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Vacuum dark fluid

Irina Dymnikova [a,b,*], Evgeny Galaktionov [b]

[a] *Department of Mathematics and Computer Science, University of Warmia and Mazury, Żołnierska 14, 10-561 Olsztyn, Poland*
[b] *A.F. Ioffe Physico-Technical Institute, Politekhnicheskaja 26, St. Petersburg 194021, Russia*

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Abstract

We address the question of unified description of dark ingredients in the Universe by a vacuum dark fluid with continuous density and pressures, which represents both distributed vacuum dark energy by a time evolving and spatially inhomogeneous cosmological term, and compact self-gravitating objects with de Sitter vacuum trapped inside. The existence of spherically symmetric globally neutral gravitationally bound vacuum objects without horizons (called G-lumps) asymptotically de Sitter at the center, is implied by the Einstein equations. Their masses are restricted by $m < m_{\rm crit}$ where $m_{\rm crit} = \alpha m_{\rm Pl}\sqrt{\rho_{\rm Pl}/\rho_0}$ with a coefficient α depending on the model. We introduce vacuum dark fluid and present the criterion of stability of G-lumps to external polar perturbations.

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1. Introduction

The idea that

$$p = -\rho \tag{1.1}$$

can be the equation of state for a superdense matter, was put forward in 1965 by Sakharov who considered it as one of possible initial states of the universe evolution [1]. In the same year Gliner suggested that (1.1) could be a final state in a gravitational collapse and interpreted a medium specified by (1.1) as a dense vacuum [2] due to the algebraic structure of its stress–energy tensor

$$T^{\nu}_{\mu} = \rho\delta^{\nu}_{\mu}. \tag{1.2}$$

It has an infinite set of comoving reference frames which makes impossible to fix a velocity with respect to it.

In 1988 Poisson and Israel stated that de Sitter vacuum can arise in place of a black hole singularity since geometry can be self-regulatory and describable semiclassically by the Einstein equations with a source term representing vacuum polarization effects [3].

The symmetry of a vacuum stress–energy tensor (1.2) can be reduced keeping its vacuum identity. In the spherically symmetric case, the anisotropic spherically symmetric vacuum is defined by [4,5]

$$T^t_t = T^r_r, \qquad T^\theta_\theta = T^\phi_\phi. \tag{1.3}$$

A stress–energy tensor with such a symmetry is invariant under radial Lorentz boosts, so that one cannot single out a preferred comoving reference frame and thus determine the velocity with respect to a medium specified by (1.3)—which is the intrinsic property of a vacuum [6].

A regular vacuum stress–energy tensor (1.3) describes a *smooth continuous* de Sitter–Schwarzschild transition by the equation of state (following from the contracted Bianchi identities) for anisotropic perfect fluid with continuous density and pressures [4]

$$p_r = -\rho, \qquad p_\perp = -\rho - \frac{r}{2}\rho'. \tag{1.4}$$

Globally regular spherically symmetric spacetime with de Sitter center [7] (for a recent review [8]) represents, dependently on the choice of observers (coordinate mapping) dis-

* Corresponding author.
E-mail address: irina@matman.uwm.edu.pl (I. Dymnikova).

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tributed or localized vacuum dark energy: Regular vacuum dominated cosmologies [9–11], vacuum nonsingular black holes [4,12] (for review [13,14]) and globally neutral self-gravitating vacuum structures without horizons [15,16], called G-lumps [17] since they are bounded by their own gravity balanced at the surface where the strong energy condition is violated and gravitational attraction becomes gravitational repulsion.

The existence of the class of solutions to the Einstein equations representing compact vacuum objects with de Sitter center follows from the requirements of regularity of density, finiteness of the ADM mass, and the weak energy condition for $T_{\mu\nu}$ which requires monotonic decrease of a density profile while the requirement of regularity leads to the obligatory de Sitter center [7,17,18].

Mass of vacuum nonsingular black hole and G-lump is generically related to smooth breaking of space–time symmetry from the de Sitter group in the origin, and to de Sitter vacuum trapped inside [5,16,17,19–22].

