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Does the NJL chiral phase transition affect the elliptic flow of a fluid at fixed η/s ? S. Plumari^{a,b}, V. Baran^{b,d}, M. Di Toro^{a,c}, G. Ferini^{a,b,c,d}, V. Greco^{a,c,*}

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ABSTRACT

We have derived and solved numerically the Boltzmann–Vlasov transport equations that includes both two-body collisions and the chiral phase transition by mean of NJL-field dynamics. The scope is to understand if the field dynamics supply new genuine effects on the build-up of the elliptic flow v_2 , a measure of the asymmetry in the momentum space, and in particular if it can affect the relation between v_2 and the shear viscosity to entropy ratio η/s . Solving the transport equation with a constant cross section for the condition of Au + Au collisions at $\sqrt{s_{NN}} = 200 \text{ A GeV}$ it is shown a sizable suppression of v_2 due to the attractive nature of the field dynamics that generates the constituent mass. However the key result is that if η/s of the system is kept fixed by an appropriate local renormalization of the cross section the v_2 does not depend on the details of the collisional and/or field dynamics and in particular it is not affected significantly by the chiral phase transition.

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1. Introduction

The ultra-relativistic heavy-ion collisions at high energy $\sqrt{s_{NN}} \sim 200$ *A* GeV represent the main tool to study the formation and the properties of the quark–gluon plasma at high temperature. The RHIC program at BNL has shown that the azimuthal asymmetry in momentum space, namely the elliptic flow v_2 , is the largest ever seen in HIC suggesting that an almost perfect fluid with a very small shear viscosity to entropy density ratio, η/s , has been created [1–3]. From simple quantum mechanical considerations [4] as well as from the study of supersymmetric Yang–Mills theory in the infinite coupling limit a lower bound for η/s of about $\sim 10^{-1}$ is predicted [5]. Such a value is much lower than any other known fluid and in particular smaller than the one of water and even than the superfluid He [6].

First developments of relativistic viscous hydrodynamics [7,8] as well as parton cascade models [9–11] indicate that even a small $\eta/s \sim 0.1$ –0.2 affects significantly the strength of $v_2(p_T)$ especially at $p_T > 1$ GeV. Therefore it has become mandatory to determine the value of η/s of the plasma created at RHIC through the study of the relation between η/s and v_2 [12]. However viscous corrections to ideal hydrodynamics are indeed large and a simple relativistic extension of first order Navier–Stokes equations is af-

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fected by causality and stability pathologies [13,14]. It is therefore necessary to go to second order gradient expansion, and in particular the Israel-Stewart theory has been implemented to simulate the RHIC collisions providing an upper bound for $\eta/s \leq 0.4$ [15]. Such an approach, apart from the limitation to (2 + 1)D simulations, has the more fundamental problem that it is based on a gradient expansion at second order that is not complete [13]. Furthermore it cannot be sufficient to describe correctly the dynamics of a fluid with large η/s as the one in the cross-over region and/or hadronic phase which at least at RHIC still gives a non-negligible contribution to v_2 [16] that affects the determination of the η/s itself [17].

A relativistic transport approach has the advantage to be a (3 + 1)D approach not based on a gradient expansion that is valid also for large viscosity and for out of equilibrium momentum distribution allowing a reliable description also of the intermediate p_T range where the important properties of quark number scaling (QNS) of $v_2(p_T)$ have been observed [18]. In this p_T region viscous hydrodynamics breaks its validity because the relative deviation of the equilibrium distribution function $\delta f / f_{eq}$ increases with p_T^2 becoming large already at $p_T \ge 3T \sim 1$ GeV [19].

In this perspective transport approaches at cascade level have already been developed [9–11,20], but they miss any effect of the field interactions responsible for the chiral phase transition or confinement. With this Letter we go one step further including a transport equation self-consistently derived from the Nambu–Jona-Lasinio (NJL) Lagrangian. This allows to study microscopically the transport behavior of a fluid that includes the chiral phase tran-

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sition looking at its impact on the relation between the v_2 and the η/s of the system. The choice of the NJL is mainly driven by its wide and renowned application to study the QCD chiral phase transition by mean of effective Lagrangians, even though the thermodynamical properties of QCD can be reproduced only qualitatively as briefly discussed in the following.

