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Dispersion of Polydisperse Droplets in a Pulsating Flow Field

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Abstract

The dispersion of polydisperse droplets in a pulsating air stream has been analyzed. The grouping and segregation of droplets in both steady and oscillating flow field has been studied. The effect of evaporation on the grouping process has also been investigated. Eulerian-Eulerian multiphase framework has been used to model the polydisperse drop phase. The model is employed to study the dynamics in a 1D plug flow evaporator. The variation in dispersion process of the droplets along the channels has been observed both without and with presence of evaporation of the droplets. It has been found that the larger size droplets responses weekly with the oscillating flow field whereas, the smaller size droplets clustered differently at different locations depending upon the phase angles of the oscillating flow field.

Keywords: population balance; multiphase flow; droplet clustering; evaporation

1. Introduction

The interaction of the droplets with a co-flowing air stream is a complicated process and is a phenomenon of fundamental importance. Real life applications are also abundant in engineering, medical and environmental sciences, in gas turbines, diesel engines, spray drying etc. In many of these applications, droplets experience forced oscillations.

Katoshevski et al. [1] studied grouping of particles in a gas flow field by assuming a 1D model. The

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analysis was performed by considering an oscillating gas flow field and the effect of particles on the gas flow field was ignored. Sazhin et al. [2] describe the dynamics of grouping of spherical particles in an oscillating Stokesian flow using a detailed analytical study. As described by Katoshevski et al. [3], particle clustering can happen in the oscillating flow field due to the change of local relative velocity of particles with respect to the oscillating carrier fluid. The non-zero relaxation time for the smaller particles results in accumulation and separation of particles. Particle clustering at various operating conditions has been reported in literatures [4, 5]. The result of various numerical and experimental studies shows that the vortex structure in shear flow causes the clustering of particles. In most of these earlier studies mono-sized droplets are considered during the analysis of grouping of droplets.

In the present work, a dispersion of polydisperse droplets in the oscillating flow field has been studied. The presence of multiple size class of droplets resulting from the secondary breakup of the liquid jet during atomization is a possible motivation for this work. The objective of the present work is to develop a model and analyze the dispersion of droplets in oscillating flow with and without the presence of evaporation. Fixed drop size methodology following the D^2 law of evaporation has been used.

2. Model description

An Eulerian-Eulerian multiphase flow has been used to describe the behaviour of the polydisperse droplets in air. The drop size distribution is discretized into a total of M size classes, where each size class is treated as a separate dispersed phase. The air is treated as a continuous phase. Continuity and momentum equations are solved for each of the phases. In addition, the phases are allowed to exchange momentum through drag. The model is used to describe the flow behaviour of droplets in a 1D channel of length 0.5 m. The velocity field of the continuous phase at the inlet is defined as,

$$u_0 = u_a - u_b sin(\omega t) \tag{1}$$

where u_a and u_b are the mean flow velocity and amplitude of the velocity oscillation respectively.

Balance equations for each dispersed phases are,

$$\frac{\partial}{\partial t}(\rho_i n_i d_i^3) + \frac{\partial}{\partial x}(\rho_i n_i d_i^3 u_i) = -\rho_i n_i d_i^3 \Gamma_i^e + \sum_{j=1}^{i-1} \rho_j n_j d_j^3 n_{ij}^e \Gamma_j^e \frac{x_i}{x_j} \tag{2}$$

The first and second term of the RHS represents evaporation to lower size class and evaporation from higher class. A fixed drop size methodology as described in Rayapati et al. [6] has been used by conserving the volume of the drop phase to model the droplet evaporation. Here Γ_i^e is the equivalent breakage frequency which determines how much fraction of the drop population moves to the next smallest phase due to evaporation. In the evaporation process,

$$\Gamma_i^e = \frac{K_e (3\pi/8)(6/\pi)^{1/3} x_i^{1/3}}{x_i - x_{i+1}} \tag{3}$$

Here K_e is the evaporation rate constant which depends on the vapor massfractions of the discrete phase and the ambient gas. In the present problem a constant value of 10^{-10} m^2/s has been chosen as the value of K_e. During evaporation of single drop of (i-1)th bin, the diameter of the (i-1)th bin decreases continuously and contributes mass to the ith bin only; hence

$$n_{ij}^e = \begin{cases} 1 & \text{for} \quad j = i\text{-}1 \\ 0 & \text{for} \quad j \neq i\text{-}1 \end{cases} \tag{4}$$

$$\frac{\partial}{\partial t}(\rho_i n_i d_i^3 u_i) + \frac{\partial}{\partial x}(\rho_i n_i d_i^3 u_i u_i) = -\rho_i n_i d_i^3 \Gamma_i^e u_i + \sum_{j=1}^{i-1} \rho_j n_j d_j^3 n_{ij}^e \Gamma_j^e \frac{x_i}{x_j} u_j + K_{0i}(u_0 - u_i) \tag{5}$$

The first two terms of the RHS represents the momentum transfer due to evaporation while the last term represents momentum exchange with continuous phase.

