Radiative and seesaw threshold corrections to the S_3 symmetric neutrino mass matrix

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ABSTRACT

We systematically analyze the radiative corrections to the S_3 symmetric neutrino mass matrix at high energy scale, say the GUT scale, in the charged lepton basis. There are significant corrections to the neutrino parameters both in the Standard Model (SM) and Minimal Supersymmetric Standard Model (MSSM) with large $\tan\beta$, when the renormalization group evolution (RGE) and seesaw threshold effects are taken into consideration. We find that in the SM all three mixing angles and atmospheric mass squared difference are simultaneously obtained in their current 3σ ranges at the electroweak scale. However, the solar mass squared difference is found to be larger than its allowed 3σ range at the low scale in this case. There are significant contributions to neutrino masses and mixing angles in the MSSM with large $\tan\beta$ from the RGEs even in the absence of seesaw threshold corrections. However, we find that the mass squared differences and the mixing angles are simultaneously obtained in their current 3σ ranges at low energy when the seesaw threshold effects are also taken into account in the MSSM with large $\tan\beta$.

1. Introduction

The neutrino oscillation experiments have enriched our knowledge of masses and mixings of neutrinos and thus flavor structure of leptons. These developments aspire theorists to construct models for unraveling the symmetries of lepton mass matrices. With the evidence of nonzero value of reactor mixing angle θ_{13} [1] we now have information of all three mixing angles contrary to earlier studies where only an upper bound on θ_{13} existed. The lepton flavor mixing matrix comprises three mixing angles and a Dirac CP violating phase, δ_{CP}. There are two additional CP phases if neutrinos are Majorana particles. The best fit values along with their 3σ ranges of neutrino oscillation parameters [2] are shown in Table 1. The strength of this leptonic CP violation is parametrized by Jarlskog rephasing invariant [3] $J = c_{12}s_{12}c_{23}s_{23}c_{13}^2 s_{13} \sin\delta_{CP}$. The two Majorana phases, however, contribute to the lepton number violating processes like neutrinoless double beta decay. The cosmological constraint on the sum of neutrino masses by the Planck Collaboration [4] is $\Sigma m_{\nu_i} < 0.23$ eV at 95% C.L. Depending on the values chosen for the priors this sum can be in the

Table 1
The experimental constraints on neutrino parameters taken from [2].

Parameter	Best fit	3σ
$\Delta m_{12}^2/10^{-5}$ eV^2 (NH or IH)	7.54	6.99–8.18
$\Delta m_{13}^2/10^{-3}$ eV^2 (NH)	2.43	2.19–2.62
θ_{12}	33.64°	30.59°–36.8°
θ_{13}	8.93°	7.47°–10.19°
θ_{23}	37.34°	35.1°–52.95°

range (0.23–0.933) eV. There are some challenges left namely to determine the absolute mass scale, mass hierarchy of neutrinos and the CP violation in leptonic sector amongst others. Evidence for nonzero θ_{13} has led to many studies for the deviation from the assumed symmetries that predict the vanishing θ_{13} value. Among many possible discrete flavor symmetries to produce the current data, S_3 has been extensively studied in the literature [5]. It is the smallest discrete non-Abelian group which is the permutation of three objects. Perturbations to S_3 symmetric leptonic mass matrices have been used to study the mass spectra of the leptons and predict well known democratic [6] and tri-bimaximal neutrino mixing scenarios [7]. The most general form of S_3 invariant Majorana neutrino mass matrix [8] is given as

$$M_\nu = pI + qD, \tag{1}$$

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where I is the identity matrix and D is democratic matrix with all elements equal to 1, p and q are in general complex parameters. There are quite a few studies on the breaking of S_3 symmetry in the leptonic sector [8,9] to produce the currently observed neutrino oscillation data. However, in the light of nonzero θ_{13} another interesting possibility can be to study radiative corrections to S_3 symmetric neutrino mass matrix between the GUT scale Λ_g and the electroweak scale Λ_{ew} in the SM and MSSM.