Another approach involving the interior de Sitter vacuum, is based on direct matching of de Sitter interior to the Schwarzschild exterior via thin transitional shell where the tangential pressure diverges and metric suffers from discontinuities [3,23–29].

In a gravastar [30], dominated by a quantum condensate which undergoes phase transition near the Schwarzschild radius where the event horizon would have been expected to form [31], an interior de Sitter condensate phase is matched to an exterior Schwarzschild geometry through a phase boundary of a stiff matter ($p = \rho$). Compensation of discontinuities in the pressure profile needs involving two additional infinitely thin shells to stabilize the resulting onion-like structure [30].

The picture of a gravastar is somewhat similar to scalar field dominated boson star [32,33] if one assumes [34] that for some field configurations the scalar field is constant in the star interior and its self-interaction potential plays the role of a cosmological constant. A boson star can have a mass comparable to that of a neutron star [35].

In the paper [36] a gravastar is related to a dark energy due to gravitational Casimir-like boundary effect at the cosmological horizon on the scale of the whole universe. A model consists of a de Sitter interior and Schwarzschild exterior separated by a thin boundary layer near $r_H \sim 10^{28}$ cm, which is a quantum transition region replacing the de Sitter and Schwarzschild horizons [36].

Dark energy particles as quanta of the cosmological constant (considered as the fundamental constant) were presented in [37] and called cosmons.[1] These particles can form stable stellar-type configurations. From the requirement of the energetic stability of configuration of the minimum density (related to the fundamental λ as absolute minimum density [39]) one obtains a mass of order of 10^{55} g concluding that the observable universe may be regarded as a dark energy dominated object with the absolute minimum density. The analysis is based on a lower bound on the mass and density of an object of a given radius [40] with using the Buchdahl identity generalized in [34] for the case of *isotropic* fluid spheres in the presence of cosmological constant.

An approach unifying dark energy and dark matter into a single dark fluid was proposed in [41–43] for the generalized Chaplygin gas model. Analysis of spherical objects dominated by an isotropic perfect fluid with a polytropic equation of state of negative index [44,45] (see also [46]), suggests that arising of Chaplygin dark stars of stellar masses can be made compatible with observational constraints [45,47]. A unified dark fluid based on a scalar field is proposed in [48]. Although the form of the scalar field potential cannot be directly derived from high energy theories, it is possible to elaborate a dark fluid model using a complex scalar field [49] (in a way resembling repulsive-attractive consideration of [37,38]).

Contained in general relativity class of spherically symmetric solutions specified by (1.3) which describe time-dependent and spatially inhomogeneous vacuum dark energy, represents actually, in a model-independent way, *anisotropic vacuum dark fluid* with continuous density and pressures, which can both be distributed and form compact objects.

In this Letter we introduce vacuum dark fluid in general setting, and investigate stability of spherically symmetric G-lumps, globally neutral gravitationally bound vacuum structures without horizons.

G-lumps can originate as possible endpoint of the Hawking evaporation of vacuum nonsingular black holes [15–17] as well as from initial inhomogeneities on the early stages of the Universe evolution by a mechanism similar to formation of primordial black holes [50]. They can be responsible for local effects related typically to dark matter, in a way similar to λ-particles of Ref. [37] or complex scalar field particles of Ref. [49].

Let us note that in the context of a vacuum fluid unification, relation dark energy–matter (not necessary dark) may appear quite nontrivial if we take into account that in nonlinear electrodynamics coupled to gravity regular charged structures must have obligatory de Sitter center [18] which for a spinning particles transforms into rotating de Sitter vacuum disk which displays superconducting behavior within a single spinning particle [51].

2. Vacuum fluid and spherical G-lumps

In the Petrov classification scheme stress–energy tensors are classified on the basis of their algebraic structure. When eigenvalues of $T_{\mu\nu}$ are real, the eigenvectors of $T_{\mu\nu}$ are nonisotropic and form a comoving reference frame with a timelike eigenvector representing a velocity.

In this classification an anisotropic fluid is specified by [IIII] and [II(II)], and an isotropic fluid by [I(III)]. The first symbol denotes the eigenvalue related to the timelike eigenvector.

