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Implications of unitarity and precision measurements on CKM matrix elements

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Abstract

Unitarity along with precision measurements of $\sin 2\beta$, V_{us} and V_{cb} allows one to find a lower bound $V_{ub} \geqslant 0.0035$ which, on using the recently measured angle α of the unitarity triangle, translates to $V_{ub} = 0.0035 \pm 0.0002$. This precise value, stable for a good deal of changes in α, along with CP violating phase δ found from unitarity allows the construction of a 'precise' CKM matrix. The above unitarity based value of V_{ub} is in agreement with the latest exclusive value used as input by UTfit, CKMfitter, HFAG, however underlines the so-called 'tension' faced by the latest inclusive $V_{ub} = 0.00449 \pm 0.00033$. Further, using this inclusive value of V_{ub} along with the latest $\sin 2\beta$, one finds $\delta = 23°$–$39°$, again in conflict with δ measured in B-decays. The calculated ranges of the elements of the CKM matrix are in excellent agreement with those obtained recently by UTfit, CKMfitter and HFAG. Also, the ratio $\frac{V_{ts}}{V_{td}}$ is in agreement with its latest measured value, whereas there is some disagreement between the 'measured' and the calculated V_{td} values.

In the last few years, extremely important developments have taken place in the context of phenomenology of Cabibbo–Kobayashi–Maskawa (CKM) matrix [1], both from theoretical as well as experimental point of view. The precise measurement of CP violating parameter $\sin 2\beta$ [2–5] and a fairly precise measurement of angle α [6] of the unitarity triangle in B-decays have allowed a precise determination of the phase of the CKM matrix. Several detailed and extensive phenomenological analyses [3–5,7] have allowed us to conclude that the single CKM phase looks to be a viable solution of CP violation not only in the case of K-decays but also in the context of B-decays, at least to the leading order. On the one hand, this situation looks highly satisfactory from the Standard Model (SM) point of view, on the other hand, it has also triggered intense amount of activity on the theoretical as well as experimental front for finding clues to New Physics (NP).

Several authors [8–11] have suggested possible strategies for deciphering NP in the context of CKM phenomenology. One possible way to observe NP is by discovering violations of unitarity, as emphasized by Buras [8]. In this context, it needs to be noted that the persistent 2σ violation of unitarity by the first row elements of the CKM matrix has been eliminated by improving the precision in the measurement of V_{us} [3,12]. This, however, has also triggered a great deal of interest in measuring the other CKM elements to better and better accuracy for testing unitarity of the CKM matrix. There are several CKM elements and phenomenological parameters, e.g., V_{us}, V_{cb}, V_{ud}, α, β, etc., wherein the error bars are limited to only a few percent, however, this is not true in the case of several other CKM elements such as V_{ub}, V_{cs}, V_{ts} and V_{td}. A precise knowledge of these elements would not only test unitarity to better and better level, but would also provide clues to the possibility of existence of NP. It may be noted that the elements V_{ts} and V_{td}, under the present circumstances, can only be measured indirectly, whereas the elements V_{ub} and V_{cs} can be measured

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through tree level decays, therefore, one would expect considerable improvements in these in the near future. At present, even after recent updating by various groups, the situation regarding V_{ub} remains largely unclear. As per PDG 2006 [7] the exclusive and inclusive values respectively are $V_{ub} = 0.00384 + 0.00067 - 0.00049$ and $V_{ub} = 0.00440 \pm 0.00029 \pm 0.00027$, whereas October 2006 (including Summer (ICHEP06) updates) updated analysis by UTfit Collaboration [3], also agreed by CKM05 Workshops [4] and HFAG [5], uses as inputs exclusive $V_{ub} = 0.0035 \pm 0.0004$ and inclusive $V_{ub} = 0.00449 \pm 0.00033$. Keeping in mind the significance of the difference between the exclusive and inclusive values of V_{ub}, the UTfit [3] carries out a separate analyses for these. This point of view has also been advocated by several authors [13].