Earlier studies have shown that there are significant RGE corrections to neutrino masses and mixing angles particularly for quasi-degenerate neutrino spectrum in the MSSM with large $\tan\beta$. The SM is extended by three heavy right handed neutrinos at high energy scale to generate neutrino masses in Type I seesaw mechanism [10]. The seesaw threshold corrections arise due to subsequent decoupling of these heavy right handed Majorana neutrinos at their respective masses. The structure of the Dirac mass matrix M_D is proportional to the neutrino Yukawa coupling matrix Y_ν. We take a general Y_ν and scan the parameter space to obtain the desired mixing pattern. The right handed Majorana mass matrix M_R is found by inverting the Type I seesaw formula at Λ_g. Above the heaviest seesaw scale (M_{R3}) there is a full theory and thus RGEs for Yukawa couplings Y_e, Y_ν and mass matrix M_R are considered. However, since our right handed neutrino mass matrix M_R is hierarchical ($M_{R1} < M_{R2} < M_{R3}$), we also consider the seesaw threshold effects and thus the respective set of effective theories in-between these scales, arising from the subsequent decoupling of heavy right handed fields at their respective masses. In the SM we find that Δm^2_{13} along with the neutrino mixing angles are generated in their present 3σ ranges at the low energy scale. However, Δm^2_{12} is greater than its allowed value ($\approx 10^{-4}$) in the SM. We find that it is possible to radiatively generate the current neutrino masses and mixing angles from the S_3 invariant neutrino mass matrix M_ν in the charged lepton basis, when the seesaw threshold effects are taken into account in the MSSM with large $\tan\beta$. In the MSSM with large $\tan\beta$, Δm^2_{12} can be produced in its current range along with the other neutrino oscillation parameters at Λ_{ew} in the presence of these threshold corrections.

In Section 2 we give the form of lepton mass matrices considered at Λ_g. In the subsequent section, we give the RGE equations governing from Λ_g to Λ_{ew}, in presence of the seesaw threshold effects both in the SM and MSSM. In Section 4 we study the order of corrections to the neutrino mass matrix in the presence of seesaw threshold effects. Section 5 gives our numerical results for both cases under consideration. We conclude in the last section.

2. Form of lepton mass matrices at the GUT scale

We consider the basis where charged lepton mass matrix (M_l) is diagonal and the effective light neutrino mass matrix (M_ν) is S_3 symmetric as given in Eq. (1). The Yukawa coupling matrix for charged leptons is given as

$$Y_e = \frac{1}{v} Diag(m_e, m_\mu, m_\tau),\qquad (2)$$

where the Higgs vacuum expectation value (VEV) v is taken to be 246 GeV in the SM and $246 \cdot \cos\beta$ GeV in the MSSM. The Yukawa coupling matrix Y_ν for the light neutrinos is taken of the form $Y_\nu = y_\nu U_\nu D$ as given in [11] where D is the diagonal matrix $Diag(r_1, r_2, 1)$. The three parameters y_ν, r_1 and r_2 are real, positive and dimensionless that characterize eigenvalues of Y_ν. The unitary matrix U_ν is the product of the three rotation matrices $R_{23}(\vartheta_2) \cdot R_{13}(\vartheta_3 e^{-i\delta}) \cdot R_{12}(\vartheta_1)$ having one CP violating phase δ. Thus, Y_ν has seven unknown parameters viz. three eigenvalues, three mixing angles and one CP phase. We vary the three hierarchy (y_ν, r_1, r_2) parameters and, though they are completely arbitrary,

but assumed to be $< \mathcal{O}(1)$. Three angles ϑ_1, ϑ_2, ϑ_3 and δ are varied in the range of $(0-2\pi)$.

The right handed mass matrix M_R is found by inverting the Type I seesaw formula at Λ_g as

$$M_R = -\frac{v^2}{2} Y_\nu M_\nu^{-1} Y_\nu^T.\qquad (3)$$

The three right handed neutrino masses M_{R1}, M_{R2} and M_{R3} are obtained by diagonalizing the right handed Majorana mass matrix M_R. The light neutrino mass matrix M_ν can be diagonalized by the unitary transformation R as $R^T M_\nu R$. One of the possible forms of R can be

$$R = U_{TBM} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}.\qquad (4)$$

The mass eigenvalues of M_ν are p, $p + 3q$ and p corresponding to the light neutrino masses m_1, m_2 and m_3, respectively. Due to the degeneracy in the mass eigenvalues m_1 and m_3, the diagonalizing matrix R is not unique. Degeneracy of masses implies that R is arbitrary up to orthogonal transformation $R_{13}(\phi)$, where ϕ is in 1–3 plane. Thus, most general diagonalizing matrix R is $U_{TBM}R_{13}(\phi)$, which implies the same physics as U_{TBM}. In this work we set $\phi = 0$ without loss of generality [8,9]. From the neutrino oscillation data we know $\Delta m^2_{12} \approx 10^{-5}$ and thus, there is small difference in the mass eigenvalues m_1 and m_2 which is a possible objection to this scenario of S_3 invariant approximation as here m_1 and m_3 are degenerate. This problem was elegantly solved in [8] where complex values of p and q are allowed. In that case we can easily achieve a situation where all three neutrinos are degenerate.

