(E_0 - E_e)^2}$ in Eq. (20). Let us note that in the region sensitive to the neutrino mass this expansion is not valid, while in the region $(E_0 - E_e) \gg m_i$ the effects of neutrino mass can be neglected.

For the neutrino mass spectrum with normal ordering, the neutrino masses are given (in the standardly used convention) by

$$\begin{aligned} \min(m_j) &= m_1, & m_2 &= \sqrt{m_1^2 + \Delta m_{12}^2}, \\ m_3 &= \sqrt{m_1^2 + \Delta m_{12}^2 + \Delta m_{23}^2}. \end{aligned} \quad (22)$$

In the case of spectrum with inverted ordering of neutrino masses we have

$$\begin{aligned} \min(m_j) &= m_3, & m_1 &= \sqrt{m_3^2 + |\Delta m_{13}^2|}, \\ m_2 &= \sqrt{m_3^2 + |\Delta m_{13}^2| + \Delta m_{12}^2}. \end{aligned} \quad (23)$$

The neutrino mass-squared differences Δm_{12}^2 and $|\Delta m_{23}^2|$ have been measured in the neutrino oscillation experiments. The existing data allow a determination of Δm_{12}^2 and $|\Delta m_{23}^2|$ at 3σ with an error of approximately 12% and 50%, respectively. These parameters will be measured with higher accuracy in the future. The highest precision in the determination of $|\Delta m_{23}^2|$ is expected to be achieved from the studies of ν_μ -oscillations in the T2K (SK) [35] experiment: the 3σ uncertainty in $|\Delta m_{23}^2|$ is estimated to be reduced in this experiment to $\sim 6\%$.

The unknown parameter in Eqs. (22) and (23) is the lightest neutrino mass m_1 (m_3). It follows from (22) and (23) that the minimal value of the heaviest neutrino mass in the cases of normal and inverted mass ordering, m_3 and m_2 , are given by

$$m_{3(2)}^{\min} = \sqrt{\Delta m_{12}^2 + |\Delta m_{23}^2|} \simeq \sqrt{|\Delta m_{23}^2|}. \quad (24)$$

As we have discussed earlier, depending on the value of the lightest neutrino mass, three types of neutrino mass spectrum are usually considered: NH (normal hierarchical), IH (inverted hierarchical) and QD (quasi-degenerate), Eqs. (9)–(11). The QD spectrum is realized if the value of the lightest neutrino mass is relatively large, $\min(m_j) \gtrsim 0.1$ eV. This spectrum requires an approximate symmetry of the neutrino mass matrix (see, e.g., [18,36]). The NH and IH spectra correspond to negligibly small value of $\min(m_j)$. The NH spectrum is typically predicted by the GUT models which unify quarks, charged leptons and neutrinos (see, e.g., [18,37]). The IH spectrum can be associated with the existence of a broken $L_e - L_\mu - L_\tau$ symmetry in the lepton sector [38] (see also, e.g., [39]).

The Mainz, Troitsk and KATRIN experiments can probe only the quasi-degenerate neutrino mass spectrum. If in the KATRIN experiment, which is under preparation at present, a positive effect due to the neutrino mass will be observed, we will have

$$m_\beta \cong m_{1,2,3} \cong m_0. \quad (25)$$

In this case no information on the $\text{sgn}(\Delta m_{23}^2)$, i.e., on the type of ordering of neutrino masses, will be obtained.

A negative result of the KATRIN experiment would imply that the neutrino mass spectrum is either NH or IH, or else is with partial normal or inverted hierarchy [21]. It will be crucial in this case to improve the sensitivity of direct neutrino mass measurement experiments by approximately a factor of 4. If the sensitivity of the β -decay experiments will allow to probe values of neutrino masses $m_3^{\min}(m_2^{\min}) \cong \sqrt{|\Delta m_{23}^2|} \cong (3.9\text{--}5.8) \times 10^{-2}$ eV, these experiments will provide fundamental information on the absolute scale of neutrino masses and on the type of neutrino mass spectrum independently of the nature of massive neutrinos, which, as is well-known, can be Dirac or Majorana particles (see, e.g., [40]).

