A KNOWLEDGE-BASED SYSTEM FOR ROBUSTNESS ANALYSIS OF LARGE-SCALE ECONOMIC SYSTEMS

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Abstract—The paper gives a conceptual framework for robustness analysis of large-scale economic systems, and its realization through interactive computer-aided software. The mismatch between the economic system and the corresponding mathematical model is discussed. The computer-aided system combines algorithmic and expert system techniques. An important feature of the present system is the modularization of the software package which allows a distributed problem solving approach. A fourth-order macroeconomic model, with typical parameters, which demonstrates the margin of power of the governmental body can exercise on the various sectoral activities, is used to illustrate some of the concepts presented in this paper.

1. INTRODUCTION

The history of attempts to introduce system-theoretic methods and in particular large-scale systems control concepts into the field of economics goes back at least twenty years. It has only been in the last decade, however, that the explosion in the technological developments in computer hardware and software, coupled with increasing sophistication in econometric modeling, have drastically altered the prospects for the successful utilization of modern control theory into the field of economics.

The problem of controlling large-scale economic systems has led to the introduction of the state space forms as an alternative representation of traditional model forms in various theoretical and empirical studies of dynamic economic systems, especially in the application of optimal decision rules for macroeconomic planning and policy models. The application of optimal control techniques to macroeconomics has demonstrated the potential of optimal control theory for macroeconomic growth theory, development and stabilization (e.g., [1-3]). To open up the field of econometric modeling to the techniques of optimal control, econometric models, in either structural, reduced or final form, have usually been translated into state space forms (e.g., [4]).

Many recent studies have been concerned with control and policy analysis in large-scale systems. A combination of structural considerations for large-scale systems, plus policy and decision analysis, in a hierarchical framework can do much to enhance our ability to cope with the complexity inherent in large-scale economic systems. The mathematical model of any large-scale system necessarily involves a number of approximations brought about by the simplification of the theory, reduction of order, elimination of nonlinearities and the assumption of parametric invariance. Each of these introduces a degree of uncertainty into any prediction of the performance of the actual closed-loop system. In this respect, the sensitivity and robustness of multi-variable systems have received considerable attention in the last few years. Both sensitivity theory and robustness theory deal with a very important issue in system theory, the preservation of various system-theory properties in the face of variations in the system model. While sensitivity (e.g., [5])

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is primarily concerned with the relationship between infinitesimal variations in nominal system parameters and the corresponding system property, robustness (e.g., [6–9]) requires the explicit delineation of finite regions about the nominal model for which the given property is preserved.

The rapid advances in modern system and control theory will not find their potential application in modeling, analysis and design of large scale economic systems unless they are puffed in computer-aided packages which are flexible, higher interactive, user friendly and easily accessible to the final users. Existing computer-aided packages are primarily focused on certain isolated parts of the overall design process (e.g., system identification, model building, dynamic simulation, model reduction, optimization, etc.) without unifying them in a comprehensive modeling, analysis and design setting. Another criticism of the existing packages nowadays is that they require considerable skill and system and control theory background for their proper use. Thus, there is a real need for the development of knowledge-based techniques in those packages to make them more user-friendly and be able to offer “intelligent” help at each step of the modeling, analysis and design process. Many researchers have already started investigation of expert systems for control systems work. The field of Artificial Intelligence is producing many tools and techniques applicable to the modeling, analysis, design and implementation of complex control systems (e.g., [10]). Much attention has been drawn to expert systems (e.g., [11]).

In this paper, we discuss a conceptual framework for robustness analysis of large-scale economic systems, and its realization through an interactive computer-aided software is presented. The system combines algorithmic and expert system techniques. The knowledge-based system is programmed in PROLOG and the algorithmic part in FORTRAN. In fact, many of the quantitative programs have been under development for many years, and with some modifications they were directly incorporated. An important feature of our knowledge-based system is the modularization of the software package.