The volume fraction of phase 0 (continuous phase) can be calculated following the constraint,

$$\alpha_0 = 1 - \sum_{i=1}^{M} \frac{\pi}{6} n_i d_i^3 \tag{6}$$

Balance equation for the continuity of continuous phase,

$$\frac{\partial}{\partial t}(\rho_0 \alpha_0) + \frac{\partial}{\partial x}(\rho_0 \alpha_0 u_0) = \alpha_M \rho_M \Gamma_M^e n_{M,M-1}^e + \sum_{k=1}^{M-1} \rho_k \alpha_k n_{k+1,k}^e \Gamma_k^e \frac{(x_k - x_{k+1})}{x_k} \tag{7}$$

The first term on the RHS of Eq. (7) represents the evaporation mass exchange from the smallest size class, during evaporation, which is added to the vapour species. The second term represents the contributions from the all other size classes except the smallest one. The mass fraction of air (j=1) and vapour (j=2) obeys the following equation.

$$\frac{\partial}{\partial t}(\rho_0 \alpha_0 Y^j) + \frac{\partial}{\partial x}(\rho_0 \alpha_0 u_0 Y^j) = -\frac{\partial}{\partial x}(\rho_0 \alpha_0 D_j \frac{\partial}{\partial x}(Y^j))$$

$$+ (j-1)[\alpha_M \rho_M \Gamma_M^e n_{M,M-1}^e + \sum_{k=1}^{M-1} \rho_k \alpha_k n_{k+1,k}^e \Gamma_k^e \frac{(x_k - x_{k+1})}{x_k}] \tag{8}$$

Here Y^j is the mass fraction and D_j is the diffusion coefficient of the jth species.

$$\frac{\partial}{\partial t}(\rho_0 \alpha_0 u_0) + \frac{\partial}{\partial x}(\rho_0 \alpha_0 u_0 u_0) = -\sum_{i=1}^{M} K_{0i}(u_0 - u_i)$$

$$+ \alpha_M \rho_M \Gamma_M^e n_{M,M-1}^e u_M + \sum_{k=1}^{M-1} \rho_k \alpha_k n_{k+1,k}^e \Gamma_k^e \frac{(x_k - x_{k+1})}{x_k} u_k \tag{9}$$

In Eq. (9) the first term of the right hand side represents total momentum exchange for all the size classes and continuous phase due to drag whereas, the last two terms are because of the momentum interaction between drop and vapour phase during evaporation. The term K_{0i} is the interphase momentum interaction coefficient. In the present problem the term is of the form,

$$K_{0i} = \frac{3}{4} d_i^2 n_i |u_0 - u_i| \rho_c f \tag{10}$$

The drag function f has been used is of the form of Schiller and Naumann,

$$f = \frac{24(1 + 0.15 \mathrm{Re}^{0.687})}{\mathrm{Re}} \tag{11}$$

Here $\mathrm{Re} = \frac{\rho_0 |u_0 - u_i| d_i}{\mu_0}$ \hfill (12)

3. Results and Discussion

The results presented here are for the values of u_a, u_b and ω are 0.5 m/s, 0.1 m/s and 40 rad/s respectively. Total 40 size classes of droplets are considered with a minimum diameter of 1 µm to maximum of 250 µm. The diameters are arranged in a geometric progression. A top-hat distribution of number density of 1e8 has been assumed at the inlet for all size classes of the droplets. The droplets are injected with an initial velocity of 1 m/s.