As shown in [9], q can be chosen completely imaginary and p is taken to be $|p|e^{-i\frac{\alpha}{2}}$. The magnitudes of p and q can be written in terms of parameter x as

$$|p| = x \sec\frac{\alpha}{2}, \qquad |q| = \frac{2}{3} x \tan\frac{\alpha}{2},\qquad (5)$$

where x is a real free parameter and allowed range of α is $0 \leq \alpha < \pi$. The magnitude of p and $p + 3q$ can be made equal by adjusting the phase α. The parameter x vanishes when $\alpha = 180°$ and thus this value is disallowed. Substituting the values of p and q given in Eq. (5), the magnitudes of the mass eigenvalues are given as

$$|m_1| = |m_2| = |m_3| = x \sec\frac{\alpha}{2}.\qquad (6)$$

This results in equal magnitude of all three mass eigenvalues and thus a degenerate spectrum of neutrinos to begin with at Λ_g. As pointed out earlier in [9] the phase α affects the rate of neutrinoless double beta decay but will not affect neutrino oscillation parameters. Thus, this phase is of Majorana type. When we run these masses from Λ_g to M_{R3}, the degeneracy of the mass eigenvalues is lifted by RGE corrections. We consider the normal hierarchical spectrum of masses where m_1 is the lowest mass. The other two masses are given as $m_2 = \sqrt{m_1^2 + \Delta m^2_{12}}$ and $m_3 = \sqrt{m_1^2 + \Delta m^2_{13}}$. Since the three mass eigenvalues at Λ_g have equal magnitude the two mass squared differences are vanishing to begin with. Once the degeneracy of three mass eigenvalues is lifted, their nonzero values are generated. In subsequent sections we will explore generation of the solar and atmospheric mass squared differences, together with the three mixing angles in their current 3σ limit at Λ_{ew} through the radiative corrections from S_3 invariant neutrino mass matrix at Λ_g.

3. RGE equations with seesaw threshold effects

In Type I seesaw the SM is extended by introducing three heavy right handed neutrinos and keeping the Lagrangian of electroweak interactions invariant under $SU(2)_L \times U(1)_Y$ gauge transformation. In this case, the leptonic Yukawa terms of the Lagrangian are written as

$$-\mathcal{L}_\nu = \bar{l}_e \phi Y_e e_R + \bar{l}_e \tilde{\phi} Y_\nu \nu_R + \frac{1}{2} \bar{\nu}_R^c M_R \nu_R + \text{h.c.} \tag{7}$$

Here ϕ is the SM Higgs doublet and $\tilde{\phi} = i\sigma^2 \phi^*$. e_R and ν_R are right handed charged lepton and neutrino singlets, Y_e and Y_ν are the Yukawa coupling matrices for charged leptons and Dirac neutrinos, respectively. The last term of Eq. (7) is the Majorana mass term for the right handed neutrinos.

Quite intensive studies have been done in the literature [12,13] regarding the general features of RGE of neutrino parameters. At the energy scale below seesaw threshold *i.e.* when all the heavy particles are integrated out, the RGE of neutrino masses and mixing angles is described by the effective theory which is the same for various seesaw models. But above the seesaw scale full theory has to be considered and thus, there can be significant RGE effects due to the interplay of heavy and light sector. The RGE equations and subsequent decoupling of heavy fields at their respective scales are elegantly given in Ref. [14,15]. The comprehensive study of the RGE and seesaw threshold corrections to various mixing scenarios is recently done in [16].

The effective neutrino mass matrix M_ν above M_{R3} is given as

$$M_\nu(\mu) = -\frac{v^2}{2} Y_\nu^T(\mu) M_R^{-1}(\mu) Y_\nu(\mu), \tag{8}$$

where $v = 246 \cdot \sin\beta$ GeV in the MSSM and μ is the renormalization scale. Y_ν and M_R are μ dependent. Since we study the evolution of leptonic mixing parameters from Λ_g to Λ_{ew} scale in a generic seesaw model we need to take care of the series of effective theories that arise by subsequent decoupling of the heavy right handed fields M_{Ri} ($i =$1,2,3) at their respective mass thresholds. The Yukawa couplings Y_ν and M_R are dependent on the energy scale Λ. At the GUT scale we consider the full theory and the one loop RGEs for Y_e, Y_ν and M_R are given as

$$\dot{Y}_{\{e,\nu\}} = \frac{1}{16\pi^2} Y_e [\alpha_{\{e,\nu\}} + C_{\{1,3\}} H_e + C_{\{2,4\}} H_\nu],$$
$$\dot{M}_R = \frac{1}{16\pi^2} C_5 [(Y_\nu Y_\nu^\dagger) M_R + M_R (Y_\nu Y_\nu^\dagger)^T], \tag{9}$$

