Available online at www.sciencedirect.com
SCIENCE DIRECT®
Physics Letters B 637 (2006) 16–20
PHYSICS LETTERS B
www.elsevier.com/locate/physletb

Cosmological expansion governed by a scalar field from a 5D vacuum

Mauricio Bellini [a,b]

[a] *Departamento de Física, Facultad de Ciencias Exactas y Naturales, Universidad Nacional de Mar del Plata, Funes 3350, (7600) Mar del Plata, Argentina*
[b] *Consejo Nacional de Ciencia y Tecnología (CONICET), Argentina*

Received 6 February 2006; received in revised form 6 April 2006; accepted 19 April 2006

Available online 2 May 2006

Editor: N. Glover

Abstract

We consider a single field governed expansion of the universe from a five dimensional (5D) vacuum state. Under an appropiate change of variables the universe can be viewed in a effective manner as expanding in 4D with an effective equation of state which describes different epochs of its evolution. In the example here worked the universe firstly describes an inflationary phase, followed by a decelerated expansion. Thereafter, the universe is accelerated and describes a quintessential expansion to finally, in the future, be vacuum dominated.
© 2006 Elsevier B.V. Open access under CC BY license.

PACS: 04.20.Jb; 11.10.kk; 98.80.Cq

1. Introduction and overview of the 5D formalism

In the last years has been an uprising interest in finding exact solutions of the Kaluza–Klein field equations in 5D, where the fifth coordinate is considered as non-compact [1]. Unlike the usual Kaluza–Klein theory in which a cyclic symmetry associated with the extra dimension is assumed, the new approach removes the cyclic condition on the extra dimension and derivatives of the metric with respect to the extra coordinate are retained. This induces non-trivial matter on the hypersurfaces with $\psi=$ const and other non-trivial frames. This theory reproduces and extends known solutions of the Einstein field equations in 4D. Particular interest revolves around solutions which are not only Ricci flat [2], but also Riemann flat [3]: $R^A_{BCD}=0$.[1] This is because it is possible to have a flat 5D manifold which contains a curved 4D submanifold, as implied by the Campbell theorem [4]. So, the universe may be "empty" and simple in 5D, but contain matter of complicated forms in 4D [5].

One of the greatest challenges of modern cosmology is understanding the nature of the observed late-time acceleration of the universe. Recent measurements of type Ia Supernovae (SNIa) [6] at redshifts $z\sim 1$ and also the observational results coming from the Cosmic Microwave Background Radiation (CMBR) along with the Maxima [7] and Boomerang data [8] indicate that the expansion of the present universe is accelerated. In fact the present day results show that supernovae are moving faster than expected from the luminosity redshift relationship in a decelerating universe. A possible explanation is that in the universe there exists an important matter component which, in its most simple description, has the characteristic of a cosmological constant as vacuum energy density which contributes to a large component of negative pressure [9,10]. The idea that the expansion of the universe could be governed by a scalar field has been developed in the quintessential models, where the dynamics of the scalar field is governed by an appropiate potential $V(\varphi)$ [11]. This paper is devoted to study the evolution of the universe which firstly suffers an inflationary expansion followed by a decelerated expansion (matter and radiation dominated) to finally be monotonically accelerated until a quasi-de Sitter expansion from a 5D vacuum state, where the (space-like) fifth dimension is considered as non-compact. We shall suppose that the universe is governed by a scalar field,

E-mail address: mbellini@mdp.edu.ar (M. Bellini).

[1] In our conventions, capital Latin indices run from 0 to 4 and Greek indices run from 0 to 3.