The NJL Lagrangian is:

$$\mathcal{L}_{NJL} = \bar{\psi} \left(i \gamma^{\mu} \partial_{\mu} - \hat{m} \right) \psi + g \left[(\bar{\psi} \psi)^2 + \sum_{\alpha=1}^{N_f^2 - 1} \left(\bar{\psi} \tau^{\alpha} i \gamma_5 \psi \right)^2 \right]$$
(1)

with ψ denoting a quark fields with N_f flavors $\psi = (u, d, ...)^t$, τ^{α} are the generators of the $SU(N_f)$ group acting in flavor space with $\alpha = 1, ..., (N_f^2 - 1)$. The $\hat{m} = \text{diag}(m_u, m_d, ...)$ is the current $N_f \times N_f$ quark mass matrix in flavor space. In the following we will refer to the $N_f = 2, N_c = 3$ for calculations. As is well known the theory is non-renormalizable, hence a cut-off Λ has to be introduced as a free parameter. The numerical results shown in the following are derived using the Buballa parametrization: $\Lambda = 588$ MeV, $g\Lambda^2 = 2.88, m = 5.6$ MeV [21] that among the variety of parameterizations entails a behavior of ϵ , P, c_s^2 closer to the IQCD results.

A transport theory for the NJL model has been derived in the closed-time-path formalism combined with the effective action method [22]. The main steps of derivation are to perform a Wigner transformation of the Dirac equation of motion and of the related gap-equation associated to the \mathcal{L}_{NJL} , Eq. (1). Then one exploits the semi-classical approximation widely used for applications in heavy-ion collisions [23,24] evaluating the expectation value of the four-point fermion interaction in the Hartree approximation (i.e. at mean field level). Finally only the scalar and vector components of the Wigner function are retained thanks to the spin saturated nature of the systems we are interested in. One finally obtains the Boltzmann–Vlasov transport equations for the (anti-)quark phase-space distribution function f^{\pm} :

$$p^{\mu}\partial_{\mu}f^{\pm}(x,p) + M(x)\partial_{\mu}M(x)\partial_{p}^{\mu}f^{\pm}(x,p) = \mathcal{C}(x,p)$$
(2)

where C(x, p) is the Boltzmann-like collision integral, main ingredient of the several cascade codes already developed [25,26,11]; M(x) represents the local value of the scalar mass that is generated by the chiral symmetry breaking, see Eq. (3). We notice that respect to the already implemented cascades the NJL dynamics introduces a new term associated to the mass generation. Also Eq. (2) is formally the same as the widely used relativistic transport approaches for hadronic matter like RBUU, uRQMD, RLV [24, 27,28], but with a vanishing vector field. However a key difference is that particles do not have a fixed mass and a self-consistent derivation couples Eq. (2) to the mass gap equation of the NJL model that extended to the case of non-equilibrium can be written as:

$$\frac{M(x) - m}{4gN_c} = M(x) \int \frac{d^3p}{(2\pi)^3} \frac{1 - f^-(x, p) - f^+(x, p)}{E_p(x)}$$
(3)

and determines the local mass M(x) at the space–time point x in terms of the distribution functions $f^{\pm}(x, p)$.

Eqs. (2) and (3) form a closed system of equations constituting the Boltzmann–Vlasov equation associated to the NJL Lagrangian that allows to obtain self-consistently the local effective mass M(x)affecting the time evolution of the distribution function $f^{\pm}(x, p)$. A seminal work on the transport equation associated to the NJL dynamics was done in Refs. [29,30], but without a collision term, not at finite η/s and never applied to the physical conditions of ultra-relativistic heavy-ion collisions. For the numerical solutions of Eqs. (2) and (3) we use a three-dimensional lattice that discretize the space as described in Ref. [11,26]. The standard test particle methods that sample the distribution function f by mean of an ensemble of points in the phase-space is employed. The normalization condition is given by: $\int d\Gamma f^{\pm} = \omega A(\tilde{A}) = N_q(N_{\tilde{q}})$ with Γ the phase space, A_{α} (\tilde{A}_{α}) the number of test particles (antiparticles) which are inside the considered cell and ω the proper normalization factor that relates the test particles to the real particle number.