The unprecedented accuracy in the measurement of $\sin 2\beta$ [2–5], V_{us} [3,12] and V_{cb} [3] provides a strong motivation for carrying out a fine grained analysis for testing CKM paradigm to better and better accuracy as well as in search for clues to situations which have potential seeds for NP. Similarly, a very recent precise measurement of ΔM_{B_s} [14] would not only have implications for CKM elements V_{ts} and V_{td} but would also have implications for other CKM phenomenological parameters [3]. In this context, unlike the several global analyses [3–5,7] carried out recently, it would perhaps be desirable to fine tune the implications of each of the vital inputs of CKM paradigm separately along with the precision measurements on the CKM matrix elements and other phenomenological parameters. To this end, an analysis emphasizing unitarity of the CKM matrix and the precisely measured CKM parameters as well as some of the over constraining measurements would be very desirable.

The purpose of the present communication is to study the implications of unitarity along with the well measured V_{us}, V_{cb}, $\sin 2\beta$, and angle α of the unitarity triangle on some of the lesser known elements of the CKM matrix such as V_{ub}, V_{cs}, V_{ts} and V_{td}. In particular, one would like to examine in detail the implications of unitarity along with recently refined $\sin 2\beta$ on exclusive and inclusive values of V_{ub} and CP violating phase δ. Using minimal inputs, e.g., unitarity and other well-measured quantities, it would also be of interest to explore the possibility of constructing a 'precise' CKM matrix. Apart from examining the compatibility of over constraining measurements, we would also like to find the unitarity based predictions for Jarlskog's rephasing invariant parameter J as well as the Wolfenstein–Buras parameters $\bar{\rho}$ and $\bar{\eta}$.

Most of the present day analyses, related to CKM phenomenology, have been carried out using the Wolfenstein–Buras parametrization [15] of the CKM matrix. However, in the present case, as the emphasis is on unitarity therefore we find it more convenient to use the PDG representation of the CKM matrix, wherein the unitarity is built-in. For ready reference as well as to facilitate discussion of results, we begin by considering the quark mixing matrix,

$$V_{\mathrm{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}, \tag{1}$$

which in the PDG representation, involving angles θ_{12}, θ_{23}, θ_{13} and phase δ [7] is given as

$$V_{\mathrm{CKM}} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix}, \tag{2}$$

with $c_{ij} = \cos\theta_{ij}$ and $s_{ij} = \sin\theta_{ij}$, for $i, j = 1, 2, 3$. In this representation, one can consider up to 4th decimal place $V_{us} = s_{12}$ and $V_{cb} = s_{23}$, whereas $|V_{ub}| = s_{13}$, henceforth $|V_{ub}|$ would be written as V_{ub}.

Unitarity of the V_{CKM} implies nine relations, three in terms of normalization conditions also referred to as 'weak unitarity conditions', and the other six are usually expressed through unitarity triangles in the complex plane. Because of the strong hierarchical nature of the CKM matrix elements as well as the limitations imposed by the present level of measurements, it is difficult to study the implications of normalization relations, therefore, the six non-diagonal relations are used to study the implications of unitarity on CKM phenomenology. Out of the six, four triangles implied by these relations are highly skewed and it is difficult to study their implications [16,17] with the present knowledge of the CKM matrix elements. The implications of the other two are usually studied through the triangle expressed by the relation

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0, \tag{3}$$

also referred to as db triangle. The angles of this triangle, in terms of V_{CKM} elements, mixing angles and CP violating phase δ [7], related to CP asymmetries, are expressed as

$$\alpha \equiv \arg\left[-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right] = \tan^{-1}\left[\frac{s_{12}s_{23}\sin\delta}{c_{12}c_{23}s_{13} - s_{12}s_{23}\cos\delta}\right], \tag{4}$$

$$\beta \equiv \arg\left[-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right] = \tan^{-1}\left[\frac{c_{12}s_{12}s_{13}\sin\delta}{c_{23}s_{23}(s_{12}^2 - c_{12}^2 s_{13}^2) - c_{12}s_{12}s_{13}(c_{23}^2 - s_{23}^2)\cos\delta}\right], \tag{5}$$

$$\gamma \equiv \arg\left[-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right] = \tan^{-1}\left[\frac{s_{12}c_{23}\sin\delta}{c_{12}s_{23}s_{13} + s_{12}c_{23}\cos\delta}\right]. \tag{6}$$

