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# Comparison of approaches to quantum correction of black hole thermodynamics

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## Abstract

There are several approaches to quantum gravitational corrections of black hole thermodynamics. String theory and loop quantum gravity, by direct analysis on the basis of quantum properties of black holes, show that in the entropy–area relation the leading order correction should be of log-area type. On the other hand, generalized uncertainty principle (GUP) and modified dispersion relations (MDRs) provide perturbational framework for such modifications. Although both GUP and MDRs are common features of all quantum gravity scenarios, their functional forms are quantum gravity model dependent. Since both string theory and loop quantum gravity give more reliable solution of the black hole thermodynamics, one can use their results to test approximate results of GUP and MDRs. In this Letter, we find quantum corrected black hole thermodynamics in the framework of GUP and MDR and then we compare our results with string theory solutions. This comparison suggests severe constraints on the functional form of GUP and MDRs. These constraints may reflect characteristic features of ultimate quantum gravity theory.

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## 1. Motivation

A common feature of all promising candidates for quantum gravity is existence of minimal observable length, which is on the order of Planck length [1–5]. There are several approaches to incorporate this finite resolution of spacetime with theoretical framework of standard model. GUP and MDRs are two of these approaches. In fact, GUP and MDRs are common features of all candidates for quantum gravity. In particular, in the study of loop quantum gravity and of models based on non-commutative geometry, there has been strong interest in some candidate modifications of the energy–momentum dispersion relations [6–10]. On the other hand, generalized uncertainty principles have been considered primarily in the literature on string theory and on models based on noncommutative geometry [1–5]. Possible relations between GUP and MDRs have been

studied recently [11]. It is natural to expect that GUP and MDRs affect black hole thermodynamics, since black hole structure is an example of extreme quantum gravity regime. Any constraint imposed on the form of GUP and MDRs in study of black hole physics, will help us to find more accurate form of ultimate quantum gravity scenario. Black hole thermodynamics in the framework of GUP and MDRs has been studied by several authors [12–23]. Recently, Amelino-Camelia et al. have studied this issue with details [9,10]. They have argued that for consistency between string theory results and the results of MDRs, the term proportional to first order of Planck length in MDRs should not be present. Here we are going to proceed further in this direction. We will show that comparison between results of string theory and MDRs, suggests that all terms proportional to odd power of energy should not be present in MDRs. On the other hand, comparison between results of string theory and GUP suggests that in GUP even power of  $\delta x$  should not be present. These two important results restrict the form of MDRs and GUP considerably. Naturally, these restrictions may show some characteristic features of underlying quantum gravity the-

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ory. In addition, our comparison between results of GUP and MDR shows that these to features of quantum gravity are not different considerably and they would be equivalent in ultimate quantum gravity theory.

In which follows we set  $\hbar = c = G = 1$ .

## 2. Preliminaries

In this section we provide some preliminaries for rest of the Letter.

### 2.1. MDR

A modified dispersion relation (MDR) can be written as [9]

$$(\vec{p})^2 = f(E, m; L_P) \simeq E^2 - \mu^2 + \alpha_1 L_P E^3 + \alpha_2 L_P^2 E^4 + O(L_P^3 E^5), \quad (1)$$

where  $f$  is the function that gives the exact dispersion relation, and on the right-hand side we have assumed the applicability of a Taylor-series expansion for  $E \ll \frac{1}{L_P}$ . The coefficients  $\alpha_i$  can take different values in different quantum-gravity proposals. Note that  $m$  is the rest energy of the particle and the mass parameter  $\mu$  on the right-hand side is directly related to the rest energy, but  $\mu \neq m$  if the  $\alpha_i$  do not all vanish.

### 2.2. GUP

A generalized uncertainty principle (GUP), can be written as follows [9]

$$\delta x \geq \frac{1}{\delta p} + \alpha l_P^2 \delta p + O(l_P^3 \delta p^2) \quad (2)$$

which has been derived within the string theory approach to the quantum-gravity problem and several alternative scenarios. This GUP is such that at small  $\delta p$  one finds the standard dependence of  $\delta x$  on  $\delta p$  ( $\delta x$  gets smaller as  $\delta p$  increases) but for large  $\delta p$  the Planckian corrections term becomes significant and keeps  $\delta x \geq L_P$ . Within string theory, the coefficient  $\alpha$  should take a value of roughly the ratio between the square of the string length and the square of the Planck length, but this of course might work out differently in other quantum-gravity proposals.