The paper is organized as follows. The preliminary concepts are given in Section 2, where we present the basic mathematical model and the concept of robustness of large-scale systems. In Section 3, the supporting knowledge-based computer-aided system is discussed. The distributed problem-solving approach is analyzed in Section 4. To illustrate some of the concepts presented in this paper, a fourth-order macroeconomic model with typical parameters, which demonstrates the margin of power the governmental body can exercise on the various sectoral activities, is used in Section 5.

2. PRELIMINARY CONCEPTS

2.1. The Basic Model

Economic systems have recently received a great deal of attention from control engineers as one of the broader and more fruitful areas of control theory applications. This arises naturally in describing both the macroeconomic and microeconomic behaviour of economic systems. More recently, the state space form as an alternative representation of traditional forms has emerged in various theoretical and empirical studies of dynamic economic systems. The realization problem and the problem of how the state space approach can be integrated into the traditional econometric models will not be considered in this paper. Various methods and results are available for the problem of realization (e.g., [12–14]). Some numerical results of an operational algorithm for constructing a minimal realization are given in [15], in which the canonical minimal realizations of some well-known economic models are illustrated numerically.

Despite the high efficiency of modern computing machinery, the formidable complexity of systems with a large number of economic variables makes a direct analysis unattractive. However, by grouping the variables of a large economy into a relatively small number of subsystems, that economy can be decomposed into various interconnected systems. In classical control and decision making problems, the system is handled in a centralized fashion. The decisions of control policies and their implementation are all made according to the preference of a single, central supervisor. However, centralized control of large-scale systems is generally unrealistic because of the excessive computational costs and because of the heavy costs of communication between a large number of information-gathering networks. Hence, an important problem of decentralization arises where the information processing and control decisions can be delegated
to a set of agents. A decentralized dynamic system is one in which two or more decision makers' actions will jointly affect the dynamic behaviour of the system. Decision makers base their actions on partial information of the various states of the dynamic system.

In order to simplify the notation, the focus of our attention in this paper is on a large-scale discrete-time system controlled by a set of $k$ agents—each having different information and control variables:

$$x(t + 1) = Ax(t) + \sum_{i=1}^{k} B_i u_i(t), \quad x(0) = x_0,$$

$$u_i(t) = F_i y_i(t), \quad i = 1, 2, \ldots, k,$$

$$y_i(t) = C_i x(t), \quad i = 1, 2, \ldots, k,$$

where $x(t), x(t) \in \mathbb{R}^n$, is the state of economy; $u_i(t), u_i(t) \in \mathbb{R}^{m_i}$, is the vector of exogenous variables (inputs); $y_i(t), y(t) \in \mathbb{R}^{r_i}$, is the vector of endogenous variables (outputs). (Reference [16] contains a useful comparison of econometric and control jargon.) Finally, $F_i$ is a time-invariant gain matrix, the time index $t$ takes values $t = 0, 1, 2, \ldots$, and $A, B_i$ and $C_i$ are known matrices of appropriate dimensions.

Equations (1)–(3) provide a basis for representing many econometric models in a control framework. From an economic standpoint, the state variable $x(t)$ includes, for example, incomes, residential construction, durable consumption, investment, inventories, etc. The control vectors $u_i(t)$ have entries such as tax rates, government expenditure, money supply and other policy variables. The output vector variables are usually a linear transformation of the state variables. Finally, the elements of the matrices $A, B_i$ and $C_i$ involve the parameters that specify alternative channels of influences and economic effects among the system variables.

A number of different methods (e.g., [17]) can be used to determine decentralized feedback matrices $F_i$, to achieve satisfactory performance of the nominal closed-loop system. The objectives of the control actions may be to optimize or stabilize the overall system. In a number of cases in economics, both objectives can be achieved through the optimization of a cost function (the term welfare function is also used), such as

$$J = \sum_{t=0}^{\infty} (x^T(t) Q x(t) + \sum_{i=1}^{k} u_i^T(t) R_i u_i(t)).$$

The weighting matrices $Q$ and $R_i$ in the decentralized quadratic performance criterion (4) are symmetric positive semi-definite and positive definite matrices, respectively, $Q = Q^T \geq 0$, $R_i = R_i^T > 0$, $i = 1, 2, \ldots, k$, and have special significance. While $Q$ accounts for the relative cost of deviating the state variables from the desired level, the matrices $R_i$ stand for the cost of operating the controls away from their desired levels. The matrices $Q$ and $R_i$ are normally diagonal and have relative magnitudes that reflect the costs of the control effort compared to output deviations.