To obtain information about the CP violating phase δ from the experimentally well determined angle β one can express Eq. (5) as

$$\tan\frac{\delta}{2} = \frac{A - \sqrt{A^2 - (B^2 - A^2C^2)\tan^2\beta}}{(B + AC)\tan\beta}, \tag{7}$$

where $A = c_{12}s_{12}s_{13}$, $B = c_{23}s_{23}(s_{12}^2 - c_{12}^2 s_{13}^2)$ and $C = c_{23}^2 - s_{23}^2$. Using $s_{12}^2 \gg c_{12}^2 s_{13}^2$ and $s_{23}^2 \ll c_{23}^2$, the above relation can be re-expressed as

$$\delta = -\beta + \sin^{-1}\left(\frac{s_{12}s_{23}}{c_{12}s_{13}}\sin\beta\right), \tag{8}$$

which can also be written as

$$\frac{\sin(\delta+\beta)}{\sin\beta} = \frac{s_{12}s_{23}}{c_{12}s_{13}}. \tag{9}$$

From Eq. (6), one can easily show that $\gamma = \delta$ with an error of around 2%, therefore, using the closure property of the angles of the triangle, $\alpha + \beta + \gamma = \pi$, the above equation can be written as

$$s_{13} = \frac{s_{12}s_{23}\sin\beta}{c_{12}\sin\alpha}, \tag{10}$$

which can also be derived from Eq. (4) by using the closure property of the triangle. Eq. (9) can be used to provide a lower bound on s_{13}, e.g.,

$$s_{13} \geqslant \frac{s_{12}s_{23}}{c_{12}}\sin\beta. \tag{11}$$

Before we discuss the details of our analysis, in Table 1 we present the PDG 2006 [7] measured values and the latest input values of some of the CKM elements and the angles of the unitarity triangle used by UTfit Collaboration [3], also agreed by CKM05 Workshops [4] and HFAG [5]. The values of angles α and γ have not been used as inputs by UTfit Collaboration, therefore we use their latest values from [6] and [4,18] respectively.

To begin with, we study the implications of unitarity and precisely measured recently improved $\sin 2\beta$ on CP violating phase δ and V_{ub}. On examining unitarity based Eq. (7), we find that δ is dependent on V_{us}, V_{cb}, angle β as well as it involves V_{ub}. Using this equation, in Fig. 1 we have plotted the CP violating phase δ versus V_{ub}, also included in the figure is the experimentally measured $\delta = (63.0 + 15.0 - 12.0)^\circ$ shown by horizontal dashed lines, inclusive of results of various global analyses. The solid central line depicts δ obtained by using the mean values of V_{us}, V_{cb} and $\sin 2\beta$ whereas the outer lines correspond to the 1σ ranges of these inputs. A general look at the figure reveals several interesting points, e.g., for values of $V_{ub} > 0.00355$, the central value of δ shows a smooth decline as well as the range of δ gets narrower and narrower with increasing V_{ub}, however for $V_{ub} < 0.00355$ it seems that there is a sharp broadening of the δ range, with no restriction on δ when $V_{ub} < 0.0035$. From the graph one finds that the 1σ range of the recent inclusive value of V_{ub}, as given in Table 1 restricts δ to 23°–39°, whereas the mean value of the recent exclusive value does not constrain δ, however the upper limit of the 1σ range of the exclusive value provides only a lower bound $\delta > 38^\circ$. In conclusion, we would like to emphasize that the precisely known $\sin 2\beta$, for the inclusive value of V_{ub} implies a narrow range for δ, whereas for the exclusive value of V_{ub} it implies only a lower bound on δ. Fig. 1 can also be used for constraining V_{ub} for particular values of δ, e.g., the range given in the table implies $V_{ub} < 0.0038$. One may wonder whether a similar analysis can be carried out using the latest measured value of angle α. We have carried out such an analysis, however it does not lead to any new conclusions.