The variation of normalized number density, normalized with respect to the inlet droplet distribution, with non dimensional time, non-dimensionalized by continuous phase velocity and channel length, is shown in Fig. 1. Due to the difference in inertia, the oscillation frequencies of different size classes are different and a phase lag can be observed from Fig. 1. The response of the smallest size droplets (i = 40) are more than the largest size droplets (i=1). The moments of size distributions can be expressed as,

$$M^{(k)} = \int_0^i D_i^k n_i dD \tag{13}$$

Here, k is the number of moment, i is the size class and n_i is the number density of size class i.
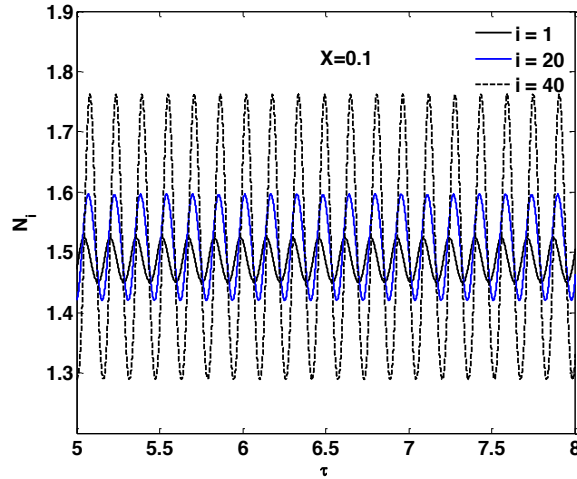
Fig. 1. Variation of number density with time for different size classes

The variation of the moments M^0 and M^3 are shown in Fig. 2 for different time (phase angles) along the length of the channel. Here, M^0 and M^3 are normalized with respect to the values at the inlet, whereas, the length of the channel is used to normalize the distance along the x axis. It can be seen from Fig. 2 that there are variation in fluctuations of M^0 between the center and outlet of the channel whereas, this fluctuations is not prominent for M^3. This is due to the weaker response of the larger sized drops that contributes more in M^3. The values of M^0 for all the phase angles are more near the inlet due to the trapping of smaller size droplets. The smaller size droplets responses more to the low continuous phase velocity and trapped near the inlet.

(a) Variation of M^0 for different phase angles

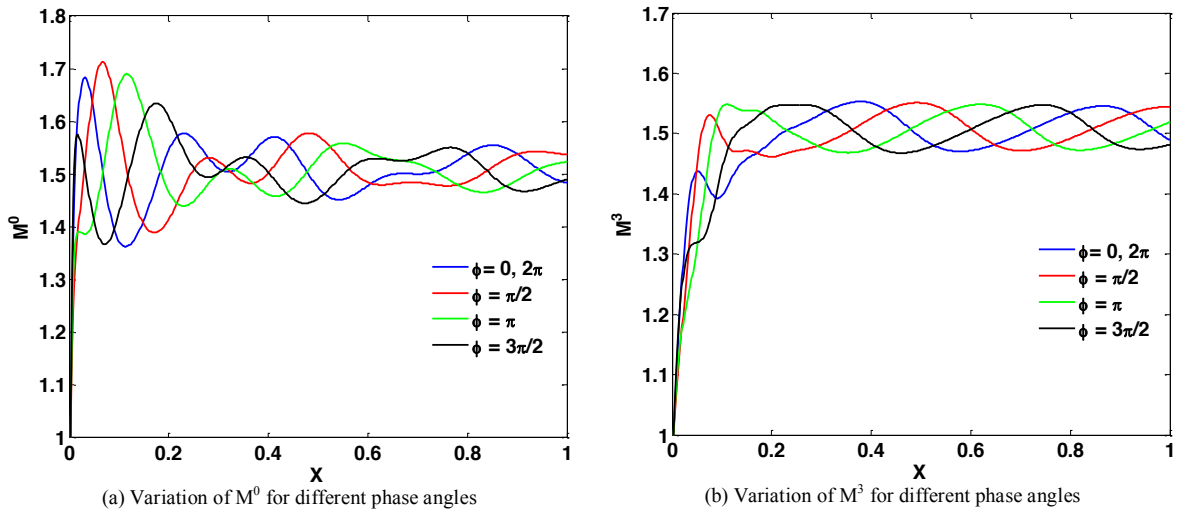(b) Variation of M^3 for different phase angles

Fig. 2. Moments at different time instants along the axial locations

The variation of normalized number density for different size classes along the channel length for different phase angles are shown in Fig. 3. The weaker response of the larger size drops with the pulsating frequency is clearly observed in Fig. 3a, which supports the observations in Fig. 2b. The clustering of droplets at different locations is clearly observed from Fig. 3. It can also be observed that at the same locations different number of droplets of same size classes can be found depending upon the phase angles.