0370-2693 © 2006 Elsevier B.V. Open access under CC BY license.
doi:10.1016/j.physletb.2006.04.033

which is minimally coupled to gravity. For the system, we shall consider that the action is

$$I = -\int d^4x\, d\psi \sqrt{\left|\frac{^{(5)}g}{^{(5)}g_0}\right|}\left[\frac{^{(5)}R}{16\pi G} + {}^{(5)}\mathcal{L}(\varphi, \varphi_{,A})\right], \tag{1}$$

where φ is a scalar (neutral) quantum field, $G = M_p^{-2}$ is the gravitational constant [being $M_p = 1.2 \times 10^{19}$ GeV the Planckian mass], and ${}^{(5)}R$ is the Ricci scalar. Furtheremore ${}^{(5)}g$ is the determinant of the covariant tensor metric g_{AB} (A, B can take the values $0, 1, 2, 3, 4$). We are interested to describe a manifold in apparent vacuum, so that the Lagrangian density $\mathcal{L}$ in (1) should be only kinetic

$${}^{(5)}\mathcal{L}(\varphi, \varphi_{,A}) = \frac{1}{2} g^{AB} \varphi_{,A}\varphi_{,B}. \tag{2}$$

To describe a 3D spatially isotropic and homogeneous universe, which is Ricci flat: $R^A_{BCD} = 0$ and describes a 5D vacuum: $G_{AB} = 0$, we shall consider the background metric [12]

$$dS^2 = \psi^2\, dN^2 - \psi^2 e^{2N}\, dr^2 - d\psi^2, \tag{3}$$

where ψ is the space-like fifth coordinate, $r(x, y, z) = \sqrt{x^2 + y^2 + z^2}$ and N, x, y, z are dimensionless coordinates. Furthermore, the metric (3) is 3D spatially isotropic, homogeneous and flat. For the metric (3), the determinant of the covariant metric tensor g_{AB} is $|{}^{(5)}g| = \psi^8 e^{6N}$ and $|{}^{(5)}g_0| = \psi_0^8 e^{6N_0}$ is a constant of dimensionalization, for the constants $\psi = \psi_0$ and $N = N_0$. On the other hand, the energy–momentum tensor is given by

$$T_{AB} = \varphi_{,A}\varphi_{,B} - \frac{1}{2} g_{AB}\varphi_{,C}\varphi^{,C}, \tag{4}$$

which is symmetric because the symmetry of g_{AB}. The dynamics for φ is described by the Lagrange equation

$$\frac{\partial^2\varphi}{\partial N^2} + 3\frac{\partial\varphi}{\partial N} - e^{-2N}\nabla_r^2\varphi - \psi\left(\psi\frac{\partial^2\varphi}{\partial\psi^2} + 4\frac{\partial\varphi}{\partial\psi}\right) = 0. \tag{5}$$

In absence of 3D spatially isotropic field fluctuations, the background field $\varphi_b(N, \psi)$ corresponding to the vacuum equation $(\frac{\partial\varphi_b}{\partial N})^2 + \psi^2(\frac{\partial\varphi_b}{\partial\psi})^2 = T_{AB}|_b = 0$ on the background metric (3), is a constant of N and ψ: $\varphi_b(N, \psi) = \text{const}$. The commutator between φ and $\overset{\star}{\varphi}$ will be

$$\left[\varphi(N, \vec{r}, \psi), \overset{\star}{\varphi}(N, \vec{r}', \psi')\right] = i\left|\frac{^{(5)}g_0}{^{(5)}g}\right|\delta^{(3)}(\vec{r} - \vec{r}')\delta(\psi - \psi'). \tag{6}$$

In order to simplify the structure of Eq. (5) we can make the transformation $\varphi = \chi e^{-3N/2}(\frac{\psi_0}{\psi})^2$. The dynamics for $\chi(N, \vec{r}, \psi)$ being described by the equation of motion

$$\overset{\star\star}{\chi} - \left[e^{-2N}\nabla_r^2 + \left(\psi^2\frac{\partial^2}{\partial\psi^2} + \frac{1}{4}\right)\right]\chi = 0, \tag{7}$$

and the commutator between χ and $\overset{\star}{\chi}$ is

$$\left[\chi(N, \vec{r}, \psi), \overset{\star}{\chi}(N, \vec{r}', \psi')\right] = i\delta^{(3)}(\vec{r} - \vec{r}')\delta(\psi - \psi'). \tag{8}$$