[1] The term *cosmon* was originally introduced in [38] for a new field which is the Goldstone boson of dilatation invariance assumed to be spontaneously broken near the Planck scale, whose task was to drive the cosmological constant to zero. The dominant component of the related force is attractive, couples to mass and should be weaker than gravity ($\alpha \sim 10^{-2}$–10^{-3}). There is also a repulsive baryon-number-dependent component of calculable strength ($\sim \alpha/20$) [38].

Parentheses combine degenerate eigenvalues. A comoving reference frame is defined uniquely if and only if none of spacelike eigenvalues λ_k $(k = 1, 2, 3)$ coincides with a timelike eigenvalue λ_0. Otherwise there exists an infinite set of comoving reference frames.

The maximally symmetric de Sitter vacuum (1.2), specified by [(IIII)] in the Petrov classification scheme (all eigenvalues equal, all reference frames comoving), represents the isotropic vacuum fluid.

A symmetry of a vacuum stress–energy tensor (1.2) can be reduced to the case when one (or two) of the spacelike eigenvalues of $T_{\mu\nu}$ coincides with its timelike eigenvalue

$$p_k = -\rho. \tag{2.1}$$

A vacuum stress–energy tensor with a reduced symmetry is invariant under Lorentz boosts in the k-direction, which makes impossible to single out a preferred comoving reference frame and thus fix the velocity with respect to a vacuum fluid.

A vacuum defined by symmetry of its stress–energy tensor, must be evidently anisotropic (except the maximally symmetric de Sitter vacuum (1.2)).

The Petrov classification scheme suggests three types of anisotropic vacuum fluid: [(II)(II)], [(II)II], [(III)I].

For the class of spherically symmetric solutions specified by (1.3), requirement of regularity and the weak energy condition lead to the existence of the class of metrics with a static spherically symmetric line element [17]

$$ds^2 = g(r)\,dt^2 - \frac{dr^2}{g(r)} - r^2\,d\Omega^2, \tag{2.2}$$

where $d\Omega^2$ is the metric on a unit 2-sphere, and the metric function $g(r)$ is given by

$$g(r) = 1 - \frac{R_g(r)}{r}, \quad R_g(r) = 2G\mathcal{M}(r); \tag{2.3}$$

with the mass function

$$\mathcal{M}(r) = 4\pi \int_0^r \rho(x)x^2\,dx. \tag{2.4}$$

The metrics are asymptotically Schwarzschild at large r

$$ds^2 = \left(1 - \frac{r_g}{r}\right) - \frac{dr^2}{\left(1 - \frac{r_g}{r}\right)} - r^2\,d\Omega^2, \quad r_g = 2Gm \tag{2.5}$$

with the mass parameter (ADM mass) $m = \mathcal{M}(r \to \infty)$.

Behavior at $r \to 0$ is dictated by the weak energy condition [17]. The equation of state near the center becomes $p = -\rho$, which gives de Sitter asymptotic

$$ds^2 = \left(1 - \frac{r^2}{r_0^2}\right)dt^2 - \frac{dr^2}{\left(1 - \frac{r^2}{r_0^2}\right)} - r^2\,d\Omega^2, \tag{2.6}$$

$$T_{\mu\nu} = \rho_0 g_{\mu\nu}, \qquad r_0^2 = 3/\Lambda, \quad \Lambda = \kappa\rho_0, \tag{2.7}$$

where $\rho_0 = \rho(r \to 0)$.

The weak energy condition defines also the form of the metric function $g(r)$: It has one minimum and the geometry can have not more than two horizons [17].

The 4-curvature scalar $R = \kappa T$ is proportional to $(\rho - p_\perp)$, and the 3-curvature scalar is given by $\mathcal{P}(r) = \kappa[(\rho - p_\perp) + \rho]$ [13]. In the case when the dominant energy condition is satisfied (e.g., [18]) both scalars remain non-negative. Spacetime of this kind specified as the DEC-subclass [13], does not exhibit changes in topology of space-like hypersurfaces.

For solutions satisfying only weak energy condition specified as WEC-subclass [13], the curvature scalars can change sign somewhere and geometry can experience changes in topology of space-like hypersurfaces.