where $\dot{Y}_i = \frac{dY_i}{dt}$ ($i = e, \nu$), $t = \ln(\mu/\mu_0)$ with $\mu(\mu_0)$ being the running (fixed) scale, and $H_i = Y_i^\dagger Y_i$ ($i = e, \nu$). The coefficients are $C_1 = \frac{3}{2}$, $C_2 = -\frac{3}{2}$, $C_3 = -\frac{3}{2}$, $C_4 = \frac{3}{2}$, $C_5 = 1$ in the SM and $C_1 = 3$, $C_2 = 1$, $C_3 = 1$, $C_4 = 3$, $C_5 = 2$ in the MSSM, respectively. The expressions for α_e and α_ν in the SM and MSSM are explicitly given as

$$\alpha_{\{e,\nu\}(SM)} = Tr(3H_u + 3H_d + H_e + H_\nu) - \left(K_{\{e,\nu\}}^1 g_1^2 + \frac{9}{4} g_2^2\right),$$
$$\alpha_{\{e,\nu\}(MSSM)} = Tr(3H_{\{d,u\}} + H_{\{e,\nu\}}) - (K_{\{e,\nu\}}^2 g_1^2 + 3g_2^2), \tag{10}$$

where $g_{1,2}$ are the $U(1)_Y$ and $SU(2)_L$ gauge coupling constants. The heavy right handed mass matrix M_R obtained from Eq. (3) is nondiagonal and thus is diagonalized by the unitary transformation U_R as $U_R^T M_R U_R = Diag(M_{R1}, M_{R2}, M_{R3})$. The Yukawa coupling Y_ν is accordingly transformed as $Y_\nu U_R^*$. At M_{R3}, the effective operator $\kappa_{(3)}$ is given by the matching condition as

$$\kappa_{(3)} = 2Y_\nu^T M_{R3}^{-1} Y_\nu, \tag{11}$$

in the basis where M_R is diagonal. The Yukawa coupling Y_ν above is a 3×3 matrix and all the variables are set to the scale M_{R3}. At the scale lower than M_{R3} ($\mu < M_{R3}$) the effective neutrino mass matrix M_ν is given as

$$M_\nu = -\frac{v^2}{4} \{\kappa_{(3)} + 2Y_{\nu(3)}^T M_{R(3)}^{-1} Y_{\nu(3)}\}. \tag{12}$$

As can be seen, M_ν is the sum of $\kappa_{(3)}$ given in Eq. (11) and the seesaw factor which is obtained after decoupling M_{R3}. Thus, $Y_{\nu(3)}$ is 2×3 and $M_{R(3)}$ is 2×2 mass matrices. RGE between the scales M_{R3} and M_{R2} is governed by the running of $\kappa_{(3)}$, $Y_{\nu(3)}$ and $M_{R(3)}$. The running of $\kappa_{(3)}$ is given as

$$\dot{\kappa}_3 = \frac{1}{16\pi^2} [(C_3 H_e + C_6 H_{\nu(3)})^T \kappa_{(3)} + \kappa_{(3)}(C_3 H_e + C_6 H_{\nu(3)}) + \alpha_{(3)} \kappa_{(3)}], \tag{13}$$

where $C_6 = \frac{1}{2}$ and $H_{\nu(3)} = Y_{\nu(3)}^\dagger Y_{\nu(3)}$. $\alpha_{(3)}$ in the SM and MSSM is explicitly given as

$$\alpha_{(3)(SM)} = 2Tr(3H_d + 3H_u + H_e + H_{\nu(3)}) - 3g_2^2 + \lambda,$$
$$\alpha_{(3)(MSSM)} = 2Tr(3H_u + H_{\nu(3)}) - \frac{6}{5} g_1^2 - 6g_2^2,$$

where all the parameters are set to seesaw scale M_{R2}. The low energy effective theory operator $\kappa_{(1)}$ is obtained after integrating out all three heavy right handed fields. The one loop RGE for $\kappa_{(1)}$ from lowest seesaw scale M_{R1} down to Λ_{ew} scale is given as

$$\dot{\kappa}_{(1)} = (C_3 H_e^T) \kappa_{(1)} + \kappa_{(1)}(C_3 H_e) + \alpha \kappa_{(1)}, \tag{14}$$

where

$$\alpha = 2Tr(3H_u + 3H_d + H_e) - 3g_2^2 + \lambda \quad \text{in the SM},$$
$$\alpha = 2Tr(3H_u) - \frac{6}{5} g_1^2 - 6g_2^2 \quad \text{in the MSSM}. \tag{15}$$

When the Higgs field gets VEV, the light neutrino mass matrix is obtained from $\kappa_{(1)}$ as $M_\nu = \frac{\kappa_{(1)} v^2}{4}$. We diagonalize M_ν to obtain neutrino masses, mixing angles and CP phases.

