ADDENDUM TO "EMBEDDING THEOREMS FOR INFINITE GROUPS"

BY

N. G. DE BRUIJN

(Communicated at the meeting of September 26, 1964)

Mr. R. Jeurissen (Nijmegen) discovered an error in one of the applications in my paper "Embedding theorems for infinite groups" (Nederl. Akad. Wetensch. Proc. Ser. A. **60** = Indag. Math. **19** (1957), 560–569).

On page 563 two examples were given for the notion of symmetrically generated groups. The second one reads: (ii) Ω is the (restricted) direct product of $\mathbf{n}$ copies of H. Mr. Jeurissen noticed that this example requires that H is abelian, for otherwise the symmetry condition is violated as soon as $\mathbf{n} > 1$.

A consequence is that in the second part of theorem 1.1 (the part about direct products) the restriction should be made that H is abelian.

The proof of theorem 4.1 is not valid, since it depends on the case that H is non-abelian. The author does not know whether the theorem remains correct, so the question whether the (restricted) direct product of $2^{\mathbf{m}}$ groups $\sum_{\mathbf{m}}$ can be embedded into $\sum_{\mathbf{m}}$ itself ($\mathbf{m}$ infinite), remains open.

Theorem 4.1 was used in the proof of theorem 4.3, but it is not difficult to replace the proof of theorem 4.3 by a correct one:

Theorem 4.3. *If $\mathbf{m}$ is an infinite cardinal number, then every abelian group of order $2^{\mathbf{m}}$ can be embedded into $\sum_{\mathbf{m}}$.*

Proof. Every abelian group can be embedded into a complete abelian group, and (assuming the axiom of choice) every complete abelian group is a direct product of groups of the type R, 2^∞, 3^∞, 5^∞, ... Here R is the rational group, and 2^∞, 3^∞, ... are the so-called quasicyclic groups (see A. G. Kurosh, The Theory of Groups, New York 1956, pp. 167 and 165). Let H be the direct product of R, 2^∞, 3^∞, 5^∞, ..., then every abelian group G can be embedded into a direct product of copies of H. If G has order $2^{\mathbf{m}}$ then we do not need more then $2^{\mathbf{m}}$ copies of H.

Since H is abelian, the direct product of $2^{\mathbf{m}}$ copies of H is symmetrically generated by these groups. Moreover, for any finite number k, the direct product of k copies of H can be embedded into $\sum_{\mathbf{m}}$ (since H^k is countable, and $\mathbf{m}$ is infinite, such an embedding is produced by the Cayley representation). Now applying theorem 3.1 we obtain that the direct product of $2^{\mathbf{m}}$ copies of H can be embedded into $(\sum_{\mathbf{m}})^{\mathbf{m}}$. The latter group can be considered as an intransitive permutation group on $\mathbf{m}^2$ objects, having $\mathbf{m}$ transitivity sets of $\mathbf{m}$ elements each. Thus $(\sum_{\mathbf{m}})^{\mathbf{m}}$ can be embedded into $\sum_{\mathbf{m}^2}$. As $\mathbf{m}^2=\mathbf{m}$, we finally obtain that G can be embedded into $\sum_{\mathbf{m}}$.

In connection with these corrections, the following changes should be made in the paper:

Page 561. Omit the first two lines, and replace them by: This result is an application of the following general theorem.

Page 561. Line 7 from top. Replace "If, for every finite k" by "If H is abelian, and if, for every finite k".

Page 563. Line 11 from top. After "$\mathbf{n}$ copies of H" add "if H is abelian".

Page 565. Delete Theorem 4.1 and its proof.

Page 566. Line 17 from top (in the Corollary): Omit the words "their direct product and".

Page 566. Replace the proof of theorem 4.3 by the one given above.

Technological University
Eindhoven, Netherlands