In such a way it is possible to get a solution of the transport equations propagating the momenta of the test (anti-)particles by mean of the relativistic Hamilton's equation. For the numerical implementation they can be written in the discretized form as:

$$\boldsymbol{p}_{i}(t^{+}) = \boldsymbol{p}_{i}(t^{-}) - 2\delta t \frac{M_{\alpha}(\boldsymbol{r}_{i}, t)}{E_{i}(t)} \vec{\nabla}_{r} M_{\alpha}(\boldsymbol{r}_{i}, t) + coll.$$
$$\boldsymbol{r}_{i}(t^{+}) = \boldsymbol{r}_{i}(t^{-}) + 2\delta t \frac{\boldsymbol{p}_{i}(t)}{E_{i}(t)}$$
(4)

with $t^{\pm} = t \pm \delta t$ and δt the numerical mesh time. The term *coll*. on the right-hand side of Eq. (4) indicates the effects of the collision integral as described in Ref. [26,11]. By mean of a reiterating procedure on time steps one gets the solutions of the transport equation coupling Eq. (4) with the gap equation, Eq. (3) that discretized on lattice and for point-like test particles becomes:

$$\frac{M_{\alpha} - m}{8gN_c} = M_{\alpha} \left[\int_{\Lambda} \frac{d^3p}{(2\pi)^3} \frac{1}{E} - \frac{\omega}{\Delta V_{\alpha}} \left(\sum_{i=1}^{A_{\alpha}} \frac{1}{E_i} - \sum_{i=1}^{\tilde{A}_{\alpha}} \frac{1}{\tilde{E}_i} \right) \right]$$
(5)

where $\Delta V_{\alpha} = \tau A_T \tanh \eta_{\alpha}$ is the volume of each cell of the space lattice given by $A_T = 0.5 \text{ fm}^2$ the area in the transverse direction and η_{α} the space-time rapidity of the center of the cell. The integral is instead the vacuum contribution to the gap-equation which is a divergent quantity and it is regularized by a cutoff, Λ and has a simple analytical expression.

The space-time dependence of the mass $M_{\alpha}(r,t) = m - 2g\langle \bar{\psi}\psi \rangle$ influences the momenta of the particles because the finite gradient of the condensate generates a force which changes the momentum of a particle proportionally to $\vec{\nabla}_r \langle \bar{\psi}\psi \rangle$, see Eq. (4). The last is negative because the phase transition occurs earlier in the surface of the expanding QGP fireball. Therefore the phase transition which take place locally results in a negative contribution to the particle momenta that makes the system more sticky respect to a free massless gas.

2. Shear viscosity to entropy density ratio

The effect of the NJL mean-field can be evaluated looking at the so-called interaction measure normalized by T^4 , $\frac{\epsilon-3P}{T^4}$, that gives the deviation from the free gas relation between the energy density ϵ and the pressure *P* that is also a measure of the breakdown of conformal symmetry. In lattice QCD it is known to be quite large with a peak at T_c and a non-negligible value up to $T \sim 2-3T_c$. In Fig. 1(left) its behavior is shown for three different NJL parameterizations. Quantitatively the QCD behavior is correctly reproduced. In the following we will use the Buballa parametrization that gives the largest values closer to IQCD where however the peak is found to be about a factor two larger at T_c . This means that the numerical results on the impact of the mean field in the dynamics of RHIC collisions is reduced respect to a more realistic case.

Our final goal is to study a fluid at fixed shear viscosity to entropy ratio η/s extending the study started at cascade level [11,10]. There the strategy was to normalize locally the cross section in order to fix the η/s according to the simple relation $\sigma \cdot \eta/s = \langle p \rangle / 15n$ valid for a massless gas. Here because of the NJL field

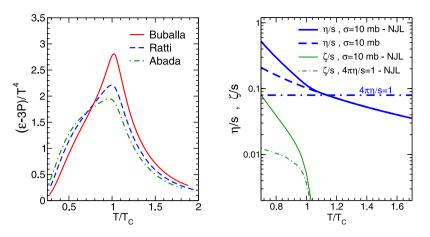


Fig. 1. Left: Interaction measure shown as a function of temperature *T* for three different NJL parameter sets. Right: The η/s and ζ/s for the Buballa parametrization as a function of *T* for various cases as indicated in the legend. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this Letter.)