4. Neutrino masses and mixings

The RGE above the M_{R3} depends on more parameters than below the lowest seesaw scale M_{R1} due to presence of the neutrino Yukawa couplings (Y_ν). The RGE equations consist of H_e, M_R and H_ν out of which latter can be large. In the basis where charged lepton mass matrix is diagonal, M_ν at two different energy scales Λ_{ew} and Λ_g are homogeneously related as [17,18]

$$M_\nu^{\Lambda_{ew}} = I_K \cdot I^T \cdot M_\nu^{\Lambda_g} \cdot I. \tag{16}$$

Here I_K is a flavor independent factor arising from gauge interactions and fermion antifermion loops. It does not influence the mixing angles. The matrix I has the form

$$I = Diag(e^{-\Delta_e}, e^{-\Delta_\mu}, e^{-\Delta_\tau})$$
$$\simeq Diag(1 - \Delta_e, 1 - \Delta_\mu, 1 - \Delta_\tau) + \mathcal{O}(\Delta_{e,\mu,\tau}^2), \tag{17}$$

where

$$\Delta_j = \frac{1}{16\pi^2} \int [3(H_j) - (H_{\nu_j})] dt, \tag{18}$$

where $j = e, \mu, \tau$. Numerically, Δ_τ^{SM} can be of the order of 10^{-3} when $Y_\tau \sim 0.01$ and $Y_\nu = 0.2$ and the scales μ and μ_0 are 10^{12} and 10^2, respectively. In the MSSM, $\Delta_\tau^{MSSM} \sim 10^{-3}(1 + \tan^2 \beta)$ for the same values of Y_ν and Y_τ. In the absence of seesaw threshold

Table 2

Numerical values of input and output parameters that are radiatively generated via the RGE and seesaw threshold effects both in the SM and MSSM. The input parameters are taken at $\Lambda_g = 2 \times 10^{16}$ GeV and $\tan\beta = 55$ in the MSSM.

Parameters Input	SM	MSSM	Observables Output	SM	MSSM
r_1	7×10^{-3}	3×10^{-4}	m_1 (eV)	9.4×10^{-2}	1.47×10^{-2}
r_2	0.33	0.34	θ_{12}	34.3°	35.2°
δ	158.1°	96.3°	θ_{13}	7.88°	9.85°
y_ν	0.72	0.49	θ_{23}	47.7°	47.4°
θ_1	225.2°	188.5°	Δm_{12}^2 (eV^2)	4.18×10^{-4}	7.14×10^{-5}
θ_2	244.6°	245.8°	Δm_{13}^2 (eV^2)	2.48×10^{-3}	2.35×10^{-3}
θ_3	345.5°	241.7°	M_{R1} (GeV)	3.1×10^5	2.86×10^3
x (eV)	7.46×10^{-2}	1.9×10^{-3}	M_{R2} (GeV)	4.43×10^8	9×10^8
α	102.5°	170.2°	M_{R3} (GeV)	2.83×10^9	4.7×10^9
			J	-2.99×10^{-2}	-3.87×10^{-2}
			$|m_{ee}|$ (eV)	9.2×10^{-2}	1.03×10^{-2}
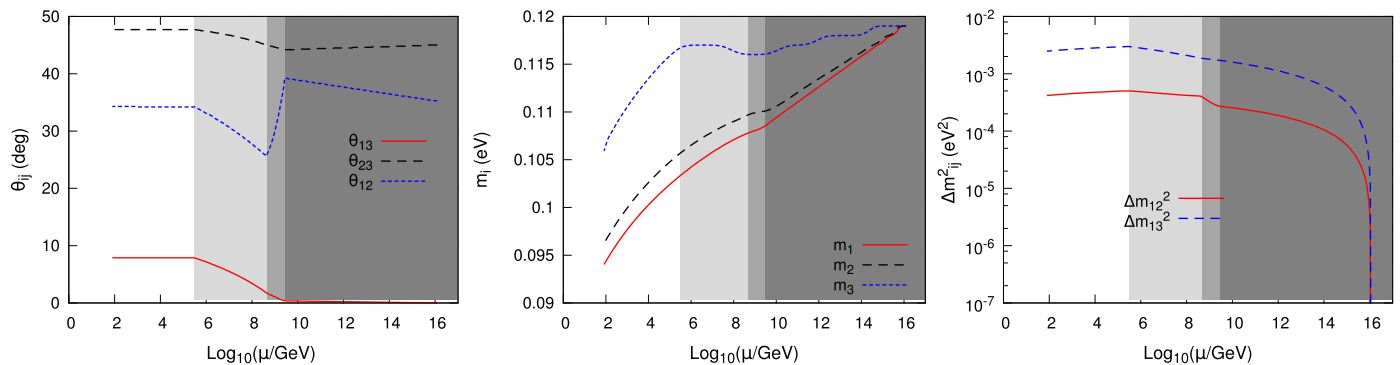
Fig. 1. The RGE of the mixing angles, masses and mass squared differences between Λ_g and Λ_{ew} in the SM. The initial values of the parameters are given in the second column of Table 2. The boundaries of three gray shaded areas, *i.e.* dark, medium and light denote the points when heavy right handed singlets M_{R3}, M_{R2} and M_{R1} are integrated out respectively.

effects, Δ_τ is small $\approx 10^{-5}$ in the SM for the above mentioned values of Y_τ.