Our conclusions about V_{ub} and δ can be sharpened further by using other unitarity based relations. To this end, Eq. (9) allows V_{ub} to be expressed in terms of the well-determined quantities V_{us}, V_{cb} and $\sin\beta$. Interestingly, in case δ is also a well-measured

Table 1
The PDG 2006 [7] measured values and the latest October 2006 (including Summer (ICHEP06) updates) input values used by UTfit Collaboration [3], also agreed by CKM05 Workshops [4] and HFAG [5]. The latest values of α and γ are from [6] and [4,18] respectively

Parameter	Latest (October 2006) values	PDG 2006 values [7]
V_{us}	0.2258 ± 0.0014	0.2257 ± 0.0021
V_{cb}	0.0416 ± 0.0007	0.0416 ± 0.0006
$\sin 2\beta$	0.675 ± 0.026	0.687 ± 0.032
β	$(21.24 \pm 1.01)^\circ$	$(21.7 \pm 1.2)^\circ$
α	$(91.0 \pm 7.0 \pm 3.0)^\circ$	$(99.0 + 13.0 - 8.0)^\circ$
γ or δ	$(63.0 + 15.0 - 12.0)^\circ$	$(63.0 + 15.0 - 12.0)^\circ$
V_{ub} (excl.)	0.0035 ± 0.0004	$0.00384 + 0.00067 - 0.00049$
V_{ub} (incl.)	0.00449 ± 0.00033	$0.00440 \pm 0.00029 \pm 0.00027$

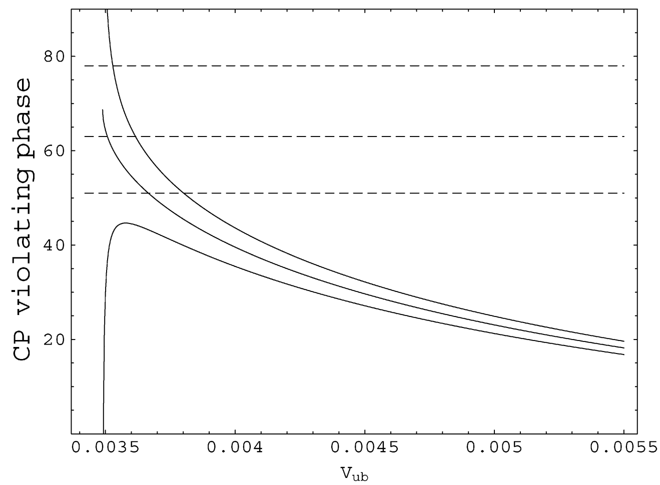

Fig. 1. Plot showing variation of V_{ub} versus CP violating phase δ, obtained by using Eq. (7). The central solid line corresponds to mean value of input parameters, whereas the other 2 lines correspond to 1σ variations. The horizontal dashed lines correspond to $\delta = (63.0 + 15.0 - 12.0)^\circ$, the central line corresponds to mean value.

quantity, then this equation immediately leads to a precise prediction for V_{ub}. However, even in the case where δ is not well determined, one can use Eq. (11) to obtain a rigorous lower bound on V_{ub}. A simple calculation using the mean values of input parameters immediately leads one to

$$V_{ub} \geqslant 0.0035. \tag{12}$$

It may be noted that this bound is independent of the value of δ as well as contamination of NP in the measurement of δ. Interestingly, Eq. (9) can also be used to show $V_{ub} \leqslant 0.00402$, found by using the lower limits of δ and β as given in the table.

Our predictions regarding V_{ub} can be refined further in case we incorporate angle α of the unitarity triangle, measured from $B \to \pi\pi$ and $B \to \rho\rho$ decays. Using its present consensus value [6], as given in Table 1, from Eq. (10) one finds

$$V_{ub} = 0.0035 \pm 0.0002. \tag{13}$$

Interestingly, this precise value is in full agreement with the recently used input value of exclusive V_{ub} by UTfit [3], however it has much smaller error bars. The above value of V_{ub} is a consequence of unitarity and the precisely measured elements V_{us}, V_{cb} and angles β and α. It may also be emphasized that this value is quite insensitive to a change in the value of angle α. In fact, even if the mean value of α changes by more than 20%, still V_{ub} would register a variation of only a few percent. Also, refinements in the measurement of δ would not affect the value of V_{ub} in Eq. (13) as δ along with $\sin 2\beta$ gives only a lower bound on V_{ub}, mentioned in Eq. (12). Therefore, the above prediction of V_{ub} can be considered as a rigorous and robust consequence of unitarity.

