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Constraining axion dark matter with Big Bang Nucleosynthesis

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ABSTRACT

We show that Big Bang Nucleosynthesis (BBN) significantly constrains axion-like dark matter. The axion acts like an oscillating QCD θ angle that redshifts in the early Universe, increasing the neutron–proton mass difference at neutron freeze-out. An axion-like particle that couples too strongly to QCD results in the underproduction of ^{4}He during BBN and is thus excluded. The BBN bound overlaps with much of the parameter space that would be covered by proposed searches for a time-varying neutron EDM. The QCD axion does not couple strongly enough to affect BBN.

The axion is a well-motivated dark-mater (DM) candidate that can arise in a variety of models [1]. The allowed mass of these light scalars is relatively unconstrained, spanning many orders of magnitude. Identifying the regions of axion parameter space that are excluded by cosmological and astrophysical constraints is of the utmost importance as it directs the focus of laboratory searches. This Letter presents a new constraint on axion dark matter arising from Big Bang Nucleosynthesis.

The axion was originally introduced to explain why the QCD θ term,

$$S = \frac{\theta}{4\pi^2} \int \operatorname{tr} G \wedge G, \tag{1}$$

is not realized in Nature, often referred to as the strong CP problem [2–4].[1] The QCD θ term in (1) induces a neutron electric dipole moment (EDM) $d_n \approx 2.4 \times 10^{-16}\theta$ e cm [5] that is in tension with experiment for $\theta > 10^{-10}$ [6,7]. The axion solves this problem by promoting the parameter θ to a dynamical field, $\theta \to (a/f_a)$, whose potential is minimized at $a = 0$.

The axion is often assumed to be the pseudo-Goldstone boson of a $U(1)$ PQ symmetry, which is spontaneously broken at some high scale, f_a [3,4,8,9]. For the axion to solve the strong CP problem, the explicit breaking of the PQ symmetry must be absent to very high accuracy in the UV [10,11]. The leading potential that the axion is allowed to receive should come from the QCD chiral anomaly. The QCD instantons break the PQ symmetry explicitly, and in the presence of bare quark masses, the axion picks up a mass [3,4]

$$f_a m_a = f_\pi m_\pi \frac{\sqrt{m_u m_d}}{m_u + m_d}, \tag{2}$$

where $m_\pi \approx 140$ MeV is the pion mass, $f_\pi \approx 92$ MeV is the decay constant, and $m_u \approx 2.3$ MeV ($m_d \approx 4.8$ MeV) is the mass of the up (down) quark.

The cosmological equation of state of axion DM is governed by the classical oscillations of the background field [12–15]:

$$a(t) = a_0 \cos(m_a t) = \frac{\sqrt{2\rho_{\rm DM}}}{m_a} \cos(m_a t). \tag{3}$$

The amplitude a_0 is fixed by requiring that the axion makes up the observed dark matter density, $\rho_{\rm DM}$. A parameter space spanning orders of magnitude in m_a and f_a is available to axion DM. Constraints on axions that come from their coupling to $G \wedge G$ arise from excess cooling of SN 1987A [16,17] and from static neutron EDM measurements [6,7,16]. Axions may also be constrained through their coupling to $E \cdot B$ (see [1] for a review).

In addition to the QCD axion, axion-like particles (ALPs) can arise in many models. ALPs do not necessarily couple to $G \wedge G$; for example, they may only couple to electromagnetism through the operator $E \cdot B$. In these models, (2) may be violated, and in particular, it is possible that

$$f_a m_a \ll \Lambda_{\rm QCD}^2. \tag{4}$$

From this point forward, we will use "axion" to refer to both the standard QCD axion and ALPs that couple to $G \wedge G$ with coupling $\propto f_a^{-1}$ and that satisfy (4).

Axions that couple to $G \wedge G$ and simultaneously satisfy (4) may be tested directly in the near future by proposed laboratory searches for an oscillating axion-induced nucleon EDM [16,18,19]. This Letter focuses on this region of axion parameter space.