The problem now is to determine the control law (2) for the system described by Equation (1), subject to the known information (measurement) constraints (3), which minimizes the cost function (4). In this paper, we assume that the system (5), Equations (1)–(3), is stabilizable and that the feedback matrices $F_i$ have been selected so that the nominal closed-loop system is stable, i.e., the characteristic roots of the closed-loop matrix $A_c$

$$A_c = A + \sum_{i=1}^{k} B_i F_i C_i,$$

are inside the unit disc $|\lambda(A_c)| < 1$. 
2.2. Robustness

2.2.1. Introduction

One of the essential roles of feedback has been to achieve a satisfactory control of systems having parameters which are either not known exactly, or are varying in time during operation. The mathematical model of any economic system involves a number of approximations brought about by the simplification of the theory, reduction of order, elimination of nonlinearities and the assumption of parametric invariance.

It is common procedure in practice to work with mathematical models that are simple, but less accurate, than the best available model of a given system. In going from the most complex to the most simplified model, the trade-off is between computational convenience and modeling adequacy. Not only should a model be faithful in terms of the physical reality which it represents, but it should also provide the planner or the analyst with enough information to enable him to act on the system in an informed manner. In other words, a satisfactory model is a good aid to decision making which at the same time achieves the high level of trade-off between accuracy and computational convenience.

Methods for the approximate control of dynamic systems have received a great deal of attention in recent years. Reduction of computation and simplification of the control system structure are of particular concern in decentralized control of large-scale systems. The methods for model simplification can be divided into two classes: aggregation methods [19] and perturbation methods. It is common practice to divide the perturbation methods into the two subclasses of singular perturbation [20] and nonsingular perturbation [21, 22] methods.

All these approximations introduce a degree of uncertainty into any prediction of the performance of the actual closed-loop system. In this light, robustness of multivariable systems has received considerable attention in the last few years in control theory (e.g., [6-9, 23-25]). This important concept of robustness can be defined as an ability of the system to maintain stability and performance in the face of uncertainty and perturbation. More precisely, the closed-loop system is robust with respect to a property $z$, if it possesses this property for every element in the set of models that can represent the real system. Within this context there are several properties that can be investigated (e.g., stability, suboptimality degree, controllability and observability.)

2.2.2. Stability Robustness

While the economic processes of interest may vary greatly and performance objectives may differ from application to application, most control strategies share the common requirement that stability be maintained in the face of significant system uncertainties and perturbations. This requirement is commonly called stability robustness. Suppose we are given a nominal system which is closed-loop stable. Then stability robustness tests give us bounds on the amount of deviations from the nominal system we can allow and still guarantee that the perturbed system is closed-loop stable.

Let the perturbed version of the nominal system $(S)$, Equations (1)-(3), satisfy

$$x(t+1) = Ax(t) + \sum_{i=1}^{k} B_i u_i(t) + \Delta A x(t) + \sum_{i=1}^{k} \Delta B_i u_i(t), \quad x(0) = x_0, \quad (\Delta S)$$

where $A, B_i, F_i$ and $C_i$ are the same as in the nominal, unperturbed system $(S)$, so that all modeling errors and parameter variations are lumped into the matrices $\Delta A$ and $\Delta B_i$, $i = 1, 2, \ldots, k$.

When large parameter variations are present in the system dynamics, then generally nothing can be inferred about the solution of the changed situation. However, a fairly fruitful technique is to attempt to find a region within which the perturbations may lie so that the perturbed system remains stable.