In the coordinates of comoving observers, the metrics (2.2) describe regular vacuum dominated cosmologies of the Lemaitré class and Kantowski–Sachs type whose dynamics depends on the number of horizons [10,11]. Vacuum density evolves smoothly from a big initial value to a small value required by observations [52].

In the coordinates of a distant observer at rest (e.g., r, t in (2.2)) the class of solutions specified by (1.3), describes compact objects dominated by anisotropic vacuum dark fluid (the word ‘dark’ in this case refers to their interiors) which can have two horizons, a black hole horizon and an internal Cauchy [3,17] horizon. For a certain value of the mass parameter, $m = m_{\rm cr}$, which puts a lower limit on a black hole mass, horizons come together (in the course of the Hawking evaporation [15]), and beyond $m_{\rm cr}$ we have self-gravitating vacuum soliton, G-lump, globally regular and globally neutral [15–17].

For any geometry with de Sitter center there exists a zero gravity surface defined by $p_\perp(r) = 0$ [5,15], beyond which the strong energy condition is violated and gravitational attraction becomes gravitational repulsion.

For geometries from the WEC-subclasss, there exist two characteristic surfaces in a self-gravitating vacuum soliton: surfaces of zero 4- and 3-curvature.

Three compact vacuum configurations (a black hole, extreme black hole and G-lump) are shown in Fig. 1 plotted for the density profile [4]

$$\rho(r) = \rho_0 e^{-r^3/r_0^2 r_g}, \quad r_0^2 = 3/\kappa\rho_0, \quad r_g = 2Gm \tag{2.8}$$

which describes a smooth de Sitter–Schwarzschild transition in a simple semiclassical model for vacuum polarization in the

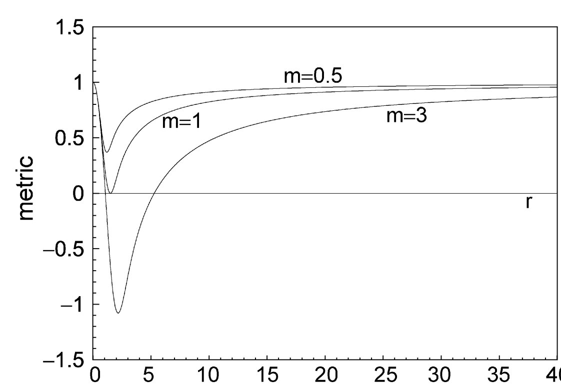

Fig. 1. The metric function $g(r)$ for compact vacuum objects with de Sitter center. Mass m is normalized to $m_{\rm cr}$.

spherically symmetric gravitational field [15]. In this case the critical mass is $m_{\rm cr} \simeq 0.3 m_{\rm Pl}\sqrt{\rho_{\rm Pl}/\rho_0}$.

Stability of G-lumps

We apply the approach of direct studying perturbations in the metric coefficients using the Einstein equations linearized about the unperturbed spacetime [53]. The perturbation analysis is fulfilled for the polar perturbations with the line element $ds^2 = e^{2\nu}\,dt^2 - e^{2\psi}\,d\phi^2 - e^{2\mu_2}\,dr^2 - e^{2\mu_3}\,d\theta^2$, in general case of an arbitrary regular density profile $\rho(r)$.

The polar perturbations in the metric coefficients $\delta\nu, \delta\mu_2, \delta\mu_3, \delta\psi$, and perturbations of a stress–energy tensor $\delta\rho, \delta p_r, \delta p_\theta, \delta p_\phi$ satisfy seven linear partial differential equations which we complete by imposing an assumption $\delta p_r = ((dp_r)/d\rho)\delta\rho$ valid for small perturbations, getting as a result the system of eight equations for eight functions which decouples into the system of four linear partial differential equations for $\delta\nu, \delta\mu_2, \delta\mu_3, \delta\psi$, the system of three algebraic equations relating $\delta p_r, \delta p_\theta, \delta p_\phi$ with $\delta\nu, \delta\mu_2, \delta\mu_3, \delta\psi$, and the equation following from the above assumption $\delta p_r = -\delta\rho$ [13].