### 2.3. String theory results for black hole thermodynamics

Bekenstein–Hawking formalism of black hole thermodynamics should be modified to incorporate quantum gravitational effects. Both GUP and MDRs provide a perturbational framework for these modifications [12–23]. On the other hand, loop quantum gravity and string theory give reliable entropy–area relation of the black holes (for  $A \gg L_P^2$ ),

$$S = \frac{A}{4L_P^2} + \rho \ln \frac{A}{L_P^2} + O\left(\frac{L_P^2}{A}\right), \quad (3)$$

where  $\rho$  might take different values in string theory and in loop quantum gravity [9,10,24]. If we use the relation

$$S = \frac{A}{4L_P^2} + \rho \ln \frac{A}{L_P^2} + \beta \frac{L_P^2}{A}, \quad (4)$$

we can derive the mass–temperature relation of the black holes as,

$$T = \frac{L_P^2}{8\pi M} \left( 1 - \rho \frac{L_P^2}{4\pi M^2} + \frac{L_P^4}{(4\pi)^2 M^4} \left( \rho^2 + \frac{\beta}{4} \right) \right). \quad (5)$$

Now the question arises: are the entropies calculated within GUP and MDR consistent with the string theory results? To answer this question, first we should calculate entropies within GUP and MDR. In this approach we will use the fact that when a quantum particle with energy  $E$  and size  $l$  is absorbed into a black-hole and  $l \sim \delta x$ , the minimum increase of area of black-hole will be

$$\Delta A \geq 4(\ln 2) L_P^2 E \delta x \quad (6)$$

and the minimum increase of entropy is  $\ln 2$ , which can be interpreted as one bit of information [25,26].

## 3. GUP and black hole thermodynamics

Consider the following GUP

$$\delta p \geq \frac{1}{\delta x} (1 + \alpha L_P^2 \delta p^2). \quad (7)$$

This relation can be written as

$$\delta p \geq \frac{1}{\delta x} \left[ 1 + \frac{\alpha L_P^2}{\delta x^2} (1 + \alpha L_P^2 \delta p^2)^2 \right]. \quad (8)$$

Considering only lowest order terms in the power of  $L_P$ , we find

$$\delta p \geq \frac{1}{\delta x} \left( 1 + \frac{\alpha L_P^2}{\delta x^2} \right). \quad (9)$$

Using standard dispersion relation  $p = E$ , we find

$$\delta E \geq \frac{1}{\delta x} \left( 1 + \frac{\alpha L_P^2}{\delta x^2} \right). \quad (10)$$

Generally this relation can be written as

$$E \geq \frac{1}{\delta x} + \frac{\alpha L_P^2}{\delta x^3} + O\left(\frac{L_P^3}{(\delta x)^4}\right). \quad (11)$$

In their analysis, Amelino-Camelia et al. have used this relation with only two first terms of the right-hand side [9,10]. Here we consider more terms to explore their effects on the black hole entropy. When we compare our results with the standard results of string theory, our comparison will suggest severe constraints on the general form of GUP.

Consider the following generalization

$$E \geq \frac{1}{\delta x} + \frac{\alpha L_P^2}{\delta x^3} + \frac{\alpha' L_P^3}{\delta x^4} + \frac{\alpha'' L_P^4}{\delta x^5} + \frac{\alpha''' L_P^5}{\delta x^6}, \quad (12)$$

which leads to

$$E \delta x \geq 1 + \frac{\alpha L_P^2}{\delta x^2} + \frac{\alpha' L_P^3}{\delta x^3} + \frac{\alpha'' L_P^4}{\delta x^4} + \frac{\alpha''' L_P^5}{\delta x^5}. \quad (13)$$

Substituting the minimum value of  $E \delta x$  in (6), we find

$$\Delta A \geq 4(\ln 2) L_P^2 \left[ 1 + \frac{\alpha L_P^2}{\delta x^2} + \frac{\alpha' L_P^3}{\delta x^3} + \frac{\alpha'' L_P^4}{\delta x^4} + \frac{\alpha''' L_P^5}{\delta x^5} \right]. \quad (14)$$