*A great variety of reduced-order modeling techniques exist for general systems (see, e.g., the bibliography of [18]).*
Stability robustness analysis considerably depends on the modeling information available to the designer. In the general case, the perturbation matrices $\Delta A$ and $\Delta B_i$, $i = 1, 2, \ldots, k$, may be classified into two types of perturbations, namely:

(i) **Unstructured perturbations**, when only a bound on the norm at the perturbation matrices is given;

(ii) **Structural perturbations**, when the structure of perturbation matrices is specified and the bounds on such structured perturbations are given.

With few exceptions, recently proposed methods to assess stability robustness of multivariable control systems are well suited for unstructured uncertainties. In these approaches, the class of "allowable perturbations" is completely unstructured except for the definition of the applicable norm. Such stability robustness criteria tends to be very conservative for structured perturbations because they fail to exploit the additional structures.

As known, in many practical situations, it may happen that there exists some a priori knowledge of the perturbations which may be encountered. In this case there is a need for a more detailed description of the "allowable perturbations". A designer, usually, following his intuition and experience, has enough information concerning the nature of the perturbations (modeling uncertainties, modeling reductions or parameter variations), which can physically occur, to select the most appropriate directions in the space of all perturbation matrices. This leads [7,26] to decomposition of the perturbations into two components, one of which along the given direction in the space of all perturbation matrices. In this spirit, one can define the model perturbation term $\Delta A$ of the open-loop matrix $A$ as

$$\Delta A = \gamma(t)A^1 + \Delta A^1,$$

and the model perturbation terms $\Delta B_i$, of the control actuating matrix $B_i$ as

$$\Delta B_i = \gamma(t)B_i^1 + \Delta B_i^1, \quad i = 1, 2, \ldots, k,$$

where $\gamma(t)$ is a scalar function and $\Delta A^1$ and $\Delta B_i^1$ represent the perturbations in system dynamic which lie out of the given directions $A^1$ and $B_i^1$, respectively.

The following result has been obtained for the unstructured perturbations case [27,28].

**Theorem 1.** If the perturbation matrices $\Delta A$ and $\Delta B_i$ satisfy the inequality

$$\left\| \Delta A + \sum_{i=1}^{k} \Delta B_i F_i C_i \right\| \left( 2\|A_c\| + \left\| \Delta A + \sum_{i=1}^{k} \Delta B_i F_i C_i \right\| \right) < \frac{\lambda_{\min}(Q_1)}{\lambda_{\max}(P)},$$

where the matrix $P$ is the positive definite solution of the following Lyapunov matrix equation

$$A_c^T PA_c + Q = P,$$

$Q_1 = Q_1^T > 0$, $A_c$ is defined by (5) and $\| \cdot \|$, $\lambda_{\min}(\cdot)$ and $\lambda_{\max}(\cdot)$ denote Euclidian norm, minimum and maximum eigenvalues of ($\cdot$), respectively, then the perturbed system, Equations (6) and (7), is asymptotically stable.

Suppose that the perturbation matrices $\Delta A$ and $\Delta B_i$, $i = 1, 2, \ldots, k$ lie entirely along the directions $A^1$ and $B_i^1$, $i = 1, 2, \ldots, k$, respectively, that is, $\Delta A^1 = 0$ and $\Delta B_i^1 = 0$. Now the perturbed system can be represented as

$$x(t+1) = Ax(t) + \sum_{i=1}^{k} B_i u_i(t) + \gamma(t) \left( A^1 x(t) + \sum_{i=1}^{k} B_i^1 F_i C_i x(t) \right),$$

$$\Delta S^1 \quad u_i(t) = F_i C_i x(t).$$

The following theorem gives the sector $(\gamma_{\min}, \gamma_{\max})$, i.e., the bounds on the scalar function $\gamma(t)$ such that the perturbed system $(\Delta S^1)$, Equations (12) and (13), remains stable.
THEOREM 2. [27,28]. If for every \( \gamma(t) \in (\gamma_{\text{min}}, \gamma_{\text{max}}) \), the inequality

\[
Q_x - \gamma(t) \left( A + \sum_{i=1}^{k} B_i F_i C_i \right)^T P \left( A + \sum_{i=1}^{k} B_i F_i C_i \right) - \gamma(t) \left( A + \sum_{i=1}^{k} B_i F_i C_i \right)^T P \left( A + \sum_{i=1}^{k} B_i F_i C_i \right) > 0,
\]

is satisfied for all \( t = 0, 1, 2, \ldots \), where \( \gamma(t) \) is a scalar function and the matrix \( P \) is the positive definite solution of Equation (11), then the perturbed system, Equations (12) and (13), remains asymptotically stable.