[1] Our conventions are $\int \operatorname{tr} G \wedge G = (1/4)\int d^4x \epsilon^{\mu\nu\alpha\beta} \operatorname{tr} G_{\mu\nu} G_{\alpha\beta}$, where $G = (1/2)G_{\mu\nu} dx^\mu \wedge dx^\nu$ is the gluon field-strength with the trace taken over gauge indices. In the following, we use also $\tilde{G}^{\mu\nu} = (1/2)\epsilon^{\mu\nu\alpha\beta} G_{\alpha\beta}$.

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First, we use chiral perturbation theory (ChPT) to show that the presence of an axion-induced nucleon EDM is in tension with (4) because the axion contribution to the nucleon EDM is associated with the irreducible QCD contribution to the axion mass in (2). As far as we know, the only way to avoid this minimum axion mass is to invoke fine-tuned cancellations, exacerbating the strong CP problem. In particular, we show in Appendix A that it is not possible to reach the parameter space (4) by invoking mixing between multiple axion states. Contrarily, $f_a m_a \gg \Lambda^2_{\rm QCD}$ is possible without fine tuning through axion mixing.

Even if one is willing to ignore fine-tuning arguments, Big Bang Nucleosynthesis (BBN) provides a strong observational constraint. The constraint arises from two simple observations. First, the QCD θ term leads to a shift in the neutron–proton mass difference, as pointed out in [20]. This nuclear mass difference is again dictated by ChPT and is directly related to the axion-induced EDM. Second, the effective θ term induced by axion DM redshifts in the early Universe, roughly as $\theta \sim (1+z)^{3/2}$. Thus, while the effect of axion DM on the neutron–proton mass difference today seems unobservably small, it can be large enough to disturb the production of light elements at the time of BBN ($z \sim 10^{10}$).

We begin by recalling the results from ChPT that relate the axion mass and some of its couplings. Considering only the axion and strongly-interacting SM fields just above the QCD scale, the most general effective Lagrangian that connects the axion to the SM and respects the axion shift symmetry is

$$\mathcal{L} = -\frac{a}{f_a}\frac{G^a_{\mu\nu}\tilde{G}^{a\mu\nu}}{32\pi^2} - \frac{\partial_\mu a}{f_a}\sum_\psi c_\psi \bar{\psi}\bar{\sigma}^\mu \psi \tag{5}$$

to leading order in f_a^{-1} [21]. The left-handed Weyl spinors ψ include u, u^c, d, d^c etc. The coefficients c_ψ are model-dependent. In general they can be off-diagonal in flavor space, but this does not affect the following discussion.

Below the QCD scale, (5) is translated to the chiral Lagrangian, and the axion couplings with pions and nucleons may be computed from ordinary ChPT. The axion enters into the chiral Lagrangian only through the quark mass spurion and through mixed derivative couplings with the neutral pion. Working in the physical basis after diagonalizing the axion–pion mass matrix and kinetic terms, we are particularly interested in the following terms in the chiral Lagrangian:

$$\begin{aligned}\mathcal{L} \supset &-\frac{1}{2}\frac{f_\pi^2 m_\pi^2 m_u m_d}{(m_u+m_d)^2}\left(\frac{a}{f_a}\right)^2 \\ &-\bar{N}\pi\cdot\sigma\left(i\gamma^5 g_{\pi NN} - 2\bar{g}_{\pi NN}\frac{a}{f_a}\right)N \\ &+\frac{f_\pi \bar{g}_{\pi NN}}{2}\frac{m_d-m_u}{m_d+m_u}\left(\frac{a}{f_a}\right)^2 \bar{N}\sigma^3 N. \end{aligned}\tag{6}$$

Here $N = \binom{p}{n}$ are the nucleons, and the numerical couplings are $g_{\pi NN} \approx 13.5$ and $\bar{g}_{\pi NN} \approx \frac{m_u m_d}{m_u+m_d}\frac{2(M_\Xi - M_\Sigma)}{(2m_s-m_u-m_d)f_\pi} \approx 0.023$ [20,22].