This relation can be written approximately as

$$\frac{dS}{dA} \approx \frac{\Delta S(\min)}{\Delta A(\min)} \approx \frac{\ln 2}{4(\ln 2)L_p^2 \left(1 + \frac{\alpha L_p^2}{\delta x^2} + \frac{\alpha' L_p^3}{\delta x^3} + \frac{\alpha'' L_p^4}{\delta x^4} + \frac{\alpha''' L_p^5}{\delta x^5}\right)}, \quad (15)$$

which leads to

$$\frac{dS}{dA} \approx \frac{1}{4L_p^2} \left[ 1 - \alpha L_p^2 \frac{1}{\delta x^2} - \alpha' L_p^3 \frac{1}{\delta x^3} + (\alpha^2 - \alpha'') \frac{L_p^4}{\delta x^4} + (2\alpha\alpha' - \alpha''') \frac{L_p^5}{\delta x^5} \right], \quad (16)$$

where we have neglected terms with order higher than  $O(\frac{L_p^5}{\delta x^5})$ . Using  $A = 4\pi R_s^2 \simeq 4\pi \delta x^2$  where  $R_s$  is radius of black hole event horizon (here we have assumed that in falling in the black hole, the particle acquires position uncertainty  $\delta x \sim R_s$  [25,26]), we can integrate to find

$$S \simeq \frac{A}{4L_p^2} - \pi\alpha \ln \frac{A}{L_p^2} + 4\pi^{\frac{3}{2}} \alpha' L_p A^{-\frac{1}{2}} - (\alpha^2 - \alpha'') L_p^2 4\pi^2 A^{-1} - \frac{16}{3} L_p^3 \pi^{\frac{5}{2}} (2\alpha\alpha' - \alpha''') A^{-\frac{3}{2}}. \quad (17)$$

Assuming that string theory result (4), is correct, we should conclude that  $\alpha' = \alpha''' = 0$ . This means that in GUP (12), all terms with even power of  $\frac{1}{\delta x}$  should be omitted. That is, only even power of Planck length could appear in GUP. Therefore, within GUP, black hole entropy is given by

$$S \simeq \frac{A}{4L_p^2} - \pi\alpha \ln \frac{A}{L_p^2} - 4\pi^2 (\alpha^2 - \alpha'') \frac{L_p^2}{A}. \quad (18)$$

Comparing this result with (4) suggests that  $\rho = -\pi\alpha$  and  $\beta = -4\pi^2(\alpha^2 - \alpha'')$ . Since according to string theory,  $\rho$  and  $\beta$  are given, then  $\alpha$  and  $\alpha''$  are determined and our GUP is well established. Using the familiar relation between black hole area and mass  $A = 16\pi M^2$  and the first law of black hole thermodynamics,  $dS = \frac{dM}{T}$ , we can easily obtain the temperature of the black hole

$$T \simeq \frac{L_p^2}{8\pi M} \left[ 1 + \frac{\alpha L_p^2}{4M^2} + \frac{\alpha' L_p^3}{8M^3} + \frac{\alpha'' L_p^4}{16M^4} + \frac{\alpha''' L_p^5}{32M^5} \right]. \quad (19)$$

Comparison between our result and string theory result (5), shows that the coefficients of even powers of  $\frac{1}{M}$  which are not present in string theory result, should be vanishing. This leads us to  $\alpha' = \alpha''' = 0$  once again. Therefore, our comparison restricts the form of GUP to having only even power of  $L_p$ , that is

$$E \geq \frac{1}{\delta x} + \frac{\alpha L_p^2}{\delta x^3} + \frac{\alpha'' L_p^4}{\delta x^5} + \frac{\alpha^{(4)} L_p^6}{\delta x^7} + \dots \quad (20)$$

Since GUP is a model independent concept, any constraint on the form of GUP (such as our finding) can be attributed to the nature of ultimate quantum gravity theory. In other words, constraints imposed on the form of GUP will help us to find deeper insight to the nature of underlying quantum gravity theory.

#### 4. MDR and black hole thermodynamics

In this section we derive the entropy and temperature of the black hole within MDR and the standard uncertainty principle. Then we compare our results with standard string theory results to find more concrete form of MDR. We use a more general form of MDR relative to (1),

$$(\vec{p})^2 = f(E, m; L_p) \simeq E^2 - \mu^2 + \alpha_1 L_p E^3 + \alpha_2 L_p^2 E^4 + \alpha_3 L_p^3 E^5 + \alpha_4 L_p^4 E^6 + O(L_p^5 E^7). \quad (21)$$