the particles acquire a mass hence both the viscosity and the entropy density are modified respect to the simple massless case. We briefly discuss the η and s for a system of massive particles deriving the pertinent formula to renormalize the cross section σ in order to keep fixed locally the η/s . Both η and s have been derived for a thermodynamical system and has been studied also for the case of the NJL model [31]. Here we derive expressions in terms of quantities that can be used easily also in the numerical solution of the transport equations. A widely used formula for η is deduced from the relaxation time approximation, like in Ref. [31]. After integration by parts it is possible to write the shear viscosity for the general case of massive relativistic particles in terms of average quantities that can be easily evaluated numerically:

$$\eta = \tau \frac{n}{15} \left[4 \left\langle \frac{p^2}{E} \right\rangle + M^2 \left\langle \frac{p^2}{E^3} \right\rangle \right] \tag{6}$$

where $\tau = [n \langle \sigma_{tr} v_{rel} \rangle]^{-1}$ is the relaxation time, i.e. the time interval between two collisions, and *n* is the total local density. One can easily see that in the ultra-relativistic limit ($M \rightarrow 0$) the well-known formula for the shear viscosity, $\eta = \frac{4}{15} \langle p \rangle / \sigma_{tr}$, is recovered. The entropy density cannot be related simply to the local den-

The entropy density cannot be related simply to the local density, $s \sim 4n$, as for the massless case. A suitable method to evaluate locally *s* during the dynamical evolution of the collision is based on the use of the thermodynamical relation $sT = \epsilon + nT$ that shift the problem to the evaluation of ϵ , *n*, *T*. These are easily calculated analytically (as for Fig. 1) but also numerically summing up the number of the test-particles and their energy in each α -cell. To evaluate the temperature we exploit the general formula for a massive gas:

$$\frac{e}{M} = \frac{3}{z} + \frac{K_1(z)}{K_2(z)}$$
(7)

with *e* the energy per particle, ϵ/n , and $K_n(z)$ the modified Bessel functions of the second kind, $z = \frac{M}{T}$. We know *e* and *M* directly from the code hence we can use Eq. (7) to extract $z = \frac{M}{T}$ and therefore the temperature. We have checked that the procedure works well performing calculation in a box with particles distributed according to a Boltzmann equilibrium distribution.

The behavior of η/s for a thermodynamical system is shown in Fig. 1(right) for a massless free gas (dashed line) and for the NJL (solid line). In both cases the cross section is fixed to $\sigma = 10$ mb. The dot-dashed line indicates the lower bound for η/s . We see that with a constant cross section $\sigma = 10$ mb the η/s is even lower than the $1/4\pi$ at high T. The lower light lines show the

behavior of bulk viscosity to entropy ratio ζ/s in the NJL model for two cases one with $\sigma = 10$ mb (green solid line) as above and the other for an η/s fixed at the $1/4\pi$ value (dot-dashed line). Of course for the massless case the bulk viscosity is zero, while for non-vanishing masses there is a link between the η and ζ through the relaxation time τ . The last results in a smaller growth of ζ/s when η/s is fixed respect to the case when σ_{tr} is fixed. However in both cases we can see that only at $T < 1.1T_c$ we have a nonvanishing ζ/s . This is due to the fact that the ζ is expected to be proportional to the deviation of the sound velocity from 1/3, $(c_s^2 - 1/3)^2$ that in the NJL model is known to occur only very close to T_c . More importantly for our purposes is that in the NJL the ζ/s remains order of magnitudes lower that first extrapolation from IQCD [32] and also much lower than the smaller values used for first studies with viscous hydrodynamics [19,33]. According to this study ζ/s has a small impact on elliptic flow [19] even if already moderate values, $\zeta / s \sim 0.04$, can better reproduce the fine structure of $v_2(p_T)$ mass-ordering [33]. However for NIL especially at $4\pi \eta/s = 1$ the value is so small that we can judge safe to discard any role of ζ/s in the following results for the elliptic flow considering also that we will not discuss the massordering.