Indeed, if the neutrino mass spectrum is of the NH type, the contribution of the heaviest neutrino mass $m_3 \cong \sqrt{|\Delta m_{23}^2|}$ to the distortion of the electron spectrum is suppressed by the factor $|U_{e3}|^2 = \sin^2 \theta_{13} < 5 \times 10^{-2}$ and will be unobservable. The distortion of the spectrum due to the mass $m_2 \cong \sqrt{\Delta m_{12}^2} \cong 9 \times 10^{-3}$ eV, which is not suppressed by the corresponding mixing matrix element, will also be unobservable. Thus, the electron spectrum that will be observed in the β -decay experiments in the case of NH neutrino mass spectrum will effectively correspond to one zero mass neutrino:

$$\frac{d\Gamma}{dE_e} \cong \frac{d\Gamma(m_i = 0)}{dE_e}. \quad (26)$$

In contrast, if the neutrino mass spectrum is of the IH type, the two heaviest neutrino masses $m_1 \cong m_2 \cong \sqrt{|\Delta m_{23}^2|}$ will enter into the expression for the electron spectrum with the coefficient $1 - |U_{e3}|^2 \simeq 1$. The spectrum will have the form

$$\begin{aligned} \frac{d\Gamma}{dE_e} &\cong (1 - |U_{e3}|^2) \frac{d\Gamma(m_{1,2})}{dE_e} + |U_{e3}|^2 \frac{d\Gamma(m_3 = 0)}{dE_e} \\ &\cong \frac{d\Gamma(\sqrt{|\Delta m_{23}^2|})}{dE_e}. \end{aligned} \quad (27)$$

It follows from the above discussion that the nonobservation of the effect of neutrino mass in a β -decay experiment having a sensitivity to $\sqrt{|\Delta m_{23}^2|}$ would imply that the neutrino mass spectrum is of the *normal hierarchical* type, i.e., that $\Delta m_{23}^2 > 0$ and $m_1 \ll m_2 \ll m_3$, independently of whether the massive neutrinos are Dirac or Majorana particles. If the spectrum of neutrino masses is of the *inverted hierarchical* type, the effect of neutrino mass must be observed in such an experiment.

The interpretation of a positive result of a β -decay experiment with a sensitivity to a neutrino mass $\sim \sqrt{|\Delta m_{23}^2|}$, how-

ever, will not be unique in what regards the $\text{sgn}(\Delta m_{23}^2)$ and the value of the lightest neutrino mass (the type of neutrino mass spectrum). Indeed, in the discussion above of the cases of NH and IH spectra it was always assumed that the lightest neutrino mass is negligible, i.e., $m_1 \ll \sqrt{\Delta m_{12}^2}$ in the NH case and $m_3 \ll \sqrt{|\Delta m_{23}^2|}$ in the IH one. However, this may not necessarily be valid. In principle, for both normal and inverted neutrino mass ordering, we can have $\min(m_j) \lesssim \sqrt{|\Delta m_{23}^2|}$, which corresponds to a spectrum with partial hierarchy [21]. Thus, the distortion of the electron spectrum in the case of positive result of a β -decay experiment under discussion could be due either to

(i) spectrum with *inverted neutrino mass ordering*, $\Delta m_{23}^2 < 0$, of two possible types:

(a) *inverted hierarchical*, $m_3 \ll m_1 < m_2$, or

(b) *with partial inverted hierarchy*, $m_3 < m_1 < m_2$ [21];

or to

(ii) spectrum with *normal neutrino mass ordering*, $\Delta m_{23}^2 > 0$, but with *partial neutrino mass hierarchy*, $m_1 < m_2 < m_3$ [21].

As an example of the possibility (ii) consider the following hypothetical spectrum: $m_1 = 5.0 \times 10^{-2}$ eV, $m_2 = \sqrt{m_1^2 + \Delta m_{12}^2} \cong 5.1 \times 10^{-2}$ eV, $m_3 = \sqrt{m_1^2 + \Delta m_{13}^2} \cong 6.9 \times 10^{-2}$ eV, where we have used the best fit values of Δm_{12}^2 and Δm_{13}^2 given in Eqs. (3) and (5). The sum of neutrino masses is equal to $\sum m_i \cong 0.17$ eV. Obviously, a β -decay experiment having the precision under discussion will not be sensitive to the difference between the masses m_1 and m_2 ; and it will not be sensitive to the distortion of the electron spectrum due to the mass m_3 since the contribution of the latter is suppressed by the factor $\sin^2 \theta_{13}$. In this case we will have for the electron spectrum

$$\begin{aligned} \frac{d\Gamma}{dE_e} &\cong (1 - |U_{e3}|^2) \frac{d\Gamma(m_{1,2})}{dE_e} + |U_{e3}|^2 \frac{d\Gamma(m_3)}{dE_e} \\ &\cong \frac{d\Gamma(m_{1,2})}{dE_e}, \end{aligned} \quad (28)$$

which practically coincides with the form of the electron spectrum predicted in the case of neutrino mass spectrum of *inverted hierarchical* type, Eq. (27).