(a) Variation of number density of size class 1 for different phase angles

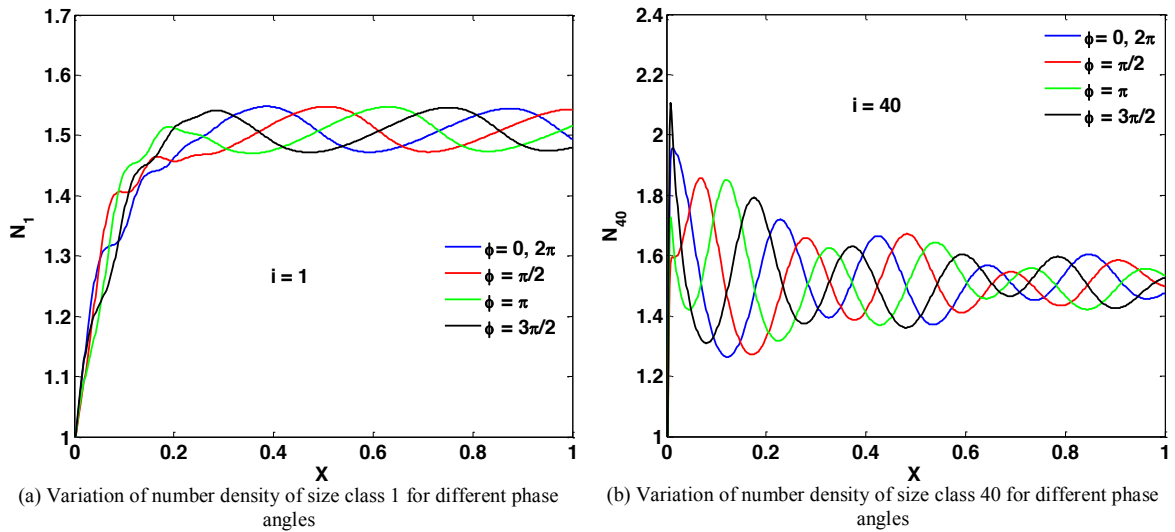(b) Variation of number density of size class 40 for different phase angles

Fig. 3. The variation of number density along the channel length for different phase angles and size classes

The effect of evaporation during the transport of droplets has also been investigated in the present work. The fixed drop size methodology following the D^2 law of evaporation has been used in this work. Due to the evaporation, the droplets of the $(i-1)^{th}$ bin decreases continuously and contributes mass to the ith bin only. The smallest size droplet evaporates and contributes mass to the vapour species. The variation of the moments M^0 and M^3 are shown in Fig. 4 for different time (phase angles) along the length of the channel during evaporation. Both M^0 and M^3 decrease continuously along the channel length as the number density of all the size classes decreases due to the evaporation. Here also, largest size droplets are less responsive with the pulsating continuous phase flow and follows almost a same decreasing trend of M^3 everywhere along the channel length after the initial rise of M^3.