REMARK 1. The robustness analysis based on Theorem 2 has been restricted to the case when the perturbations lie along the given directions \( A^1 \) and \( B^1 \) in the space of all perturbation matrices. The conditions which guarantee that perturbed closed loop system with \( \gamma(t) \) satisfying (14) will remain stable in the face of the perturbations \( \Delta A^1 \) and \( \Delta B^1 \) out of the given directions, whenever the norm of \( \Delta A^1 \) and \( \Delta B^1 \) remains appropriately bounded, can be directly derived from Theorem 1.

REMARK 2. Various algorithms can be developed to evaluate the criterion (14). In [28] a sequential numerical algorithm has been proposed to evaluate (14).

REMARK 3. In the case of weakly coupled systems [7,29], the perturbation directions in the open loop matrix \( A \) and in the control actuating matrix \( B \) are defined as

\[
A^1 = \begin{bmatrix}
0 & A_{12} & \cdots & A_{1p} \\
A_{21} & 0 & \cdots & A_{2p} \\
\vdots & \vdots & \ddots & \vdots \\
A_{p1} & A_{p2} & \cdots & 0
\end{bmatrix}, \quad B^1 = \begin{bmatrix}
0 & B_{12} & \cdots & B_{1p} \\
B_{21} & 0 & \cdots & B_{2p} \\
\vdots & \vdots & \ddots & \vdots \\
B_{p1} & B_{p2} & \cdots & 0
\end{bmatrix},
\]

where the matrices \( A^1 \) and \( B^1 \) are known. In a similar way, in the case of singularly perturbed systems [30]

\[
A^1 = \begin{bmatrix}
0 & 0 \\
A_{21} & A_{22}
\end{bmatrix}, \quad B^1 = \begin{bmatrix}
0 \\
B_{21}
\end{bmatrix}.
\]

The stability robustness analysis of weakly coupled systems and singularly perturbed systems, in case of continuous systems, is given in [29] and [30], respectively.

2.2.3. Other Performances

As mentioned, several system-theoretic properties can be evaluated in the context of robustness analysis, e.g., controllability, observability and suboptimality degree.

DEFINITION 1. The control law (2) is said to be suboptimal for the perturbed system (\( \Delta S \)), Equations (6) and (7), with degree \( \beta \) if there exists a positive number \( \beta \) such that

\[
J_2(x_0) < \beta^{-1} J_1(x_0),
\]

for all initial states \( x_0 \), where \( J_1 \) is the value of the decentralized performance index (4) for the nominal, unperturbed system, and \( J_2 \) is the value of the performance index of the perturbed system.

Obviously, the value of \( J_2 \) depends on the perturbation matrices \( \Delta A \) and \( \Delta B_i \), \( i = 1, 2, \ldots, k \). The "contribution" of the perturbations on \( J_2 \) depends on whether \( \beta \in (0, 1) \) or \( \beta \in [1, \infty) \). The former case occurs when perturbation matrices are "beneficial" and the perturbed system has better properties, in terms of the value of performance index, than the nominal system. More detailed discussion on suboptimal performance index sensitivity of large scale continuous time decentralized systems is given in [31]. Similar results can be easily developed for discrete-time case.

Finally the robustness analysis of controllability and observability can be performed following the results presented in [32].
2.9. Conceptual Framework for a Knowledge-Based System

To design a control law, i.e., to determine the decision strategy for complex systems such that a set of performance requirements are satisfied, a user first creates the model (S) of the system. Together with the model (S) is a set of performance specifications that constrain the final design. Then, experts from different fields are required to determine the control law. Therefore, typically, a design environment consists of a set of model building and control design experts that cooperate with each other. In addition to the experts, there is a large body of experience gained through a number of complex systems that have already been designed.