Let us note that neutrino mass spectrum with partial hierarchy can possibly be probed by future cosmological/astrophysical observations (see, e.g., [29]).

3. Conclusion

The knowledge of neutrino mass spectrum is decisive for the understanding of the origin of neutrino masses and mixing. From the data of neutrino oscillation experiments the two independent neutrino mass-squared differences in the case of 3-neutrino mixing, which are responsible for the solar and atmospheric neutrino oscillations, were determined. However, the

existing data does not allow to determine the sign of the neutrino mass squared difference driving the atmospheric neutrino oscillations, $\text{sgn}(\Delta m_{23}^2)$. Thus, we can have $\Delta m_{23}^2 > 0$, corresponding (in the standardly used convention) to *neutrino mass spectrum with normal ordering of neutrino masses*, $m_1 < m_2 < m_3$; or $\Delta m_{23}^2 < 0$, which is associated with *neutrino mass spectrum with inverted ordering of neutrino masses*, $m_3 < m_1 < m_2$. In both cases the minimal value of the heaviest neutrino mass is given by $m_{3(2)}^{\min} \simeq \sqrt{|\Delta m_{23}^2|} \sim 5 \times 10^{-2}$ eV. We do not know at present the mass of the lightest neutrino $\min(m_j)$ as well. The existing data from β -decay experiments and the cosmological data allow us to obtain only an upper bound on $\min(m_j)$.

Depending on the value of the mass of the lightest neutrino, there are three possible characteristic types of neutrino mass spectrum: *quasi-degenerate*, in which the lightest neutrino mass is relatively large, $m_{1(3)}^2 \gg |\Delta m_{23}^2|$; and two spectra in which the lightest neutrino mass is negligibly small: *normal hierarchical* with $\Delta m_{23}^2 > 0$ and $m_1 \ll \sqrt{|\Delta m_{12}^2|}$, and *inverted hierarchical*, with $\Delta m_{23}^2 < 0$ and $m_3 \ll \sqrt{|\Delta m_{23}^2|}$. The future KATRIN β -decay experiment will probe the quasi-degenerate neutrino mass spectrum. If effect of neutrino mass will be observed in this experiment, the lightest neutrino mass will be measured. However, the $\text{sgn}(\Delta m_{23}^2)$, and thus the character of neutrino mass spectrum—with normal or inverted ordering of neutrino masses, will not be determined in this case.

If in the KATRIN experiment the effect of nonzero neutrino mass will not be observed, it will be crucial to improve the sensitivity of the β -decay experiments by approximately a factor of four, which would permit to probe values $m_{3(2)}^{\min} \cong \sqrt{|\Delta m_{23}^2|}$. As we have shown, the nonobservation of the effect of neutrino mass in a β -decay experiment having a sensitivity to $\sqrt{|\Delta m_{23}^2|}$ would imply that the neutrino mass spectrum is of the *normal hierarchical* type, i.e., that $\Delta m_{23}^2 > 0$ and $m_1 \ll m_2 \ll m_3$, independently of whether the massive neutrinos are Dirac or Majorana particles. If the spectrum of neutrino masses is of the *inverted hierarchical* type, the effect of neutrino mass must be observed in such an experiment.

It follows from our analysis, however, that the interpretation of a positive result of a β -decay experiment with a sensitivity to a neutrino mass $\sim \sqrt{|\Delta m_{23}^2|}$ will not be unique in what regards the $\text{sgn}(\Delta m_{23}^2)$ and the type of neutrino mass spectrum. The distortion of the electron spectrum observed in such an experiment could be due either to (i) spectrum with *inverted neutrino mass ordering* ($\Delta m_{23}^2 < 0$) of two possible types, (a) *inverted hierarchical*, $m_3 \ll m_1 < m_2$, or (b) *with partial inverted hierarchy*, $m_3 < m_1 < m_2$; or to (ii) spectrum with *normal neutrino mass ordering* ($\Delta m_{23}^2 > 0$), but with *partial neutrino mass hierarchy*, $m_1 < m_2 < m_3$.

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