The perturbations in the metric coefficients are presented as the series [53]

$$\delta\nu(r,\theta,t) = \sum_{l=2}^{+\infty} N_l(r) P_l(\cos\theta) e^{i\sigma_l t}, \tag{3.1a}$$

$$\delta\mu_2(r,\theta,t) = \sum_{l=2}^{+\infty} L_l(r) P_l(\cos\theta) e^{i\sigma_l t}, \tag{3.1b}$$

$$\delta\mu_3(r,\theta,t) = \sum_{l=2}^{+\infty} \big[T_l(r) P_l(\cos\theta) + V_l(r) P_{l,\theta\theta}(\cos\theta)\big] e^{i\sigma_l t}, \tag{3.1c}$$

$$\delta\psi(r,\theta,t) = \sum_{l=2}^{+\infty} \big[T_l(r) P_l(\cos\theta) + V_l(r) P_{l,\theta}(\cos\theta)\mathrm{ctg}\theta\big] e^{i\sigma_l t}. \tag{3.1d}$$

The Einstein equations give the linear system of three differential equations for $N_l(r)$, $L_l(r)$, $T_l(r)$, $V_l(r)$ and the relation $T_l = V_l - L_l$. Introducing $X_l(r) = L_l(r) + nV_l(r)$ where $n = l(l+1)/2 - 1$, we obtain the system of three differential equations in the normal form [13]

$$xg^2(x)N_{l,x} = (n+1)gN_l + g\left(\frac{x}{2}g' - (n+1)\right)L_l + x^2\left(\frac{1}{4}(g')^2 - \frac{1}{2}gg'' + \sigma_l^2\right)X_l, \tag{3.2a}$$

$$xgL_{l,x} + \left(\frac{x}{2}g' + g\right)L_l = xgN_{l,x} + \left(\frac{x}{2}g' - g\right)N_l, \tag{3.2b}$$

$$xgX_{l,x} = -gL_l + \left(\frac{x}{2}g' - g\right)X_l, \tag{3.2c}$$

where $x = r/r_1$ is the dimensionless radial coordinate; $r_1 = (r_0^2 r_g)^{1/3}$ is the characteristic scale of de Sitter–Schwarzschild geometry; $\rho(x)$ is normalized to ρ_0; the prime denotes differentiation with respect to x.

By the linear transformation

$$N_l = \left[\frac{1}{x}z_{1l} - \left(\big(b(x) - n - 1\big)\frac{g'}{2g} + \sigma_l^2\frac{x}{g}\right)z_{2l} + z_{3l}\right]\sqrt{g(x)}, \tag{3.3a}$$

$$L_l = (-b_{,x}z_{2l} + z_{3l})\sqrt{g(x)}, \tag{3.3b}$$

$$X_l = \frac{b(x)\sqrt{g(x)}}{x}z_{2l}, \tag{3.3c}$$

where

$$b(x) = n + 1 + \frac{x}{2}g'(x) - g(x) = n + \frac{3\alpha}{2x}\left(M(x) - x^3\rho\right) \tag{3.4}$$

and α is the characteristic parameter of the problem

$$\alpha = r_g/r_1 \tag{3.5}$$

we transform the normal system (3.2) to the form

$$z_{1l,x} = \left(\frac{2}{x} - \frac{g'}{g}\right)z_{1l} - \left(\frac{1}{2}x^2g''' + xg'' - g'\right)z_{2l} + \left[2 + \frac{x^2}{b(x)}\left(\frac{g''}{2} - \frac{(g')^2}{4g} - \sigma_l^2\frac{1}{g}\right)\right]z_{3l}, \tag{3.6a}$$

$$z_{2l,x} = -\frac{1}{b(x)}z_{3l}, \tag{3.6b}$$

$$z_{3l,x} = \frac{b(x)g^{-1}}{x^2}z_{1l} - \left[\frac{2}{x} + \frac{(xg'' - g')}{2b(x)}\right]z_{3l} + \frac{1}{x}\left(\frac{x^2}{2}g''' + xg'' - g'\right)z_{2l}. \tag{3.6c}$$