Therefore, a powerful computer-aided package must not only have a user-friendly interface, robust numerical software, interactive graphics capabilities and good database management, but also an extensive knowledge base for a particular application. This viewpoint of using expert systems as a knowledge-based tool would enhance the power of most current computer-aided packages for system and control-theoretic methods application in economic modeling and decision making.

One criticism of the computer-aided packages nowadays is that they require considerable skill and system and control theory background for their proper use. The rapid advances in modern system and control theory will not find their potential applications unless they are conveniently packaged in computer-aided packages which are easily accessible to the practicing users. Today the major stumbling block to the wide acceptance of advanced system and control concepts is not the computer hardware but the development of interactive knowledge-based computer-aided software which would help the user to formulate and solve the right set of modeling and design problems in a computationally efficient way.

Time-critical and numeric in nature, many algorithms capable for modeling, identification and optimization of complex economic systems exhibit computational characteristics which are radically different from those exhibited by existing expert systems. This is one of the most critical factor that complicate the use of expert system ideas in modeling and design oriented computer-aided packages for complex economic systems. As known, the strength of expert systems comes from symbolic processing capabilities, i.e., their ability to reason with non-numerical data. Therefore, the challenge is to integrate symbolic and numeric computation.

Most knowledge-based modeling and design oriented computer-aided packages developed to date can be characterized by

(i) a supervisory role for the expert system, usually involving modeling, monitoring, diagnosing, control system designing and planning;
(ii) a separation within the design system of the symbolic and numeric processing environments (software and/or hardware).

In this paper, we follow a similar line, although more recently the issue of highly integrated symbolic and numeric processing in real-time control-oriented expert systems has been addressed [33]. In this new approach symbolic and numeric processing occur in one environment, allowing the expert system to become an integral, rather than supervisory, part of the modeling and design system.

3. KNOWLEDGE-BASED SYSTEM

3.1. Introduction

As mentioned, we are in the process of developing a knowledge-based system for complex systems modeling, analysis and design. As this computer-aided system is a dynamic one, it was necessary to write modular programs so that additional knowledge (rules and data) can be added easily. The application is still primarily restricted to the modeling and robustness analysis and design.

It is not our intention to describe in detail all the rules and equations used in this knowledge-based system. Some of the basic concepts and methods for robustness analysis have been discussed in the previous section, and therefore, only those elements necessary for understanding the software implementation will be given here.
The knowledge-based computer-aided software possesses the following essential characteristics:

(i) It is interactive. The user has the capability to interact with the modeling, analysis and design process and consider all pertinent trade-off before reaching a decision.
(ii) It is modular and flexible. It is easy to modify as new system and control theoretical results become available and as design objectives go through evolutionary changes.
(iii) The knowledge base is separated into two main parts:
   * system-independent (generic)
   * system-specific rules.
   System-independent knowledge includes information about different modeling, identification and optimization approaches as well as different criteria for robustness analysis. System-specific knowledge covers the design specification and data of the considered economic system.
(iv) It has graphical capabilities. Visual information is vital to the decision-making process of the analyst and designer, making it fast and more efficient.
(v) It uses reliable computational methods.

3.2. Decision-Making System Environment

In order to reduce the complexity involved in large-scale economic systems analysis and design the decision-making problem could be divided into a number of sub-problems. In our case it was done by breaking the modeling, analysis and design process down into three basic levels:
(1) data handling;
(2) control law design;
(3) analysis;
which are typical for any decision-making system environment. These levels are organized in such a way that they can be represented and structured in computer(s). In other words, each level is organized as an object-oriented system. The partition of the decision-making problem in the above manner enables well-defined sub-procedures to be built. Each level consists of various modules to accomplish the task of that particular level. This has the significant advantage that the associated knowledge base is extremely modular. As known, this is a very effective key to addressing the complex issue.

3.2.1. Data Handling Level

This level accepts data and specifications (e.g., modeling data, dynamic performance specifications, etc.) of the given economic system. The Modeling Module and Identification Module which are part of this level, manipulate the data performing model identification, linearization, model reduction, transformations between traditional econometric models and state-space models, etc. As a result, models with different levels of complexity can be derived for the same economic system.