We notice that Eqs. (6) and (7) supply the formula for the normalization of the cross section in each α -cell in order to keep fixed η/s of the system:

$$\sigma_{tr,\alpha} = \frac{1}{15} \frac{T_{\alpha}}{\langle v_{rel} \rangle} \frac{4 \langle p^2 / E \rangle_{\alpha} + M_{\alpha}^2 \langle p^2 / E^3 \rangle_{\alpha}}{(\epsilon_{\alpha} + n_{\alpha} T_{\alpha}) \eta / s}$$
(8)

for a massless gas $M \to 0$ (p = E, $\epsilon \sim 3nT$) and Eq. (8) reduce to the simple relation $\sigma_{tr,\alpha} = \frac{4\pi}{15} \langle p \rangle_{\alpha} / n_{\alpha}$ used in Refs. [11,34,10,14] for $\eta/s = 1/4\pi$. Eq. (8) will allow to extend such studies of a fluid at finite viscosity to the case of partons with finite mass.

3. Elliptic flow

We have run the simulations for Au + Au at $\sqrt{s_{NN}} = 200 \text{ A GeV}$ and b = 7 fm. The density distribution in coordinate space is given by the standard Glauber model. The maximum initial temperature is T = 340 MeV and the initial time is $\tau_0 = 0.6 \text{ fm}/c$ as usually done also in hydrodynamical calculations. We follow the dynamical evolution of quarks, anti-quarks and gluons. The last has been included, even if they are not explicitly present in the NJL model, with the aim of using a realistic density for both the total and the (anti-)quark density in the simulation of the collisions. However gluons do not actively participate in the evaluation of the

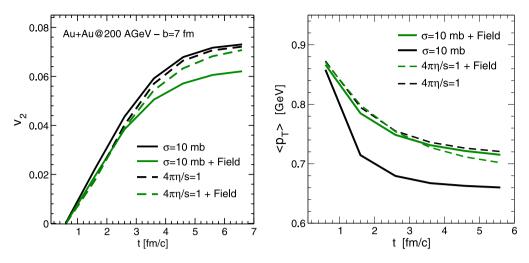


Fig. 2. Left: Average elliptic flow as a function of time for Au + Au collisions in the mid-rapidity region |y| < 1 at b = 7 fm. Right: time evolution of the average transverse momentum for the same case as in the left panel. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this Letter.)

chiral phase transition, but they simply acquire the mass of the quarks not contributing to its determination according to the NJL model. The justification for this choice relies on the quasi-particle models that are fitted to lQCD thermodynamics [35,36]. One finds a similar behavior of M(T) for both gluons and quarks approximately. Of course for a more quantitative calculation a more careful treatment would be needed but it is not relevant to the main objective of the present seminal work, considering also that anyway the NJL model cannot be used for an accurate quantitative study.

In Fig. 2(left) it is shown the time evolution of the average elliptic flow $\langle v_2 \rangle$ for a constant transport cross section of $\sigma_{tr} = 10$ mb a typical value that is able to reproduce the amount of v_2 observed in experiments [20]. Comparing the two solid lines (black and green) we can see that the NJL mean field cause a decrease of $\langle v_2 \rangle$ of about 15%. The reduction of $\langle v_2 \rangle$ can be expected considering that the NJL field produce a scalar attractive field that at the phase transition results in a gas of massive particles. In Fig. 3, we show also the elliptic flow at freeze-out as a function of the transverse momentum p_T . One can see that the role of the mean field even increases with momentum affecting also particles at a p_T quite larger than the energy scale of the scalar condensate $\langle \psi \bar{\psi} \rangle \sim 300$ MeV. This is due to the fact that a high p_T particle collides mainly with the much more abundant particles in the bulk. These have an average momentum comparable to the strength of the scalar field: $p_T \sim 2T \sim M_c$. Therefore the effect of the scalar field extends thanks to collisions into a range quite larger than one would naively think and the interplay between collisions and mean field is fundamental. We find that the presence of an NJL-field that drives the chiral phase transition suppress the $v_2(p_T)$ by about 20% at $p_T > 1$ GeV. This would imply the need of a parton scattering cross section σ_{tr} even larger than that estimated with the cascade model which was already quite larger than the pQCD estimates [25,37]. On the other hand the mean field modifies both the local entropy density reduced by the mass generation, and the shear viscosity that increases as shown in Fig. 1(right). Therefore even if the cross section is the same with and without the NJL field the system evolves with a different η/s . Considering that one of the main goal is to determine the η/s of the QGP we have investigated what is the action of the mean field once the η/s of the system is fixed to be the same by mean of the cross section renormalization according to Eq. (8). Therefore we have run simulations with and without the NIL-field but keeping constant locally the η/s .