The first line in (6) is the irreducible contribution to the axion mass quoted in (2). We know of no way to eliminate this contribution for an axion with decay constant f_a besides to cancel it with some unrelated mass correction associated with some new Lagrangian term $\Delta\mathcal{L}(a) \propto \delta m^2(a+\delta\theta)^2$. Such a cancelation would involve fine-tuning the parameter δm^2 by an amount

$$\Delta_{\rm mass} \sim \frac{f_a^2 m_a^2}{f_\pi^2 m_\pi^2} \sim 10^{-14}\left(\frac{f_a m_a}{10^{-9}\ {\rm GeV}^2}\right)^2. \tag{7}$$

Moreover, $\delta\theta$ must also be tuned to avoid CP violation, thereby restoring the strong CP problem on top of the mass fine-tuning in (7). We comment that it is not possible to reach the parameter space (4) by introducing multiple axion-like states and invoking mixing between them, as might be conceived in some string-inspired models [23] (see Appendix A).

The second line in (6) gives the dominant contribution to the axion-induced neutron EDM [18,22],

$$d_n \approx \left(\frac{a}{f_a}\right)\frac{e g_{\pi NN}\bar{g}_{\pi NN}}{4\pi^2}\frac{\ln(4\pi f_\pi/m_\pi)}{m_n}, \tag{8}$$

with m_n the neutron mass.

The third line in (6) gives the axion-induced neutron–proton mass splitting,

$$\begin{aligned} m_n - m_p &= Q_0 + \delta Q, \\ \delta Q &\approx \frac{f_\pi \bar{g}_{\pi NN}}{2}\left(\frac{m_d-m_u}{m_d+m_u}\right)\left(\frac{a}{f_a}\right)^2 \\ &\approx (0.37\ {\rm MeV})\left(\frac{a}{f_a}\right)^2, \end{aligned}\tag{9}$$

when evaluated on a classical axion-field background. $Q_0 \approx 1.293$ MeV is the measured mass difference between the neutron and proton. Thus, an axion field that induces a nuclear EDM also affects the neutron–proton mass splitting in a directly related way. Moreover, the relation between the two effects does not depend on the model-dependent c_ψ coefficients, to leading order in $1/f_a$.[2] We now explore the consequence of the shift in the nuclear mass difference on nucleosynthesis.

For $m_a \gg H(z)$, where $H(z)$ is the proper Hubble expansion rate at redshift z, the axion DM may be treated as an ensemble of Bose–Einstein condensed non-relativistic particles [15]. Neglecting any temperature dependence in m_a, the time-dependent effective θ angle in this limit is

$$\begin{aligned}\theta_{\rm eff}(t) &= \left(1+z(t)\right)^{3/2}\frac{\sqrt{2\bar{\rho}_{\rm DM}}}{f_a m_a}\cos(m_a t) \\ &\approx 5\times 10^{-9}\left(\frac{{\rm GeV}^2}{f_a m_a}\right)\left(\frac{1+z(t)}{10^{10}}\right)^{3/2}\cos(m_a t), \end{aligned}\tag{10}$$

where $\bar{\rho}_{\rm DM} \approx 2.7\times 10^{-27}$ kg/m^3 is the mean cosmological DM energy density today [24]. Neutron freeze-out occurs at temperatures of order 1 MeV, meaning that (10) is adequate for calculating a BBN bound as long as $m_a \gg (1\ {\rm MeV})^2/m_{\rm pl} \approx 10^{-16}$ eV. We begin by discussing m_a in this regime and extend the calculation to the ultra-light regime, $m_a \ll 10^{-16}$ eV, later.

Substituting (10) into (9) shows that axion DM increases the mass difference between the neutron and proton at BBN. This reduces the relative occupation number of neutrons compared to that of protons in thermal equilibrium just before neutron freeze-out, reducing the resulting mass fraction, Y_p, of ^{4}He. The net effect is stronger at smaller $f_a m_a$. We now provide an analytic estimate of the dependence of Y_p on $f_a m_a$, subsequently moving on to a more precise numerical calculation.