A simple calculation (neglecting rest mass) gives

$$dp = dE \left[ 1 + \alpha_1 L_p E + \left( \frac{3}{2}\alpha_2 - \frac{3}{8}\alpha_1^2 \right) L_p^2 E^2 + \left( 2\alpha_3 - \alpha_1\alpha_2 + \frac{1}{4}\alpha_1^3 \right) L_p^3 E^3 + \left( -\frac{5}{4}\alpha_1\alpha_3 + \frac{15}{16}\alpha_1^2\alpha_2 - \frac{5}{8}\alpha_2^2 - \frac{25}{128}\alpha_1^4 \right) L_p^4 E^4 + \left( -\frac{3}{2}\alpha_2\alpha_3 + \frac{9}{8}\alpha_1^2\alpha_3 + \frac{9}{8}\alpha_1\alpha_2^2 + \frac{21}{128}\alpha_1^5 - \frac{45}{48}\alpha_2\alpha_1^3 \right) L_p^5 E^5 \right], \quad (22)$$

then we find

$$dE = dp \left[ 1 - \alpha_1 L_p E + \left( -\frac{3}{2}\alpha_2 + \frac{11}{8}\alpha_1^2 \right) L_p^2 E^2 + (4\alpha_1\alpha_2 - 2\alpha_1^3 - 2\alpha_3) L_p^3 E^3 + \left( \frac{23}{8}\alpha_2^2 + \frac{21}{4}\alpha_1\alpha_3 - \frac{137}{16}\alpha_1^2\alpha_2 + \frac{379}{128}\alpha_1^4 \right) L_p^4 E^4 + \left( \frac{15}{2}\alpha_2\alpha_3 - \frac{97}{8}\alpha_1\alpha_2^2 - \frac{89}{8}\alpha_1^2\alpha_3 - \frac{565}{128}\alpha_1^5 + \frac{801}{48}\alpha_1^3\alpha_2 \right) L_p^5 E^5 \right]. \quad (23)$$

Within quantum field theory, the relation between particle localization and its energy is given by  $E \geq \frac{1}{\delta x}$ , where  $\delta x$  is particle position uncertainty. Now it is obvious that within MDRs, this relation should be modified. In a simple analysis based on the familiar derivation of the relation  $E \geq \frac{1}{\delta x}$  [27], one can obtain the corresponding generalized relation. This generalization is

$$E\delta x \geq 1 + \frac{-\alpha_1 L_p}{\delta x} + \frac{(\frac{11}{8}\alpha_1^2 - \frac{3}{2}\alpha_2)L_p^2}{\delta x^2} + \frac{(4\alpha_1\alpha_2 - 2\alpha_3 - 2\alpha_1^3)L_p^3}{\delta x^3} + \frac{(\frac{23}{8}\alpha_2^2 + \frac{21}{4}\alpha_1\alpha_3 - \frac{137}{16}\alpha_1^2\alpha_2 + \frac{379}{128}\alpha_1^4)L_p^4}{\delta x^4} + \frac{(\frac{15}{2}\alpha_2\alpha_3 - \frac{97}{8}\alpha_1\alpha_2^2 - \frac{89}{8}\alpha_1^2\alpha_3 - \frac{565}{128}\alpha_1^5 + \frac{801}{48}\alpha_1^3\alpha_2)L_p^5}{\delta x^5}. \quad (24)$$

In the same manner as previous section, the entropy of black hole would be

$$\begin{aligned}
 S \simeq & \frac{A}{4L_p^2} + \frac{\alpha_1 \pi^{\frac{1}{2}}}{L_p} A^{\frac{1}{2}} + \pi \left( \frac{3}{2} \alpha_2 - \frac{3}{8} \alpha_1^2 \right) \ln \frac{A}{L_p^2} \\
 & - 4\pi^{\frac{3}{2}} L_p \left( -\alpha_1 \alpha_2 + \frac{1}{4} \alpha_1^3 + 2\alpha_3 \right) A^{-\frac{1}{2}} \\
 & - 4\pi^2 L_p^2 \left( -\frac{5}{4} \alpha_1 \alpha_3 - \frac{5}{8} \alpha_2^2 + \frac{15}{16} \alpha_1^2 \alpha_2 - \frac{25}{128} \alpha_1^4 \right) A^{-1} \\
 & - \frac{16}{3} \pi^{\frac{5}{2}} L_p^3 \left( \frac{9}{8} \alpha_1^2 \alpha_3 - \frac{45}{48} \alpha_1^3 \alpha_2 + \frac{9}{8} \alpha_1 \alpha_2^2 \right. \\
 & \left. + \frac{21}{128} \alpha_1^5 - \frac{3}{2} \alpha_2 \alpha_3 \right) A^{-\frac{3}{2}}. \tag{25}
 \end{aligned}$$