Above seesaw scale appreciable deviations may occur only for large values of Y_τ or Y_ν. In the absence of seesaw threshold effects *i.e.* when there is no H_ν term in Eq. (18) the radiative corrections are governed by Δ_τ term as $\Delta_{e,\mu}$ is too small. On the other hand, due to the presence of large H_ν, $\Delta_{e,\mu}$ can have comparable contributions as Δ_τ. Below the seesaw scales, deviations are obtained from $Y_\tau \sim \sqrt{2}m_\tau/v \approx \mathcal{O}(10^{-2})$ in the SM, and $\sim \frac{\sqrt{2}m_\tau}{v\cos\beta}$ in the MSSM. There can be significant deviations in the MSSM with large $\tan\beta$ which enhances Y_τ. The analytic expressions for the running neutrino mixing angles, masses and CP phases are quite long and have been earlier derived in literature [19].

At Λ_g we have seven free parameters in Y_ν and two free parameters α and x in M_ν. The three mixing angles in ϑ_1, ϑ_2 and ϑ_3 and phase δ are allowed to take the values in the range $(0–2\pi)$. The hierarchy parameters y_ν, r_1, r_2 and x are randomly varied and are expected to be $< \mathcal{O}(1)$. The physical range of phase α is from $(0–\pi)$. The mass spectrum at the high scale is degenerate and thus we have vanishing solar and atmospheric mass squared difference to begin with. The parameter space at Λ_g with which the low energy neutrino data is obtained at Λ_{ew} is illustrated in Table 2. The set of input parameters in that particular parameter space is also given in the table. We choose the set of input parameters in parameter space at the high scale for which maximum value of θ_{13} is obtained and the other mixing angles and mass squared differences are simultaneously obtained in current 3σ range at Λ_{ew}. However, the parameter space under consideration is only for illustration and not unique. Search for complete parameter space is an elaborate study and thus independent future work.

5. Radiative and threshold corrections in the SM

Study of radiative corrections to S_3 symmetric neutrino mass matrix can be divided into three regions that are governed by different RGE equations. The first region is above the highest seesaw scale M_{R3} to Λ_g, where there can be considerable contribution of Y_ν. The second region is in-between the three seesaw scales and the third region is below the lowest seesaw scale M_{R1}, where all heavy fields are decoupled. The solar mixing angle θ_{12} can have large RGE corrections as the running is enhanced by the factor proportional to $\frac{m^2}{\Delta m_{12}^2}$ at the leading order, which can be large for degenerate spectrum. The RGE is comparatively small for other two mixing angles θ_{23} and θ_{13}, where the RGE is proportional to $\frac{m^2}{\Delta m_{13}^2}$. However, for the degenerate neutrino mass spectrum there can be considerable corrections for these mixing angles, too. Below the seesaw scale the RGE corrections to the mixing angles in the SM are negligible as they get contributions only from Y_τ. In Fig. 1, we show the RGE corrections to the mixing angles and masses in the SM for the set of input parameters given in second column of Table 2. Fig. 1 shows that below M_{R1} scale there are no significant corrections to the mixing angles. Below M_{R1}, running of the mass eigenvalues is significant even in the SM for degenerate as well as hierarchical neutrinos [13] due to the factor α given in Eq. (15), which is much larger than Y_τ^2. The running of masses is given by a common scaling of the mass eigenvalues [20]. Clearly, the RGE of each mass eigenvalue is proportional to the mass eigenvalue itself. The running of masses in Fig. 1 can be seen to start from the degenerate values of masses at Λ_g and there are significant corrections to the masses below M_{R1}. Earlier analysis [21] studied the successful generation of mass squared differences and mixing angles for degenerate neutrinos in the SM. In their analysis it is
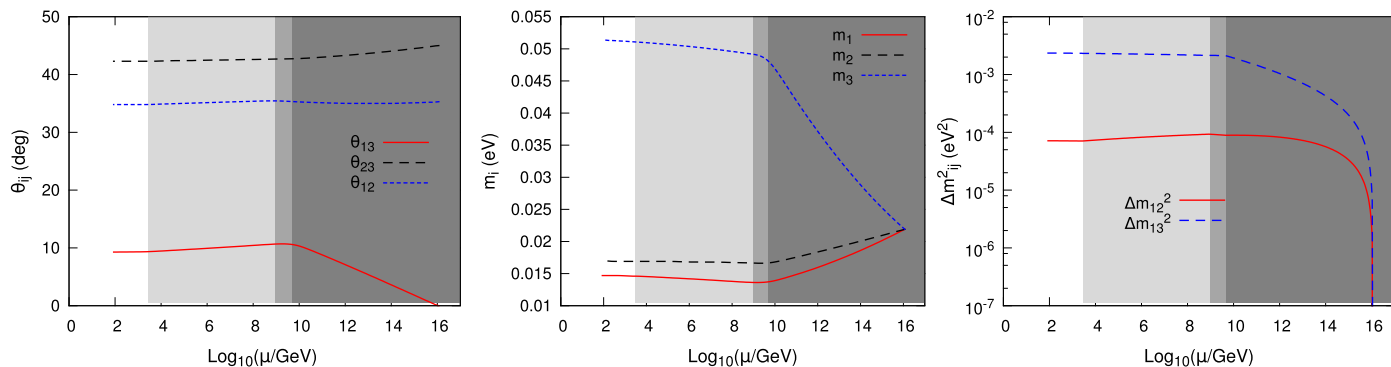
Fig. 2. The RGE of the mixing angles, masses and mass squared differences between Λ_g and Λ_{ew} in the MSSM with $\tan\beta = 55$. The initial values of the parameters are given in the third column of Table 2. The boundaries of three gray shaded areas, *i.e.* dark, medium and light denote the points when heavy right handed singlets M_{R3}, M_{R2} and M_{R1} are integrated out respectively.