We can make a Fourier expansion for χ

$$\begin{aligned}\chi(N, \vec{r}, \psi) &= \frac{1}{(2\pi)^{3/2}}\int d^3k_r \int dk_\psi \left[a_{k_r k_\psi} e^{i(\vec{k}_r\cdot\vec{r} + k_\psi\cdot\psi)}\xi_{k_r k_\psi}(N, \psi)\right.\\ &\quad \left. + a^\dagger_{k_r k_\psi} e^{-i(\vec{k}_r\cdot\vec{r} + k_\psi\cdot\psi)}\xi^*_{k_r k_\psi}(N, \psi)\right],\end{aligned} \tag{9}$$

where the asterisk denotes the complex conjugate and ($a_{k_r k_\psi}$, $a^\dagger_{k_r k_\psi}$) are respectively the annihilation and creation operators. They satisfy the following commutation expressions

$$\left[a_{k_r k_\psi}, a^\dagger_{k_r k_\psi}\right] = \delta^{(3)}(\vec{k}_r - \vec{k}'_r)\delta(\vec{k}_\psi - \vec{k}'_\psi), \tag{10}$$

$$\left[a^\dagger_{k_r k_\psi}, a^\dagger_{k'_r k'_\psi}\right] = [a_{k_r k_\psi}, a_{k'_r k'_\psi}] = 0. \tag{11}$$

The expression (8) complies if the modes are normalized by the following condition:

$$\xi_{k_r k_\psi}(\overset{\star}{\xi}_{k_r k_\psi})^* - (\xi_{k_r k_\psi})^*\overset{\star}{\xi}_{k_r k_\psi} = i. \tag{12}$$

This equation provides the normalization for the complete set of solutions on all the spectrum (k_r, k_ψ). As was demonstrated in a previous work [13], $\xi_{k_r k_\psi}(N, \psi) = e^{-i\vec{k}_\psi\cdot\vec{\psi}}\bar{\xi}_{k_r}(N)$, where $\bar{\xi}_{k_r}(N)$ is a solution of

$$\overset{\star\star}{\bar{\xi}}_{k_r} + \left(k_r^2 e^{-2N} - \frac{1}{4}\right)\bar{\xi}_{k_r} = 0, \tag{13}$$

such that the normalization condition for $\bar{\xi}_{k_r}(N)$ becomes

$$\bar{\xi}_{k_r}(\overset{\star}{\bar{\xi}}_{k_r})^* - (\bar{\xi}_{k_r})^*\overset{\star}{\bar{\xi}}_{k_r} = i, \tag{14}$$

where the overstar denotes the derivative with respect to N. Hence, the field χ in Eq. (9) can be rewritten as

$$\begin{aligned}\chi(N, \vec{r}) &= \frac{1}{(2\pi)^{3/2}}\int d^3k_r \int dk_\psi \left[a_{k_r k_\psi} e^{i\vec{k}_r\cdot\vec{r}}\bar{\xi}_{k_r}(N)\right.\\ &\quad \left. + a^\dagger_{k_r k_\psi} e^{-i\vec{k}_r\cdot\vec{r}}\bar{\xi}^*_{k_r}(N)\right].\end{aligned} \tag{15}$$

Furthermore, the functions $\bar{\xi}_{k_r}(N)$ are given by [13]

$$\bar{\xi}_{k_r}(N) = \frac{i\sqrt{\pi}}{2}\mathcal{H}^{(2)}_{1/2}\left[k_r e^{-N}\right]. \tag{16}$$

Finally, the field φ is given by

$$\varphi(N, \vec{r}, \psi) = e^{-\frac{3N}{2}}\left(\frac{\psi_0}{\psi}\right)^2\chi(N, \vec{r}), \tag{17}$$

with $\chi(N, \vec{r})$ given by Eq. (15). It is very important to notate that exponentials $e^{\pm i\vec{k}_\psi\cdot\vec{\psi}}$ disappear in $\chi(N, \vec{r})$ and there is not dependence on the fifth coordinate ψ in this field. This fact is an evidence of that the field $\varphi(N, \vec{r}, \psi)$ propagates only on the 3D spatially isotropic space $r(x, y, z)$, but not on the additional space-like coordinate ψ.