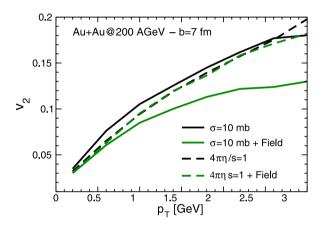


Fig. 3. As in Fig. 2 for the elliptic flow as a function of p_T . (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this Letter.)

The results for $4\pi \eta/s = 1$ are shown by dashed lines in Figs. 2 and 3, the (black) dashed line is the case with only the collision term (cascade) while the (green) dashed line is the case with the field. We can see that once the η/s is fixed there is essentially no difference in the calculations with and without a field dynamics included. This is seen for both the average $\langle v_2 \rangle$ and the $v_2(p_T)$ in Figs. 2(left) and 3 respectively. This is a key result that shows that even in a microscopic approach that distinguishes between the mean field and the collisional dynamics the $v_2(p_T)$ is mainly driven by the n/s of the fluid. In other words we have found that in a microscopic approach the η/s is the pertinent parameter and the language of viscous hydrodynamics is appropriate. Of course this does not mean that $v_2(p_T)$ in the transport theory is the same of the viscous hydrodynamics one, but that, and even more importantly, the direct relation between $v_2(p_T)$ and η/s is quite general and their relation is not modified by the NJL field dynamics. We have checked that this is valid also at other impact parameter (b = 3, 5, 9 fm) and for larger η/s up to π^{-1} . This is of course very important for the determination of η/s by mean of the data on elliptic flow and confirms the validity of the studies pursued till now even if they miss an explicit mean field dynamics and/or the chiral phase transition.

However we notice that while the $v_2(p_T)$ appears to be totally independent on the presence of the NJL-field once the η/s is kept fixed, the time evolution of $\langle v_2 \rangle$ still shows a slightly reduced elliptic flow at $t \sim 3-5$ fm/c. A similar difference can be observed also in the time evolution of the transverse momentum $\langle p_T \rangle$ shown in Fig. 2(right). One may ask what is the physical origin of such differences. In principle there are two parameters affecting the $v_2(p_T)$ and the $\langle p_T \rangle$: the sound velocity c_s and the bulk viscosity ζ . As discussed previously it is safe to discard the possibility of any significant influence of the finite ζ/s on our results considering its tinv value in the NIL see Fig. 1(right). It is instead reasonable that the weak decrease is due to the decrease of the sound velocity for NJL at $T < 1.1T_c$. It is well known that c_s^2 decreases from 1/3 that is the value of a massless free gas and that this cause already in ideal hydrodynamics a decrease of the elliptic flow [38]. On the other hand when the bulk of the system reaches this region most of the $v_2(p_T)$ has already been built-up hence the effect of a moderate decrease of c_s is quite weak and could explain the small difference still visible in the time evolution of $\langle v_2 \rangle$ and $\langle p_T \rangle$.

4. Conclusion

The novelty of the present work is to be the first study within a transport approach of a fluid at finite n/s that includes the field dynamics of the chiral phase transition. Generally we find that at fixed cross section the effect of the NJL field is to reduce the elliptic flow by about a 20%. More importantly we can state that the presence of the NJL dynamics does not change the relation between the elliptic flow and the η/s that remains the same as in the cascade models and at low p_T is very close to the one from hydrodynamics [9,11,10,34]. If such a finding is confirmed also for a more general class of interacting quasi-particle models it will make much safer and solid the determination of η/s by $v_2(p_T)$. In fact as we have shown the relation is independent on the microscopic details of the interaction once the EoS and/or the $c_s^2(T)$ has been fixed. This will be investigated in the next future, in fact the kinetic theory and the numerical implementation presented here can be easily extended to quasi-particle models that are fitted to reproduce the energy density and pressure of IQCD results. In such a case it will be possible also to study the elliptic flow with a realistic behavior of $c_s(T)$ and the effect of a finite and sizable ζ/s on the elliptic flow complementing the study from viscous hydrodynamics that are subject to problems for not too small ζ/s and/or for $p_T > 3T$ [19].

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