Differentiating (3.6c), expressing z_{1l} from (3.6c) and using (3.6a) we come to the system which includes one second-order equation, and one first-order equation

$$z_{3l,xx} + 2\left(\frac{g'}{g} + \frac{1}{x}\right)z_{3l,x} + q_l(x)z_{3l} = r_l(x)z_{2l}, \tag{3.7a}$$

$$z_{2l,x} = -\frac{1}{b(x)}z_{3l}, \tag{3.7b}$$

where

$$q_l(x) = \sigma_l^2\frac{1}{g^2} - \frac{2(n+1)}{x^2g} - \frac{1}{2}\frac{g''}{g} + \frac{1}{4}\left(\frac{g'}{g}\right)^2 + \frac{3}{x}\frac{g'}{g} - \frac{(xg'' - g')}{b(x)}\left[\frac{(xg'' - g')}{2b(x)} - \frac{g'}{g} + \frac{1}{x}\right] + \frac{xg'''}{2b(x)} + \frac{3\alpha x}{b(x)}p'_\perp \tag{3.8}$$

and

$$r_l(x) = -3\alpha p'_\perp\left[\frac{(n+1)}{g} - \frac{3x}{2}\frac{g'}{g} + \frac{x}{2b(x)}(xg'' - g')\right] + \frac{3\alpha}{x}\left(x^2p'_\perp\right)'. \tag{3.9}$$

Introducing the "tortoise" coordinate x_* by $\frac{d}{dx} = g(x)^{-1}\frac{d}{dx_*}$ and the new function

$$w_l(x_*) = x\sqrt{g(x)}z_{3l}(x_*) \tag{3.10}$$

we reduce the problem to the integro-differential equation

$$w_{l,x_*x_*} + \left[\sigma_l^2 - W_l(x)\right]w_l(x_*) = -K_l w_l(x_*), \tag{3.11}$$

where

$$K_l u(x_*) = x(x_*) g^{5/2}(x_*) r_l(x_*) \int_{d_*}^{x_*} \frac{\sqrt{g(z_*)}u(z_*)\,dz_*}{x(z_*)b(z_*)} \tag{3.12}$$

is the integral Volterra operator.

The local potential has the form

$$\begin{aligned} W_l(x) = g\bigg[&\frac{l(l+1)}{x^2} + g'' - \frac{1}{x}g' \\ &+ \frac{g(xg''-g')}{b}\left(\frac{xg''-g'}{2b} - \frac{g'}{g} + \frac{1}{x}\right) - \frac{xgg'''}{2b} \\ &- \frac{g}{xb}\left(\frac{1}{2}x^2g''' + xg'' - g'\right)\bigg]. \end{aligned} \tag{3.13}$$

In the particular case specified by the condition [13]

$$\left(x^2g''/2 - g\right)_{,x} = 0 \quad \text{or equivalently } \left(x^3\rho'\right)_{,x} = 0, \tag{3.14}$$

the function $r_l(x)$ in (3.12) vanishes identically, and the spectral problem reduces to the standard Schrödinger equation with the potential which in the case of the Schwarzschild black hole coincides with the potential in Zerilli equation [53]. The condition (3.14) is the necessary and sufficient condition for reducing the spectral problem to the canonic Schrödinger equation [13].

The asymptotic behavior of perturbations can be seen directly from the normal system (3.2). As $x \to \infty$ the limiting system for (3.2) reads

$$xN_{l,x} = (n+1)N_l - (n+1)L_l + x^2\sigma_l^2 X_l, \tag{3.15a}$$

$$xL_{l,x} + L_l = xN_{l,x} - N_l, \tag{3.15b}$$

$$xX_{l,x} = -L_l - X_l. \tag{3.15c}$$

The linear transformation

$$\begin{aligned} N_l &= \frac{1}{x}z_{1l} - \sigma_l^2 x z_{2l} + z_{3l}, \\ L_l &= z_{3l}, \qquad X_l = \frac{n}{x}z_{2l} \end{aligned} \tag{3.16}$$

reduces the problem to the second-order equation

$$z_{3l,xx} + \frac{2}{x}z_{3l,x} + \left[\sigma_l^2 - \frac{l(l+1)}{x^2}\right]z_{3l} = 0, \tag{3.17}$$

which is the radial equation for a particle in a spherically symmetric field (see, e.g., [54]).