The above discussion also underlines the fact that precisely measured $\sin 2\beta$ along with V_{us} and V_{cb} does not lead to any well-defined conclusion regarding δ because of the persistent difference between exclusive and inclusive values of V_{ub}. Therefore, to find unitarity based δ one has to use the closure property of the angles of the unitarity triangle. Using the well measured angles α and β, one obtains

$$\delta = 67.8^\circ \pm 7.3^\circ. \tag{14}$$

This unitarity based value of δ is compatible with the directly measured value in $B^\pm \to DK^\pm$ decays [18] as well as with the recently obtained δ [19] from the $B \to \pi\pi$ and $B \to \pi K$ decays. It may also be mentioned that this value is compatible with the δ bound given by exclusive V_{ub}, as obtained from Fig. 1, however does not agree with the δ range obtained for inclusive V_{ub}.

After having found V_{ub} and δ from unitarity, one would like to construct the entire CKM matrix which is obtained at 1σ C.L. as follows

$$V_{\mathrm{CKM}} = \begin{pmatrix} 0.9738\text{–}0.9745 & 0.2244\text{–}0.2272 & 0.0033\text{–}0.0036 \\ 0.2243\text{–}0.2270 & 0.9730\text{–}0.9736 & 0.0409\text{–}0.0423 \\ 0.0082\text{–}0.0091 & 0.0401\text{–}0.0415 & 0.9990\text{–}0.9991 \end{pmatrix}. \tag{15}$$

It may be mentioned that this matrix is free from contamination by NP to the extent that the measured values of angles α and β are free from NP effects. Also, it needs to be emphasized that this has been constructed by using minimal inputs such as V_{us}, V_{cb}, V_{ub}, $\sin 2\beta$ and the unitarity based PDG parametrization, however without incorporating the full constraints due to unitarity. A general look at the matrix reveals that the ranges of CKM elements obtained here are quite compatible with those obtained by recent global analyses. In particular, the ranges found here are in excellent agreement with those emerging from global fits by UTfit, CKMfitter and HFAG. This perhaps indicates that unitarity plays a key role even in the case of global analyses. However, it must be mentioned that although the matrix presented here agrees well with the one given by PDG, yet there is a slight disagreement in the case of V_{ub} and V_{td}. The discrepancy in V_{ub} can be easily understood as the V_{ub} value used here is somewhat lower than the average V_{ub} value considered by PDG 2006. The disagreement in the value of the element V_{td}, sensitive to both loop and NP effects, suggests the need for further experimental scrutiny in this case. An experimental confirmation of the values of the CKM elements would strengthen the present unitarity based analysis as well as its predictions regarding V_{ub} and δ.

For the sake of completeness and better appreciation of the present results, we have evaluated the Jarlskog's rephasing invariant parameter J and the Wolfenstein–Buras parameters $\bar{\rho}$ and $\bar{\eta}$ by expressing these in terms of the mixing angles and the CP violating phase δ. Using the experimental values of V_{us}, V_{cb} as well as the unitarity based values of V_{ub} and δ found above, we obtain

$$J = (2.95 \pm 0.22) \times 10^{-5}, \tag{16}$$

$$\bar{\rho} = 0.14 \pm 0.04 \quad \text{and} \quad \bar{\eta} = 0.34 \pm 0.02. \tag{17}$$

Interestingly, the value of $\bar{\eta}$ is in complete agreement with those found by recent global analyses [3–5,7], whereas in case of $\bar{\rho}$ the value found here agrees with the one obtained by UTfit Collaboration [3].

The present analysis brings out several points which need to be emphasized. The so-called 'tension' between the precisely known $\sin 2\beta$ and the inclusive value of V_{ub}, as has already been observed by several authors [3,11,20], becomes quite evident in the present analysis. In the present context, this tension gets depicted in the form of disagreement between the value of δ implied by inclusive V_{ub} and the measured value of δ. From Fig. 1, one immediately finds that the δ value corresponding to inclusive V_{ub} comes out to be much smaller than the experimentally measured δ. This 'tension' is also visible in the form that the present unitarity based V_{ub} is much smaller than the inclusive value of V_{ub}, however is in excellent agreement with the latest exclusive V_{ub}. Therefore, the so-called 'tension' can also be seen as a disagreement between the unitarity based/exclusive and inclusive values of V_{ub}. It also becomes clear that in the case of PDG 2006, the exclusive value used by them is somewhat higher than the one used by other global analyses, therefore, they find corresponding reduction in the so-called 'tension'.