2. An effective 4D model of expansion for the universe

In order to develope an effective 4D model for an universe which is governed by the scalar field φ, we shall use the following change of variables on the metric (3)

$$dt = \psi\, dN, \qquad dR = \psi\, dr, \quad \psi = \psi \tag{18}$$

so that the resulting metric becomes

$$dS^2 = dt^2 - e^{2\int \frac{dt}{\psi}}\, dR^2 - d\psi^2. \tag{19}$$

On hypersurfaces $\psi = 1/H(t)$ the metric (19) becomes

$$dS^2 = dt^2 - e^{2\int H\,dt} dR^2 - \left[d\left(H^{-1}\right)\right]^2, \tag{20}$$

which also can be rewritten as an effective 4D metric

$$dS^2 \to ds^2 = \left[1 - \left(\frac{\dot H}{H^2}\right)^2\right] dt^2 - e^{2\int H\,dt}\, dR^2, \tag{21}$$

which is well defined when $1 - (\frac{\dot H}{H^2})^2 \neq 0$. Notice that, from the mathematical point of view, the change of variables (18) do not describes a transformation of coordinates. This change of variables describes a map from the particular frame $U^\psi = 0$ [of the metric (3)], to the particular frame $u^R = 0$ of the effective 4D metric (21). Here, $U^A = \frac{dx^A}{dS}$ are the penta velocities of the original metric (3) and $u^\mu = \frac{dX^\mu}{ds}$ are the tetra velocities of the effective 4D metric (21), such that $g^{AB} U^A U^B = 1$ and $g^{\mu\nu} u^\mu u^\nu = 1$, respectively.

Furthermore, the density Lagrangian $\mathcal{L}$ of Eq. (2) can be expanded as

$$\mathcal{L}(\varphi, \varphi_{,A}) = \frac{1}{2}\left[g^{\mu\nu}\varphi_{,\mu}\varphi_{,\nu} + g^{\psi\psi}\varphi_{,\psi}\varphi_{,\psi}\right], \tag{22}$$

so that we can make the following identification for the scalar potential on the effective 4D metric (21):

$$V(\varphi) = \frac{1}{2} g^{\psi\psi}\varphi_{,\psi}\varphi_{,\psi}|_{\psi=1/H} = 2H^2\varphi^2(t, \vec R). \tag{23}$$

Using the change of variables (18), the equation (5) can be rewritten on the 4D effective submanifold (21), as

$$\ddot\varphi + \left(3H - \frac{\dot H}{H}\right)\dot\varphi - e^{2\int H\,dt}\nabla_R^2\varphi + V'(\varphi) = 0, \tag{24}$$

with

$$V'(\varphi) = -H^2\left[\psi^2\varphi_{,\psi\psi} + 4\psi\varphi_{,\psi}\right]\Big|_{\psi=H^{-1}} = 2H^2\varphi. \tag{25}$$

2.1. The effective 4D equation of state

Using the effective 4D metric (21), we obtain the following Einstein's equations

$$\frac{3H^6}{\dot H^2 - H^4} = 8\pi G\rho, \tag{26}$$

$$\frac{H^6}{(\dot H^2 - H^4)^2}\left[3H^4 - 3\dot H^2 + 2\dot H H^2 - 6\frac{\dot H^3}{H^2} + 2\ddot H\frac{\dot H}{H}\right] = 8\pi G p, \tag{27}$$

where ρ and p are, respectively, the energy density and the pressure. The effective 4D equation of state for the universe becomes

$$\frac{p}{\rho} = \omega_{\rm eff}(t) \tag{28}$$

with

$$\omega_{\rm eff}(t) = -\left[1 + \frac{(2\dot H H^2 - 6\frac{\dot H^3}{H^2} + 2\frac{\dot H\ddot H}{H})}{3(H^4 - \dot H^2)}\right], \tag{29}$$