After the quark–hadron transition, neutrons and protons are kept in equilibrium through the weak interactions

$$\begin{aligned} &n \longleftrightarrow p + e^- + \bar{\nu}_e, \\ &\nu_e + n \longleftrightarrow p + e^-, \\ &e^+ + n \longleftrightarrow p + \bar{\nu}_e. \end{aligned}\tag{11}$$

[2] The relation between the nuclear EDM and the neutron–proton mass splitting could be modified if we allow for other sources of explicit PQ symmetry breaking beyond the mass-tuning term. We do not consider such possibilities in this Letter.

The rates of these reactions become smaller than the Hubble parameter around the freeze-out temperature $T_F \approx 0.8$ MeV. Below this temperature, neutrons and protons fall out of equilibrium, and the neutron to proton ratio is approximately fixed to the ratio of n/p at freeze-out:

$$\left(\frac{n}{p}\right)_{\text{freeze-out}} \approx e^{-Q_F/T_F}, \tag{12}$$

where $Q_F = Q_0 + \delta Q_F$ is the neutron–proton mass difference at freeze-out.

For $m_a > 10^{-16}$ eV, the axion oscillation frequency m_a is greater than the rate of the weak interactions in (11) when $T \approx T_F$. Each weak scattering event therefore sees a different value for Q_F, and Q_F in (12) should be averaged over times of order m_a^{-1}. This amounts to replacing the factor $\cos^2(m_a t)$ by a 1/2 when using (10).

In addition to the direct effect on Q_F, decreasing $f_a m_a$ also decreases the freeze-out temperature T_F itself. This effect is small in the range of $f_a m_a$ of interest here. For now, we assume that $T_F \sim 0.8$ MeV is unchanged and relax this assumption later in the numerical calculation.

Most of the neutrons left over at the end of the deuterium bottleneck, which occurs at a temperature $T_{\text{Nuc}} \approx 0.086$ MeV, are converted into ^{4}He. The mass fraction of ^{4}He is approximately

$$Y_p \approx \frac{2(n/p)_{\text{Nuc}}}{1+(n/p)_{\text{Nuc}}}. \tag{13}$$

Between freeze-out and nucleosynthesis, a small fraction of neutrons are lost by free decay. To a first approximation, we neglect neutron decay and estimate the fractional change in Y_p as a result of the axion DM,

$$\frac{\delta Y_p}{Y_p} \equiv \frac{Y_p^0 - Y_p(f_a m_a)}{Y_p^0}, \tag{14}$$

by taking $(n/p)_{\text{Nuc}} \approx (n/p)_{\text{freeze-out}}$ and using (12). Y_p^0 denotes the value of Y_p in the absence of axion DM.

The mass fraction Y_p^0 of ^{4}He is measured to be in the range 0.227–0.266 [1], to 95% confidence. Taking the conservative bound $\delta Y_p/Y_p < 10\%$, we find the constraint $f_a m_a \gtrsim 10^{-9}$ GeV^2.

A more accurate numerical method for calculating Y_p as a function of $f_a m_a$ involves integrating the rate equation for neutrons and protons (see, for example, [25,26]). The rates for neutron $\Leftrightarrow$ proton conversion are modified in the presence of axion DM because of the correction δQ to the neutron–proton mass difference. For $m_a > 10^{-16}$ eV, we solve the rate equation using the time-averaged rates, where the averaging is performed over the axion oscillation time $\sim m_a^{-1}$.

Below the freeze-out temperature, it is also important to include the effect of the deuterons, because ^{4}He production proceeds through reactions that involve deuteron production. The deuteron fraction is highly suppressed until the temperature goes sufficiently below $T \approx T_{\text{Nuc}} \sim \epsilon_D/26$, where ϵ_D is the deuteron binding energy. The dependence of ϵ_D on θ is not known, and this means that we do not know if axion DM delays or speeds up the end of the deuterium bottleneck. However, we expect this effect to be sub-leading. The reason is that by the time the Universe has cooled to temperatures of order 0.1 MeV, θ_{eff} is only $\sim 4\%$ of its value at T_F. In addition, the effect of free neutron decay on the ^{4}He abundance is small.