shown that the generation of mass squared differences is very sensitive to the value of $\sin^2\theta_{12}$, which should be greater than 0.99 to fit the mass squared differences simultaneously with the consistent angles. This limit is, however, completely ruled out by the current oscillation data. The degeneracy of three mass eigenvalues is lifted by the RGE running from Λ_g to M_{R3}. Potentially significant breaking of neutrino mass degeneracy is provided by the RGE effects. The seesaw threshold effects in addition increase the mass splitting between the masses m_2 and m_3 required to fit the masses with the current data in terms of mass squared differences.

Running between and above the seesaw scales is modified by the contribution of Yukawa couplings Y_ν. The contribution of Y_ν in the RGE can result all the three mixing angles in their currently allowed ranges at the EW scale in the SM. Thus, there are significant corrections to mixing angles even in the SM in presence of the seesaw threshold effects when there is exactly the degenerate mass spectrum to begin with. As can be seen from Fig. 1, the mixing angles at the GUT scale are $\theta_{23} = 45°$, $\theta_{12} = 35.3°$ and $\theta_{13} = 0°$. For the set of parameters given in the second column of Table 2 we get the mixing angles in the allowed range at the electroweak scale in the SM. θ_{23} is found to have values below maximality. The presence of Y_ν in the RGE equations makes this possible even in the SM, as can be seen in Fig. 1. The gray shaded area in Fig. 1 and Fig. 2 illustrates the ranges of effective theories that emerge when we integrate out heavy right handed singlets. At each seesaw scales, *i.e.* M_{R1}, M_{R2} and M_{R3}, one heavy singlet is integrated out and thus $(n-1) \times 3$ sub-matrix of Y_ν remains. Therefore, the running behavior between these scales can be different from running behavior below or above these scales. Between these scales the neutrino mass matrix comprises two terms $\kappa_{(n)}$ and $2Y_{\nu(n)}^T M_{R(n)}^{-1} Y_{\nu(n)}$, as given in Eq. (12). It is shown in [13] that in the SM these two terms between the thresholds are quite different which can give dominant contribution to the running of mixing angles in this region. Both θ_{12} and θ_{13} in Fig. 1 get large corrections between three seesaw scales. θ_{23} gets the deviation of $\approx 2.7°$ in the upper direction. In the SM, Δm_{13}^2 is generated within the current oscillation data limit ($\approx 2.48 \times 10^{-3}$ eV^2) starting from the vanishing value at the Λ_g since all masses are degenerate, as seen from Fig. 1. The solar mass squared difference Δm_{12}^2 of the order of $\approx 10^{-4}$ eV^2 is simultaneously generated at the Λ_{ew}, as shown in Fig. 1 which is larger than its present value. The byproduct of this analysis is the masses of right-handed neutrinos that are determined from Eq. (3) and are not free parameters. The values of $|M_{ee}|$ and J at the Λ_{ew} are also calculated for particular set of parameters given in Table 2. The value of $|M_{ee}|$ will be useful to provide limit on unknown absolute neutrino mass.