which, for a Hubble parameter $H = p/t$ (with constant p) agrees exactly with that of a spatially flat 4D Friedmann–Robertson–Walker (FRW) metric $ds^2 = dt^2 - e^{2\int H\,dt} dR^2$: $\omega_{\rm eff}(t)|_{H=p/t} = \frac{2-3p}{3p} = \omega_{\rm FRW}(t)$. Some particular cases $p \to \infty$, $p = 2/3$ and $p = 1/2$, give us respectively $\omega_{\rm eff} \to -1$, $\omega_{\rm eff} = 0$ and $\omega_{\rm eff} = 1/3$, which describes respectively expansions dominated by vacuum, matter and radiation. Note that for $p > 1$ the effective 4D metric (21) is always Lorentzian in nature. However for $p = 1$ this metric is not well defined. On the other hand, for $p(t) < 1$ the metric (21) is Euclidean and hence losses its relativistic nature. Hence, a well defined model for the expansion of the universe must be developed using $p > 1$ in the metric (21). For a more realistic model with a time dependent $p(t)$, one obtains $\omega_{\rm eff}(t)|_{H=p(t)/t} \neq \omega_{\rm FRW}(t)$. In Section 3 we shall study with more detail this last case.

2.2. Comoving frame

In order to describe the evolution of the universe on a comoving frame, which is the relevant for cosmological models, we can make use of the hyperbolic condition $g_{\mu\nu}u^\mu u^\nu = 1$ on the 4D effective metric (21). Here, $u^\mu = \frac{dx^\mu}{dS(t)}$, are the tetravelocities being $u^R = 0$ in the comoving frame. In this frame the velocity u^t is

$$u^t = \frac{1}{\sqrt{1 - (\dot H/H^2)^2}}. \tag{30}$$

Note that when

$$-\dot H/H^2 \ll 1, \tag{31}$$

we obtain $u^t \simeq 1$ and the metric (21) describes an asymptotic 4D FRW metric, which usually is used to describe the universe in cosmological models.

2.3. Evolution of the background field φ_b

The effective 4D evolution of the background field $\varphi_b(t)$ is given by Eq. (24) with $\nabla_R^2\varphi = 0$

$$\ddot\varphi_b + \left(3H - \frac{\dot H}{H}\right)\dot\varphi_b + 2H^2\varphi_b = 0, \tag{32}$$

which has the general solution

$$\varphi_b(t) = \frac{\varphi_b(0)}{2} e^{-\int H(t)\,dt}\left[1 + e^{-\int H(t)\,dt}\right], \tag{33}$$

where $\varphi_b(0) = \varphi_b(t = 0)$. It is evident that $\varphi_b(t)$ decreases monotonically with the time and rolls down the minimum of the potential, so that $\varphi_b(t \to \infty) \to 0$.

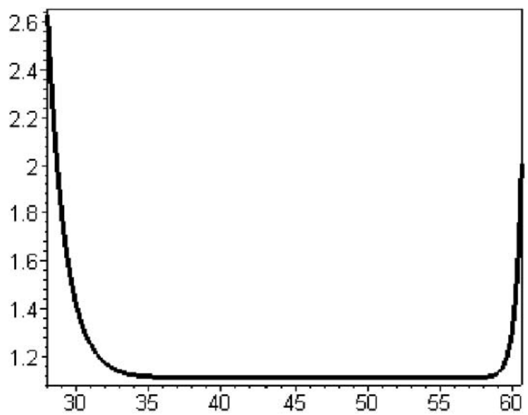

Fig. 1. Evolution of $p[x(t)]$ as a function of $x(t) = \log_{10}(t/t_0)$.

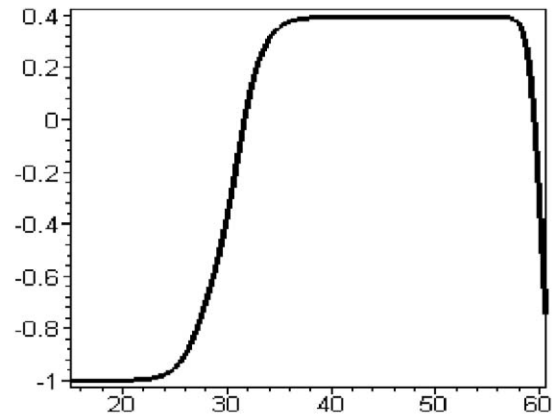

Fig. 2. Evolution of ω_{eff} as a function of $x(t) = \log_{10}(t/t_0)$.