The results of the numerical calculation are as follows. At large $f_a m_a$, we find $Y_p \approx 0.247$, which is consistent both with the observed abundance and with the more precise numerical calculations [1]. We find that $\delta Y_p/Y_p \approx 10\%$ when $f_a m_a \approx 1.3 \times 10^{-9}$ GeV^2, confirming the analytical estimate.

The results above pertain to $m_a > 10^{-16}$ eV. For $m_a \ll 10^{-16}$ eV, the axion field is approximately constant during BBN and does not redshift up with increasing temperature, due to the Hubble friction.[3] In this regime, the ^{4}He BBN bound is approximately

$$f_a m_a \gtrsim \sqrt{2} \times \left(1.3 \times 10^{-9}\ \text{GeV}^2\right)\left(\frac{1+z_m}{1+z_F}\right)^{3/2} \approx \left(1.8 \times 10^{-9}\ \text{GeV}^2\right)\left(\frac{m_a}{10^{-16}\ \text{eV}}\right)^{3/4}, \tag{15}$$

where z_F is the redshift at neutron freeze-out and z_m is defined via $H(z_m) = m_a$.

We now discuss the implications for the axion DM-induced EDM experiments proposed in Refs. [16,18,19]. The authors point out that the oscillating background field of axion DM induces an effective, oscillating neutron EDM,

$$d_n(t) = g_d a(t) = \frac{\sqrt{2\rho_{\text{DM}}}}{g_d^{-1} m_a}\cos(m_a t), \qquad g_d \approx \frac{(2.4 \times 10^{-16}\ \text{e cm})}{f_a}. \tag{16}$$

For the QCD axion, the amplitude of the oscillating EDM is $d_n \sim 10^{-34}$ e cm, assuming a local DM density of $\rho_{\text{DM}} \approx 0.3$ GeV/cm^3. The experiment proposed in [16,19] detects this small, oscillating nuclear EDM using NMR techniques, and the prospective sensitivity is shown in the right panel of Fig. 1 by the regions above the blue dashed and red solid lines.

Fig. 1 shows the region (orange) of the g_d, m_a parameter space that is excluded by the ^{4}He abundance from BBN. The width of the solid orange line takes into account the roughly 40% uncertainty in the expression for g_d in (16) [5]. The solid black line shows the prediction for the QCD axion, which lies safely below the BBN bound. Static EDM searches exclude the region to the left of the blue dashed line [6,7,16], and a conservative bound from SN 1987A excludes the region above the green dashed line (see e.g., [16,17]). Note that thermal production of axions, increasing N_{eff} at the time of BBN [27], provides a bound on f_a that is comparable to the SN 1987A result. Model-dependent constraints also arise from the axion's coupling to $E \cdot B$ (not shown, see [1]).

Our BBN analysis neglects the temperature dependence of the axion mass. If such temperature dependence is important, then in the parameter space defined by (4) the axion mass may go negative at some time between now ($z = 0$) and BBN ($z \sim 10^{10}$). In this case, BBN would see a value of θ dependent on the extra PQ breaking dynamics, regardless of its value today. Thus θ would naturally be $O(1)$, strengthening the bound. Alternatively, if the QCD-induced contribution to the axion mass increases between today and T_F, the BBN bound is weakened. Resolving this issue requires understanding the axion mass at temperatures significantly below T_F.

To conclude, axion DM that couples to QCD induces operators in the chiral Lagrangian that redshift up in the early Universe. For $m_a f_a \sim 10^{-9}$ GeV^2, the perturbation parameter a/f_a that controls these operators approaches order unity at the time of BBN, even though it is negligible today. We showed that the production of ^{4}He during BBN provides a novel constraint on the coupling of axion DM to QCD. In particular, BBN excludes a large region of axion DM parameter space, with implications for current and future searches for axion DM-induced nuclear EDMs. Our bound is

[3] Of course, when m_a is similar in size to the Hubble parameter at freeze-out ($m_a \sim 10^{-16}$ eV), the calculation is more complicated. We save the details for future work.