(a) Variation of M^0 for different phase angles

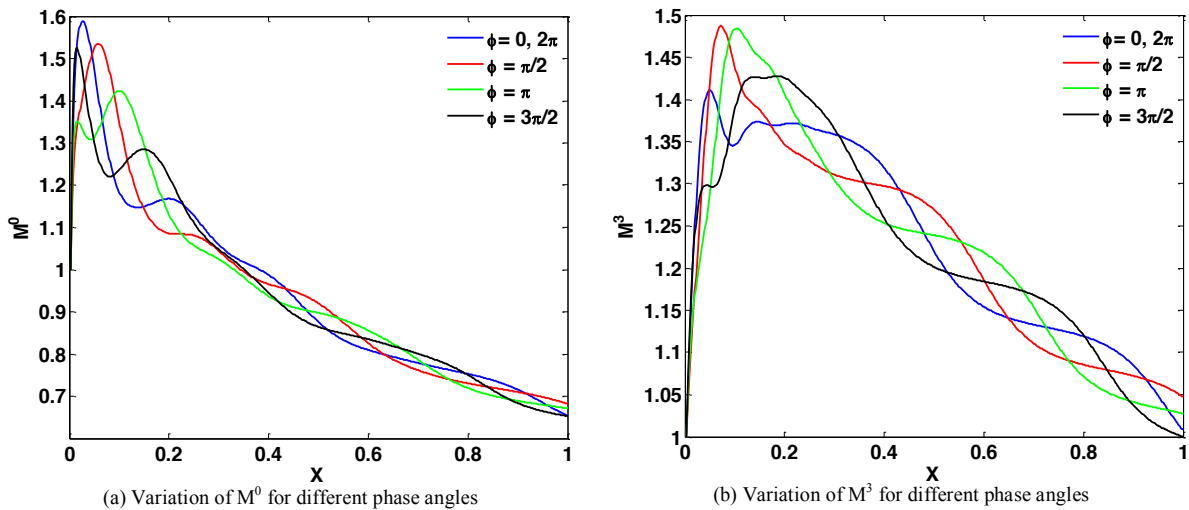

(b) Variation of M^3 for different phase angles

Fig. 4. Moments at different time instants along the axial locations during evaporation

This can be understood clearly from the variation of number density for different phase angles during evaporation. The decrease of number density along the channel length is almost uniform for the largest size class of the droplets as shown in Fig. 5a whereas, the slope of the decrease of the number density saturates after a certain length of the

channel for the smallest size classes of the droplets (i=40). The effects of evaporation on the smallest size classes of the droplets are almost independent of the continuous phase pulsation after the certain length of channel, as shown in Fig. 5b. Therefore, the clustering of droplets is totally different in presence of evaporation. The clustering can be observed for the larger size droplets but the smaller size droplets evaporate more to the continuous vapour phase and almost a uniform distribution can be found.
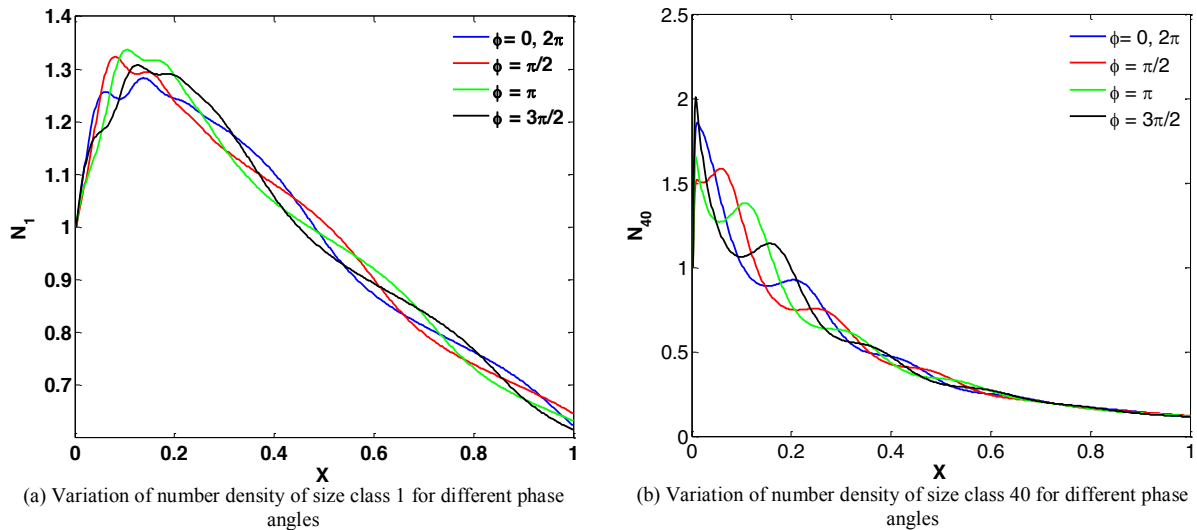

(a) Variation of number density of size class 1 for different phase angles

(b) Variation of number density of size class 40 for different phase angles

Fig. 5. The variation of number density along the channel length for different phase angles and size classes during evaporation

4. Conclusions

The clustering of different size of droplets is brought into light in the present work in the context of a pulsating flow field with and without the presence of evaporation. Eulerian-Eulerian framework of multiphase methodology is used to model the dispersion of the polydisperse droplets. The polydisperse drop phase itself is modeled following the continuum hypothesis. It has been seen that the grouping of different size classes at a particular location is possible at different time instants due to the oscillating flow field. In presence of the oscillating flow field the variation in fluctuations of M0 between the centre and outlet of the channel is observed whereas, this fluctuation is not prominent for M3. The phenomena of clustering of droplets during evaporation are totally different. In the polydisperse droplets the number density of the larger size droplets decreases continuously along the length of the channels, whereas, for the smaller size droplets the number density becomes constant after a certain length of the channel due to evaporation.

Acknowledgments

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