6. Radiative and Threshold corrections in the MSSM

As stated earlier we divide the radiative corrections to S_3 symmetric neutrino mass matrix into three regions governed by different RGE equations in the MSSM. In the region below the lightest seesaw scale, for the SM and MSSM with small $\tan\beta$, Yukawa coupling $Y_\tau \sim 0.01$ is small, and thus there are small corrections. For large $\tan\beta$ these corrections can be larger due to the presence of factor $Y_\tau^2(1 + \tan^2\beta)$ in the MSSM. We show the RGE of mixing angles and masses in the MSSM with $\tan\beta = 55$ in Fig. 2 for the set of input parameters given in third column of Table 2. In the region above the energy scale M_{R1}, we get contributions from another Yukawa coupling Y_ν which brings in more free parameters in the analysis. In the region for the MSSM with large $\tan\beta$ the presence of seesaw threshold effects can enhance the RGE of the mixing angles significantly. As can be seen from Fig. 2, we can have all three mixing angles simultaneously in the current limit at the EW scale starting from S_3 symmetric neutrino mass matrix at the Λ_g. Fig. 2 shows that there are large corrections to θ_{13} ($\sim 9.85°$) between the Λ_g and M_3 scale due to the presence of Y_ν. As mentioned earlier, the running of neutrino mass matrix between the seesaw thresholds gets contributions from two terms $\kappa_{(n)}$ and $2Y_{\nu(n)}^T M_{R(n)}^{-1} Y_{\nu(n)}$ given in Eq. (12). In the MSSM, as can be seen from Fig. 2, there are not much deviations in the mixing angles between the energy scales. It is because the two contributions $\kappa_{(n)}$ and $2Y_{\nu(n)}^T M_{R(n)}^{-1} Y_{\nu(n)}$ are almost identical and thus cancel each other resulting in minimum deviation in those regions. The only significant correction occurs in the region above M_{R3} due to relatively large Y_ν.

The mixing angle θ_{12} does not have much corrections and θ_{23} receives the correction of 2.5° in the upper direction and is thus above maximal. The running of masses in the MSSM (Fig. 2) is much larger than the SM due to the presence of $\tan\beta$ which in our case is large. The dominant effect, however, is the corrections in the range $M_{R3} \leq \mu \leq \Lambda_g$ where the flavor dependent terms (Y_l and Y_ν) can be large. The interesting dependence of α_ν (MSSM) and $\tan\beta$ on the running contributions of flavor dependent terms is given in [13]. For large $\tan\beta$ the contribution of Y_e and Y_ν becomes important. We also show the radiative corrections to the two mass squared differences from the Λ_g to the Λ_{ew} for degenerate masses at the GUT scale in the MSSM with $\tan\beta = 55$. To begin with both the mass squared differences are zero at the GUT scale. From the mass squared differences shown in Fig. 2, we see that the RGE in combination with seesaw threshold corrections can result both mass squared differences in their current 3σ ranges at the low scale. For given set of input parameters in Table 2, the value of $|M_{ee}|$ is $\approx 10^{-2}$ eV and that of J is $\approx 3.87 \times 10^{-2}$. Thus, we find that it is possible to simultaneously obtain the neutrino

oscillation parameters at the electroweak scale for S_3 mass matrix at the Λ_g in the MSSM with large $\tan \beta$.

7. Conclusions

We studied the RGE corrections to the S_3 symmetric neutrino mass matrix in the presence of seesaw threshold corrections both in the SM and MSSM. In the absence of seesaw threshold effects there are negligible corrections to the mixing angles in the SM and MSSM with low $\tan \beta$. However, significant corrections are possible in neutrino parameters once the seesaw threshold effects are taken into consideration both in the SM and MSSM. In the SM we found that the mixing angles can be obtained in their current 3σ range at the electroweak scale when we begin with the S_3 symmetric neutrino mass matrix at the GUT scale Λ_g. The significant running occurs between and above the seesaw threshold scales. Below lowest seesaw scale there are no significant corrections as the only contribution comes from Y_τ which is small. However, in this case of exactly equal magnitude of mass eigenvalues, the solar mass squared difference is not simultaneously generated with other neutrino parameters in the current range at the electroweak scale in the SM. There can be large radiative corrections in the MSSM with $\tan \beta = 55$ when threshold effects are taken into consideration. The large corrections to the mixing angles occur at the scale above the seesaw threshold where the Yukawa coupling, Y_ν, is present and has large free parameters which can enhance running for large $\tan \beta$. Thus, in the MSSM with large $\tan \beta$ we can simultaneously generate all the masses and mixing angles in the currently allowed range at the electroweak scale, starting from exactly degenerate mass spectrum at high scale.

Acknowledgements

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