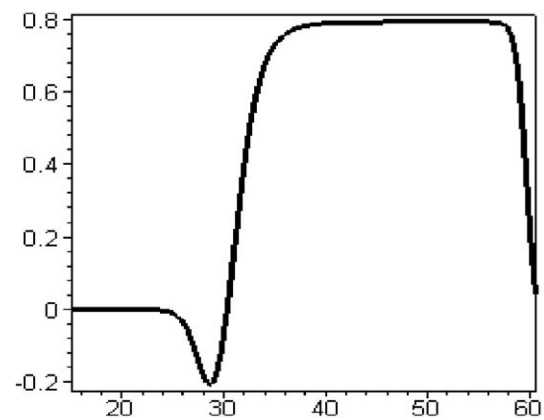

Fig. 3. Evolution of $\omega_{\mathrm{eff}} - \omega_{\mathrm{FRW}}$ as a function of $x(t) = \log_{10}(t/t_0)$.

3. Model of expansion of the universe

In order to describe all the evolution of the universe we can propose the following expression for the Hubble: $H(t) = p(t)/t$, such that

$$p(t) = 1.8at^{-n} + \left(\frac{b^2}{4a} + 0.62\right) + ct, \tag{34}$$

where $a = 1/6 \times 10^{30n} G^{n/2}$, $b = 8/7 \times 10^{15n} G^{n/4}$, $c = 2.0 \times 10^{-61} G^{-1/2}$ and $n = 0.352$. The general solution (33) in the example we are worked assume the explicit expression

$$\varphi_b(t) = \varphi_b(0) e^{-\frac{3ct}{2}} \left(\frac{t}{t_0}\right)^{-31/25} \left\{ \left(\frac{t}{t_0}\right)^{-b^2/(2a)} e^{-[\frac{36at^{-n}-5cnt}{10n}]} + \left(\frac{t}{t_0}\right)^{\frac{62a-25b^2}{100a}} e^{\frac{18at^{-n}+5cnt}{10n}} \right\}. \tag{35}$$

In Fig. 1 is ploted the parameter $p[x(t)]$ [where $x(t) = \log_{10}(t)$], which always remains with values $p > 1$. In the Fig. 2 is shown the evolution for the cosmological parameter $w_{\mathrm{eff}}[x(t)]$. Note that for $x(t) < 20$ the universe is governed by vacuum and describes an inflationary expansion, but later $w_{\mathrm{eff}}[x(t)]$ increases to thereafter describe a phase with positive pressure: $\omega_{\mathrm{eff}} > 0$ (in the range $30 < x(t) < 60$). After it, the pressure decreases and for $x(t) > 60$ the universe expands with negative pressure until the present day, when $\omega_{\mathrm{eff}}[x \simeq 60.652] \simeq -0.7$. This result agrees with the experimental data [14]. In the Fig. 3 is ploted $(\omega_{\mathrm{eff}} - \omega_{\mathrm{FRW}})[x(t)]$. It is very clear that the discrepance between ω_{eff} and ω_{FRW} becomes more notorious in epochs where the universe expands with positive pressure. However, in both epochs (in the very early universe and the present day universe), the equation of state for the universe agrees with that predicted by a 4D spatially flat FRW metric: $ds^2 = dt^2 - e^{2\int H\,dt}\, dR^2$. More exactly, the condition (31) holds for