It is assumed that the user starts with nonlinear models (in traditional or state-space form) of the economic system and progress through the following range of activities: nonlinear simulation (e.g., model validation and behavioral analysis, equilibrium determination, linearization, etc.). In many applications, and in particular when large-scale economic systems are considered, such activity develops a substantial data-base. In the case of complex economic systems, a typical data-base may contain one nonlinear model and over ten linearised models with different levels of complexity.

The Pre-design Analysis Module has a function to analyze the model of the system by checking the open-loop stability, controllability and observability of the system, calculating the uncertainties bounds, etc. The sources of modeling errors are classified into two categories:
(i) uncertain parameters,
(ii) uncertainty in the model structure (e.g., model order reduction, neglecting secondary economic and physical phenomena in state-space models, etc.).

A high level of expertise is needed to separate parameter variations and modeling uncertainties into several subclasses and to apply the most appropriate characterization of perturbations and
uncertainties in order to use more effectively the robustness criteria, as well as the most appropriate corrective control actions. The Pre-design Analysis Module can also classify the models automatically (e.g., to determine if it is a single-input single-output system or multivariable system) such that users do not have to memorize the details of the system. All these results are made available to the designer and to the next level of control law design.

3.2.2. Control Law Design Level

This level consists of various design techniques in time and frequency domain, i.e., it consists of a set of the control system design experts in single-input single-output systems, multivariable systems, decentralized and hierarchical control systems. The systematic and modular nature of these design techniques facilitates the development of the corresponding knowledge-based system. Much of this activity is exploratory and iterative in nature, and ultimately produces a control law. If the decision maker is satisfied with the control system (e.g., acceptable complexity of the control system), he or she proceeds with the analysis level.

3.2.3. Analysis Level

The analysis level consists of various time-domain and frequency-domain techniques and provides additional information so that the designer can determine whether the current control law is acceptable or not, i.e., it advises the user on the closed-loop system characteristics, whether or not the selected control algorithm was in fact the one most appropriate to the given system specifications.

If the control algorithm is not acceptable, the designer having access to all levels can decide at which point he or she should try to make modifications so that an acceptable closed-loop system is created. As mentioned, much of this activity is exploratory and iterative in nature. In some cases, for example, it is quite possible that the initial performance specifications may not be achievable by any feasible control algorithm, in which case the designer has to relax some of these specifications.

The human interface is provided by a Language Processor that allows problem-oriented communications between the user(s) and the knowledge-based computer-aided package. When a user asks for the justification of a conclusion, the expert system works backward to show the logic behind its conclusion.

4. DISTRIBUTED PROBLEM SOLVING

An important problem associated with the application of knowledge-based systems to large scale economic systems lies in their computational complexity. Complex problems require a large knowledge base. Therefore, search space for problem-solving algorithms tend to be large. The corresponding knowledge-based system may thus require a fair amount of time to reason before reaching conclusions and, thus, fail to provide timely solutions.

Real-life complex problems often require distributed problem-solving approaches, that is, approaches that involve the collaborative efforts of several problem-solving agents with different fields of expertise. Collaboration is necessary when no single agent can solve the entire problem. The agents must often work in parallel for reasons of speed and feasibility.

In order to reduce the complexity involved in modeling, analysis and design of large-scale economic systems, as shown in the previous section, the decision-making process has been divided into a number of sub-processes. The advantages of organizing the modeling, analysis and design knowledge into different levels of complexity are as follows:

- This provides the necessary basis for the application of parallel processing algorithms.
- The performances of the closed-loop system, particularly those whose evaluation is very time consuming (e.g., robustness analysis) can be evaluated in parallel. If an acceptable level of the overall system behaviour is estimated by numerically simpler algorithms, the use of more complex algorithms can be avoided.
- The lower level of design algorithms and techniques can be tried first before a more complex algorithm is used to tackle the design problem. This approach will give solutions which are no more complicated than necessary.
The explanation of how a particular control strategy has been reached can be presented in a manner reflecting the level of complexity required by the design. When an unacceptable level of control complexity has been reached, the decision-maker may want to modify his specifications rather than to continue to more complex design techniques.