The CKM matrix constructed above allows us to calculate the ratio $\frac{V_{ts}}{V_{td}}$ which is expected to be free from hadronic uncertainties. The present calculated value 4.69 ± 0.23 looks to be quite precise and has an excellent overlap with 4.7 ± 0.4 [21], found recently from precision measurements of ΔM_{B_s}. Also, the measured value of the ratio $\frac{V_{ts}}{V_{td}}$ can be considered as an over constraining check on the above unitarity based predictions. Interestingly, this measurement also provides an indirect check on the unitarity based δ value used here which can be seen as follows. One can easily check that V_{ts} is essentially independent of δ and V_{ub}, therefore can be predicted quite accurately from unitarity. The element V_{td} has hardly any dependence on V_{ub} while it is known to be very much δ dependent, therefore the ratio $\frac{V_{ts}}{V_{td}}$ measurement can be considered to imply a precise value of δ which would fully agree with the value considered here. This possibility also ensures the validity of our δ dependent construction of CKM matrix even if the error bars in α become larger leading to larger error in the δ value found from the closure relationship.

One would also like to emphasize that the present $V_{td} = 0.0087 \pm 0.0004$ looks to be at variance with $V_{td} = 0.0072 \pm 0.0008$ [22] found from ΔM_{B_d}. In case one takes the present values of hadronic factors used in the calculation of ΔM_{B_d} seriously, then this difference may indicate the presence of NP in B^0–$\bar{B}^0$ mixing. The above mentioned conflict in the V_{td} values gets further sharpened in case we consider the recently obtained $\delta = 74^\circ \pm 6^\circ$ [19] from the $B \to \pi\pi$ and $B \to \pi K$ decays. While this value would be compatible with the implied δ bound found from the present unitarity based V_{ub}, however would be in conflict with the one found using inclusive V_{ub}, as well as would imply $V_{td} \sim 0.0091$, aggravating the above conflict further.

It also needs to be mentioned that a further precision in the measurement of $\sin 2\beta$, needless to say, would have far reaching implications for CKM phenomenology, particularly for CP violating phase δ and V_{ub}. It should also be noted that in case the value of V_{ub} is found to be ~ 0.0035 then it will need a careful scrutiny for studying its implications as around this value the behaviour of δ and V_{ub} in Fig. 1 depicts sharp changes.

A summary of our principal conclusions is as follows. Unitarity along with precisely measured V_{us}, V_{cb} and $\sin 2\beta$ leads to $V_{ub} \geqslant 0.0035$. In case one uses the measured value of the angle α of the unitarity triangle, one finds $V_{ub} = 0.0035 \pm 0.0002$, this precise value can be considered as a rigorous prediction of unitarity along with the other precisely measured quantities as it is almost independent of good deal of changes in α. This is in agreement with the latest exclusive $V_{ub} = 0.0035 \pm 0.0004$ used as input by UTfit, CKMfitter and HFAG, however is in conflict with the latest inclusive $V_{ub} = 0.00449 \pm 0.00033$, bringing out the so-called 'tension' faced by inclusive V_{ub}. Further, when this inclusive V_{ub} is used along with $\sin 2\beta$ one finds $\delta = 23^\circ$–39°, again in conflict with the γ or δ measured in B-decays.

Using unitarity based closure property of the angles of the unitarity triangle, one can find almost precise δ which along with other precisely known elements allows one to construct an almost 'precise' CKM matrix. The ranges of CKM elements of the present matrix, constructed by using 'minimal inputs', are in excellent agreement with those emerging from global fits by UTfit, CKMfitter and HFAG. Also, the ratio $\frac{V_{ts}}{V_{td}}$ [21] is in full agreement with its latest measured value, whereas in the case of V_{td} there is some disagreement with its recent measured value [22], perhaps indicating the presence of NP. The unitarity based values of J, $\bar{\rho}$ and $\bar{\eta}$ found here are in agreement with those found by some latest global analyses.

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