It is easily seen that the entropy corrected by MDR has some terms very different from string theory result. According to string theory, the terms which include the half-odd power of  $A$  or  $A^{-1}$  are not present in the entropy relation. Looking back to our general form of MDR, (21), we see that if coefficients of the odd power of energy in the modified dispersion relation were vanishing ( $\alpha_1 = \alpha_3 = 0$ ), then unwanted terms in entropy–area relation will disappear. Comparison between results of MDR and string theory, suggests that in MDR, black hole entropy should be

$$S \simeq \frac{A}{4L_p^2} + \frac{3}{2} \pi \alpha_2 \ln \frac{A}{L_p^2} + \frac{5}{2} \pi^2 \alpha_2^2 \frac{L_p^2}{A}. \tag{26}$$

We conclude that in MDR, all odd powers of energy should be omitted. In other words, MDRs should contain only even power of energy. Using Eq. (25), we find for temperature of black hole

$$\begin{aligned}
 T \simeq & \frac{L_p^2}{8\pi M} \left[ 1 - \frac{\alpha_1 L_p}{2M} + \frac{(\frac{11}{8} \alpha_1^2 - \frac{3}{2} \alpha_2) L_p^2}{4M^2} \right. \\
 & \left. + \frac{(4\alpha_1 \alpha_2 - 2\alpha_3 - 2\alpha_1^3) L_p^3}{8M^3} \right]. \tag{27}
 \end{aligned}$$

Naturally, the presence of the even powers of the  $\frac{1}{M}$  which are not present in string theory mass–temperature relation (5), is due to  $\alpha_i$ , where  $i$  is odd. When we set  $\alpha_i = 0$  for all odd  $i$ , we find usual string theory result.

Now we answer the following question: what is the relation between results of GUP and MDR? First we consider corresponding relations for entropy. These are Eqs. (18) and (26),

$$\begin{aligned}
 S \simeq & \frac{A}{4L_p^2} - \pi \alpha \ln \frac{A}{L_p^2} - 4\pi^2 (\alpha^2 - \alpha'') \frac{L_p^2}{A} \quad \text{GUP result,} \\
 S \simeq & \frac{A}{4L_p^2} + \frac{3}{2} \pi \alpha_2 \ln \frac{A}{L_p^2} + \frac{5}{2} \pi^2 \alpha_2^2 \frac{L_p^2}{A} \quad \text{MDR result.}
 \end{aligned}$$

If we require these two results be consistent, we should have, for example,  $\alpha = -\frac{3}{2} \alpha_2$  and  $\alpha'' = \frac{23}{8} \alpha_2^2$ . This arguments show that actually GUP and MDRs are not independent concepts. Since  $\alpha$ ,  $\alpha''$ , and ... are quantum gravity model dependent parameters, it seems that in ultimate theory of quantum gravity, GUP and MDRs may be equivalent concepts. Now, using string theory entropy–area relation, (4), we see that  $\rho = -\pi \alpha$  and

$\beta = -4\pi^2 (\alpha^2 - \alpha'')$  for GUP–string theory correspondence, and  $\rho = \frac{3}{2} \pi \alpha_2$  and  $\beta = \frac{5}{2} \pi^2 \alpha_2^2$  for MDR–string theory correspondence.

Note that we have considered only a few terms of GUP and/or MDRs for rest of our calculations, but considering more generalized form of GUP and MDRs do not change our results regarding the form of GUP and/or MDRs.

### 5. Summary

In this Letter we have compared GUP and MDRs quantum corrections of black hole thermodynamics with more reliable string theory results. Our comparison suggests that

- In GUP, only even power of Planck length (or equivalently, only odd power of  $\frac{1}{\delta x}$ ) should be present.
- In MDRs, only even power of energy should be present.
- GUP and MDRs are not independent. It seems that they could be equivalent concepts in ultimate quantum gravity theory.
- Constraints on the form of GUP and/or MDRs may reflect inherent features of underlying quantum gravity theory.

One may argue that our conclusions regarding GUP and/or MDRs functional form, are not general since we have considered only a few terms in GUP and/or MDRs. Actually calculations based on more terms in GUP and/or MDRs support our results. This is reasonable at least on symmetry grounds. Note that our arguments are based on the assumption that today, string theory and loop quantum gravity results are more reliable than other alternatives of quantum gravity.

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