The functions $z_{1l}(x)$ and $z_{2l}(x)$ are calculated from

$$z_{1l} = \frac{x}{n}(xz_{3l,x} + 2z_{3l}), \qquad z_{2l,x} = -\frac{1}{n}z_{3l}. \tag{3.18}$$

The function $w_l(x)$ satisfies the limiting ($x \to \infty$) equation

$$-w_{l,xx} + \frac{l(l+1)}{x^2}w_l = \sigma_l^2 w_l, \tag{3.19}$$

which gives asymptotic behavior at infinity

$$w_l(x) \sim A_l \sin(\sigma_l x - l\pi/2) + B_l \cos(\sigma_l x - l\pi/2). \tag{3.20}$$

In a small neighborhood of $x = 0$, the limiting system for (3.2) differs from (3.15) by that in (3.15a) we have $\sigma_l^2 + \text{const}$ in place of σ_l^2. Acting as above, we obtain the restricted asymptotic as $x \to 0$

$$w_l(x) = O\left(x^{l+1}\right). \tag{3.21}$$

This asymptotic behavior allows us to impose in the case of G-lump the condition $|w_l(0)| < \infty$ as $x \to 0$ and the boundary condition for the spectral problem (3.11)

$$w_l(x_*) \to e^{i\sigma_l x_*} + R_l e^{-i\sigma_l x_*} \quad \text{as } x_* \to \infty. \tag{3.22}$$

Ultimately the case reduces to investigation of the eigenvalue problem with integro-differential operator (3.11) which is the Schrödinger equation with non-local potential. The spectrum of eigenvalues contains all the values of the parameter σ_l^2, at which solutions exist which satisfy the imposed boundary conditions (3.22).

A considered static configuration is stable if there are no integrable modes with negative σ_l^2. The appearance of negative eigenvalues σ_l^2 would lead to the existence of exponentially growing modes of perturbations.

The local potential (3.13) is restricted from below and vanishes at infinity. In such a case its spectrum consists of the essential spectrum (positive semi-axis) and may be isolated negative eigenvalues which are absent if the potential is non-negative [55].

The non-local operator (3.12) can be considered as operator which perturbs the spectrum of the local operator (3.13). The Weyl theorem on self-conjugated operators states that the essential spectrum conserves under relatively compact perturbations [56].

In the case without horizons we can neglect in a small neighborhood of $x = x_* = 0$ the non-local contribution as compared with the contribution from the local potential. At the rest of the positive semi-axis the kernel of the Volterra operator (3.12) is square integrable smooth function [13]. Hence the non-local perturbation in this case is relatively compact and the essential spectrum of the problem (3.11) is the same as for the local potential (3.13). Perturbation of the spectrum due to (3.12) can lead only to appearance of isolated negative values σ_l^2.

For the case of a black hole the lower limit in the Volterra integral is $d_* = -\infty$ (the event horizon). In the case when the Killing horizons are absent, the coordinates r, t cover the whole manifold, and the non-local contribution is formed over the whole way of an initial perturbation wave from infinity (ingoing wave) through the center where an ingoing wave becomes outgoing, to the observation point x_* so that its contribution is given by $\int_\infty^0 + \int_0^{x_*}$. When $x_* \to \infty$, these two components are evidently cancelled so that non-local contribution does not lead to existence of isolated negative values σ_l^2.