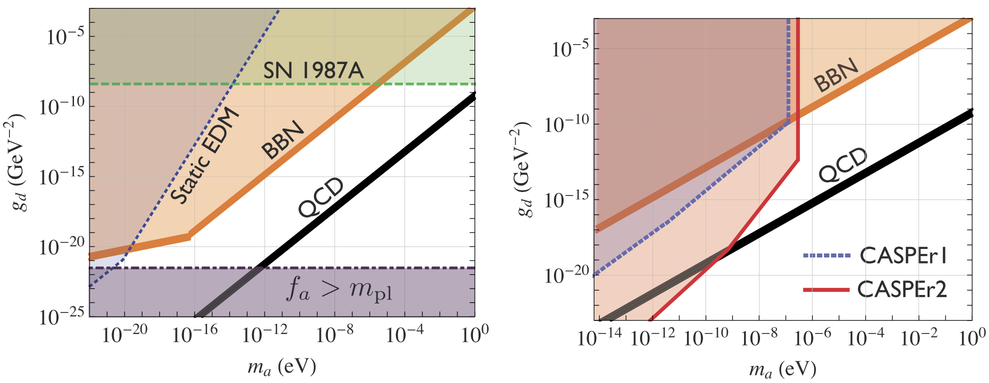

Fig. 1. Left panel: BBN excluded region in the (g_d, m_a) plane is shown in orange. Other constraints include static EDM searches (blue shaded region, dashed blue boundary) and the bound from SN 1987A estimated conservatively in [16,17] (green shaded region). The shaded purple region with dot-dashed boundary denotes $f_a > m_{\rm pl}$. Right panel: The future projected sensitivity of the oscillating EDM search of Refs. [16,19]. CASPEr1 and CASPEr2 are the first and second generations of the experiments, respectively. The black line in both panels represents the QCD axion, $f_a m_a \approx \Lambda^2_{\rm QCD}$. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

conservative, allowing for a 10% deviation in the predicted amount of ^{4}He and ignoring deviations in the abundances of other light elements, such as deuterium. Moreover, we showed that if an axion lives anywhere above the black line in Fig. 1, then the strong CP problem is reintroduced and made worse. It would be interesting to investigate other constraints on axion-chiral Lagrangian operators that may arise from astrophysics.

1. Note added in proof

After our paper was completed, a measurement of the primordial B-mode power spectrum was announced by the BICEP2 Collaboration [28]. If the BICEP2 result holds upon further scrutiny, then it would imply that $g_d \geq 4 \times 10^{-17}$ GeV^{-2} [29,30].

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Appendix A. Axion mixing

The usual QCD axion satisfies the constraint (2). It would be phenomenologically interesting if one could build a (potentially more complicated) model, in which (2) is violated and, instead, $f_a m_a$ satisfies (4).

We show that is not possible to reach the parameter space (4) by introducing multiple axion-like states and invoking mixing between them. This situation might be conceived in certain string-inspired models [23]. To see this, consider an additional axion A coupled to $G \wedge G$ with PQ breaking scale F_A, so that

$$\mathcal{L} \supset -\left(\frac{a}{f_a} + \frac{A}{F_A}\right)\frac{G^a_{\mu\nu}\tilde{G}^{a\mu\nu}}{32\pi^2}. \tag{A.1}$$

We can allow an extra potential for A, generated by some UV physics, without spoiling strong CP, so that the low energy effective potential contains the terms

$$V = \frac{1}{2}m_a^2\left(a + \frac{f_a}{F_A}A\right)^2 + \frac{1}{2}m_{\rm UV}^2 A^2 + \ldots \tag{A.2}$$

The new mass eigenstates and their couplings to $G \wedge G$ depend on the dimensionless ratios $\alpha = f_a/F_A$ and $\beta = m^2_{\rm UV}/m^2_a$. We are free to choose $|\alpha| \leq 1$. Avoiding a tachyonic state requires $\beta > 0$. With these constraints, it is straightforward to verify that $fm \geq f_\pi m_\pi \sqrt{m_u m_d}/(m_u + m_d)$ for both mass eigenstates. We learn that while it is not possible to reach the parameter space (4) by mixing multiple axions, it may be possible to build a model with $f_a m_a > \Lambda^2_{\rm QCD}$ while preserving the solution to the strong CP problem.

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