$$\frac{1}{p(t)} - \frac{t\dot{p}(t)}{p(t)^2} \ll 1. \tag{36}$$

4. Final comments

In this Letter we have studied the evolution of the universe which is considered as governed by a single scalar field from a 5D vacuum state, where the fifth dimension is considered as non-compact [see the metric (3)]. However when we make the change of variables (18) the universe describes an effective 4D evolution which can be described by the metric (21). This metric is well defined for $(\dot{H}/H^2)^2 \neq 1$, which is the case we are considered in this Letter. If we consider a Hubble parameter $H(t) = p(t)/t$, this metric remains Lorentzian for $p > 1$. In the example here considered, with a power law expansion (34), the universe initially describes an equation of state with $\omega_{\mathrm{eff}} \simeq -1$, which increases until take values of the order of $\omega_{\mathrm{eff}} \simeq 0.4$. Thereafter, this cosmological parameter begins to decrease to take values which agree very good with the present day experimental data $\omega_{\mathrm{eff}} \simeq -0.7$. This result is consistent with a quintessential expansion. The model predicts that, in the future, the universe will be more and more accelerated to finally describe an effective 4D vacuum dominated expansion $\omega_{\mathrm{eff}} \simeq -1$.

Acknowledgements

M.B. acknowledges CONICET (Argentina) and UNMdP for financial support.

References

[1] P. Wesson, Gen. Relativ. Gravit. 22 (1990) 707;
P.S. Wesson, Phys. Lett. B 276 (1992) 299;
P.S. Wesson, J. Ponce de Leon, J. Math. Phys. 33 (1992) 3883;
P. Wesson, H. Liu, P. Lim, Phys. Lett. B 298 (1993) 69;
H. Liu, P.S. Wesson, J. Math. Phys. 33 (1992) 3888;
M. Bellini, Nucl. Phys. B 660 (2003) 389;
D.S. Ledesma, M. Bellini, Phys. Lett. B 581 (2004) 1;
J.E. Madriz Aguilar, M. Bellini, Phys. Lett. B 596 (2004) 116;
B. Chang, H. Liu, L. Xu, Mod. Phys. Lett. A 20 (2005) 923;
L. Xu, H. Liu, B. Chang, astro-ph/0507397;
C. Romero, F. Dahia, Int. J. Mod. Phys. A 20 (2005) 2237.

[2] A. Davidson, J. Sonnenshein, A.H. Vozmediano, Phys. Rev. D 32 (1985) 1330;
J. Ponce de Leon, Gen. Relativ. Gravit. 20 (1988) 539;
D.J. McManus, J. Math. Phys. 35 (1994) 4889;
G. Abolghasem, A.A. Coley, D.J. McManus, J. Math. Phys. 37 (1996) 361;
H. Liu, P.S. Wesson, Gen. Relativ. Gravit. 30 (1998) 509.

[3] T. Liko, P.S. Wesson, J. Math. Phys. 46 (2005) 062504;
M. Bellini, Phys. Lett. B 609 (2005) 187;
J.E. Madriz Aguilar, M. Bellini, Phys. Lett. B 619 (2005) 208.

[4] J.E. Campbell, A Course of Differential Geometry, Clarendon Press, Oxford, 1926;
L. Margaard, PhD. Tesis, Kiel, 1963;
N.I. Katzourakis, math-ph/0407067;
N.I. Katqourakis, math.DG/0411630;
P.S. Wesson, gr-qc/0507107.

[5] J.M. Overduin, P.S. Wesson, Phys. Rep. 283 (1997) 303;
P.S. Wesson, Space–Time–Matter, World Scientific, Singapore, 1999.

[6] S. Permutter, et al., Astrophys. J. 517 (1999) 565.

[7] C.B. Netterfield, et al., Astrophys. J. 571 (2002) 604.

[8] A. Balbi, et al., Astrophys. J. 545 (2000) L1.

[9] O. Bertolami, Nuovo Cimento B 93 (1986) 36.

[10] M. Bellini, Phys. Lett. B 632 (2006) 610.

[11] S.M. Carroll, Phys. Rev. Lett. 81 (1998) 3067.

[12] D.S. Ledesma, M. Bellini, Phys. Lett. B 581 (2004) 1.

[13] J.E. Madriz Aguilar, M. Bellini, Phys. Lett. B 632 (2006) 6.

[14] Particle Data Group, Review of Particle Physics, Phys. Lett. B 592 (2004) 206–215.