To obtain the condition of non-negativity of the local potential $W_l(x)$ we write it, introducing the function $p(x) = xg''(x) - g'(x)$, in the form

$$W_l(x) = g\left[\frac{1}{2}g\left(\frac{p}{b}\right)^2 + \frac{1}{2b(x)}(g')^2 + \frac{2(n+1)}{x^2} - \frac{I_l(x)}{bx}\right], \tag{3.23}$$

where

$$I_l(x) = \alpha\left[-4\alpha\frac{M^2}{x^3} + \frac{9}{2}\alpha x^4\rho\rho' + 3(n-1)x^2\rho' + \frac{9}{2}\alpha x\rho' M - 3nx\rho - 3gx\left(x^2\rho'' + 2\rho\right) + \frac{3(n+2)}{x^2}M\right], \tag{3.24}$$

The function $b(x)$ in (3.23) can be written as $b(x) = n + 3\alpha(M(x) - \rho x^3)/2x$. With taking into account $\int_0^x \rho_{,z} z^3 dz = x^3\rho - M(x)$ and $\rho_{,x} \leqslant 0$, we find that $b(x) \geqslant n$ for all values of x. One negative contribution from $I_l(x)$, given by $3\alpha(n+2)M(x)/x^2$, is compensated by the term with $2(n+1)/x^2$ in (3.23). Another gives the sufficient condition

$$x^2\rho''(x) + 2\rho(x) \geqslant 0, \tag{3.25}$$

which guarantees the positivity of the local potential $W_l(x)$. The sufficient condition (3.25) constraints the growth of the derivative of $p_\perp + \rho$ by

$$x(p_\perp + \rho)' \leqslant \rho + (p_\perp + \rho) \tag{3.26}$$

and represents the criterion of stability of G-lumps. For the density profile (2.8) the condition (3.26) is satisfied.

4. Summary and discussion

G-lumps, globally neutral compact vacuum structures with the de Sitter center are stable to the external polar perturbations if they satisfy the condition (3.26) on the equation of state. We expect stability of G-lumps to axial perturbations under reasonable condition on $\rho(r)$ [57].

Mass of G-lumps is related to de Sitter vacuum in the center and smooth breaking of spacetime symmetry. Their masses are constrained by $m < m_{\mathrm{cr}}$. Characteristic mass scale which puts an upper limit on G-lump mass is $m_{\mathrm{cr}} = \alpha m_{\mathrm{Pl}}\sqrt{\rho_{\mathrm{Pl}}/\rho_0}$ where $\alpha < 1$ depends on the detailed particular model for the density profile $\rho(r)$. The limiting density at the center, ρ_0 is related to the scale of symmetry restoration to the de Sitter group in the origin. For globally neutral vacuum structures the relevant scale is M_{GUT}, then $m_{\mathrm{cr}} \sim 10^3$ g, so that G-lumps can have masses in the wide range below this value. G-lump can be viewed as model-independent image of mini-gravastar with continuous density and pressures[2] applying the term 'gravitating vacuum star' literally to a self-gravitating object made of a vacuum with the reduced symmetry.

The class of regular solutions to the Einstein equations specified by (1.3), includes configurations with vacuum density evolving smoothly from the big value at the center to a small value at infinity [5,11,12,17].

A positive cosmological constant may decelerate the onset of gravitational collapse but does not prevent its completion [59,60]. G-lump in a cosmological background can arise in a way similar to primordial black hole formation from quantum fluctuations resulting in a local density increase. It can emerge in a quantum tunnelling process as an object without BH horizon ($m < m_{\mathrm{cr}}$), either can appear as an end-product of the Hawking evaporation of a vacuum nonsingular black hole ([17] and references therein). The semiclassical nucleation rate of primordial black holes in de Sitter space [61–63] decreases with the BH mass and reaches the minimum when the event horizon is equal to cosmological horizon. On the other hand, the nucleation rate grows with increasing of background cosmological constant λ since higher Gibbons–Hawking temperature makes quantum fluctuations stronger [64]. Analysis of dynamics of BH evaporation in an inflationary universe with the spatially flat de Sitter metric revealed slight decrease in the evaporation process [65]. Quantum evolution of black holes in the de Sitter background depends on a BH size: Near maximal black holes (size comparable to de Sitter horizon) anti-evaporate but there is also evaporating mode [66]. The study of primordial black hole dynamics with the one-loop effective action for conformal matter suggests that cosmological PBHs may survive much longer than expected [67]. In the case of G-lumps the only difference is that a singularity is replaced with the interior de Sitter vacuum, so that one can expect the existence of a population of primordial G-lumps. Analysis of their stability at the background of small λ will be presented elsewhere.

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