The new computing architectures that are available might enable the decision-maker to consider knowledge-based systems as a powerful means for modeling, analysis and design of complex economic systems. A variety of radically new architectures, such as parallel, fully or partially distributed (coupled or uncoupled) are now available to decision-makers. Earlier restriction on memory, and the amount of time that can be allocated to a given process, may be overcome with these new architectures. With a new multiprocessor parallel and distributed software architecture many formally intractable problems associated with the modeling, analysis and design of large-scale economic systems can be successfully solved.

In order to provide tractability, to facilitate maintenance of large bodies of knowledge, and to spare users from memorizing every detail of the system, one effective possibility is to apply a deductive knowledge-based [34] approach. The essential idea of this approach is to bridge the modeling, analysis and design system and the user(s) of the system with a deductive knowledge base, which is a map, or a summary, of the system.

5. ROBUSTNESS OF A MACROECONOMIC MODEL

To illustrate some of the concepts developed in this paper, a fourth order macroeconomic stabilization model, with typical parameters, is used throughout. A brief description of the model considered is given in [35]. The model has been intentionally built in order to demonstrate the margin of power the governmental body can exercise on the various sectoral activities. The dynamic behaviour of the system is described by Equations (1)-(3), where:

\[
A = \begin{bmatrix}
0.5021 & 0.3083 & 0.3083 & 0 \\
0.2806 & -0.3819 & 0.2806 & 0 \\
0.1406 & 0.1406 & 0.4403 & -0.2198 \\
0.1109 & 0.1109 & 0.1109 & 0 
\end{bmatrix},
\]

\[
B_1 = \begin{bmatrix}
0.4079 \\
0.1683 \\
0 \\
-0.7389 \\
\end{bmatrix}, \quad B_2 = \begin{bmatrix}
0.0783 \\
0 \\
0 \\
0.1872 \\
\end{bmatrix},
\]

\[
F_1C_1 = [-0.062 - 0.062 0.51 0], \quad F_2C_2 = [0.13 0.13 1.91 0].
\]

The robustness of the perturbed system is first studied for case $\Delta A^1 = 0$, $\Delta B_1^i = 0$ and $B_2^i = 0$, $i = 1, 2$. From the results of Theorem 1, Equation (10), it follows that the perturbed system will remain stable for all $\Delta A^1$ satisfying

$$
\| \Delta A^1 \| < 0.0537.
$$

The result shows that the robustness criteria in form of matrix norm are very conservative, and for the considered macroeconomic model are of limited practical use. The measure of the "size" of the perturbations used in this approach does not distinguish the "direction" of the perturbations, thus this approach tends to be very conservative.

To realistically evaluate the stability robustness properties we will employ the results of Theorem 2, for the case $A^1 = A$ and $B_1^i = 0$, $i = 1, 2$. In this case the perturbation matrix $\Delta A$ is defined by

$$
\Delta A = \gamma(t)A + \Delta A^1.
$$

For $\Delta A^1 = 0$, from Theorem 2, Equation (14), it follows that the perturbed system will remain stable as long as

$$
\gamma(t) \in (-1.5, 0.85),
$$

for all $t = 0, 1, 2, \ldots$. Therefore, the results of Theorem 2 are less conservative than the results of Theorem 1.
6. CONCLUSIONS

A conceptual framework for robustness analysis of large-scale economic systems, and its realization through a knowledge-based computer-aided system have been presented. Particular attention has been given to the modularization of the corresponding software package and the need for distributed problem solving. The impact of the mismatch between the economic system and the corresponding mathematical model, due to the modeling errors and parameter variations on the system stability, suboptimality, controllability and observability, has been discussed. A macroeconomic model, with typical parameters, has been used to demonstrate the power of the considered knowledge-based system.

REFERENCES

34. C. Kellogg, From data management to knowledge management, *Computer* (January, 1986).