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A model to forecast the response of concrete under severe loadings the μ damage model

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Abstract

Among the "theories" applied to model concrete behavior, damage mechanics has proven to be efficient. One of the first models for concrete introduced within such a framework is Mazars' damage model. A new formulation of such a model, called the "μ model", is proposed herein for the purpose of 3D cyclic and dynamic loading, based on a coupling of elasticity and damage within an isotropic formulation. Unilateral behavior (i.e. the opening and closure of cracks) is introduced by the use of two thermodynamic variables. A threshold surface is then associated with each of these variables and strain rate acts on the initial threshold. Applications are presented on plain or reinforced concrete elements subjected to various loading (uni- and multi-axial, cyclic, dynamic). A comparison with experimental results demonstrates the effectiveness of the various selected options.

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1. Introduction

Structural dynamic analysis generates, for RC structures, complex and large numerical problems. Discretization technique and material modelling are important key points to control the size. For concrete, damage mechanics is a good candidate to model in a simple way its behaviour (Simo and Ju 1987, La Borderie et al. 1994, Jirasek 2004).

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Based on previous work (Mazars 1986, Pontiroli et al. 2010), the μ *damage model* is a new model, as complete and simple as possible, which implies formulating the set of main assumptions listed below:

- Concrete behaviour is considered as the combination of elasticity and damage.
- The damage description is assumed to be isotropic and directly affects the stiffness evolution of the material. Let $\boldsymbol{\Lambda}$ be the stiffness matrix of the original material, then the matrix for the damaged material is given by:

$$\boldsymbol{\Lambda}_d = \boldsymbol{\Lambda}(1-d) \tag{1}$$

- As opposed to classical damage models, *d* denotes the effective damage. Classically speaking, damage is a variable that describes the microcracking state of the material (Lemaitre, Chaboche 1990). While, *d* describes the effect of damage on the stiffness activated by loading. In a cracked structure, *d* must then be able to describe the effects of crack opening and closure (i.e. unilateral effects).
- Two principal damage modes are considered (tension and compression), to be subsequently associated with two thermodynamic variables Y_t and Y_c, which characterize the extreme loading state reached in the tensile part and compressive part, respectively, of the strain space.

2. Modelling concepts

In the μ model, the thermodynamic variables are defined from two equivalent deformations ε_t and ε_c (scalar indicators of the state of local strain). ε_t favours tensile strain and ε_c favours compression strains.

$$\varepsilon_t = \frac{I_\varepsilon}{2(1-2\upsilon)} + \frac{\sqrt{J_\varepsilon}}{2(1+\upsilon)} \qquad \varepsilon_c = \frac{I_\varepsilon}{5(1-2\upsilon)} + \frac{6\sqrt{J_\varepsilon}}{5(1+\upsilon)} \tag{2}$$

ν is the Poisson's ratio, I_ε is the first invariant of the strain tensor and J_ε the second invariant of the deviatoric part of the strain tensor.

$$I_\varepsilon = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 \qquad J_\varepsilon = \frac{1}{2}\left[(\varepsilon_1-\varepsilon_2)^2 + (\varepsilon_2-\varepsilon_3)^2 + (\varepsilon_3-\varepsilon_1)^2\right] \tag{3}$$

ε_i are the principal strains. From this, the expressions for the two internal variables are:

$$Y_t = max(\varepsilon_{t0}, max(\varepsilon_t)) \qquad Y_c = max(\varepsilon_{c0}, max(\varepsilon_c)) \tag{4}$$

They are the maximum equivalent strain reached, beyond a threshold (ε_{t0} and ε_{c0}), during the loading history. They are each associated with a loading surface. As Drucker Prager model (1952), the two loading surfaces are cones centred on the trisector of the strain space. They operate independently of each other.

$$f_t = \varepsilon_t - Y_t = 0 \qquad f_c = \varepsilon_c - Y_c = 0 \tag{5}$$

The second new feature, compared to the original model, is managing the damage variable, denoted *d*. This variable is controlled by the variable Y, a combination of both internal variables Y_t and Y_c:

$$Y = rY_t + (1-r)Y_c \tag{6}$$

r is the triaxiality factor, which provides information on the actual loading state (Lee, Fenves 1998). It is equal to 1 in the tension area and to 0 in the compression area. It takes intermediate values for tension-compression situations. Its expression is:

$$r = \frac{\sum_i \langle\tilde{\sigma}_i\rangle_+}{\sum_i |\tilde{\sigma}_i|} \quad \text{with} \quad \tilde{\sigma}_i = \frac{\sigma_i}{(1-D_\mu)} \text{ (effective stress – Lemaitre, Chaboche 1990)} \tag{7}$$

|x| and <x>$_+$ are the absolute and the positive values of x respectively.

The damage evolution law is the same type as the original model. Its expression is:

$$d = 1 - \frac{(1-A)Y_0}{Y} - A\,exp\left(-B\left(Y - Y_0\right)\right) \tag{8}$$

Y_0 is the initial threshold of the driven variable Y , function of the equivalent strain thresholds defined equations (2) and (3):

$$Y_0 = r\varepsilon_{t0} + (1-r)\varepsilon_{c0} \tag{9}$$

A and B are two variables which depend on the actual loading state through the r value. They allow to reproduce the different behaviours from quasi brittle to hardening including mixed situation due to shear effects.

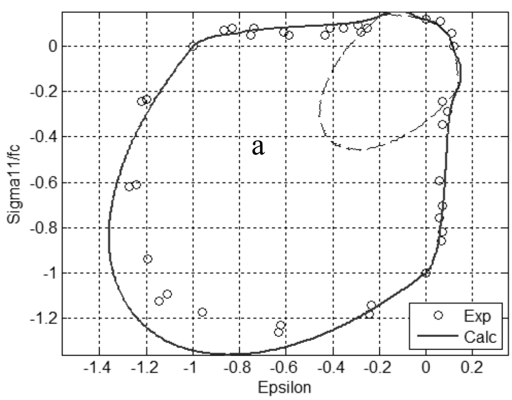
The following evolution for A and B has been chosen:

$$A = A_t\left(2r^2(1-2k) - r(1-4k)\right) + A_c\left(2r^2 - 3r + 1\right) \tag{10}$$

$$B = r^{(r^2-2r+2)}B_t + (1 - r^{(r^2-2r+2)})B_c \tag{11}$$

A_t, B_t, A_c and B_c are all material parameters, just like in the original model and k the ratio A/At used to calibrate pure shear behaviour. For more detailed on the model see (Mazars et al 2013).

(6) and (8) induce, and this is the new compared to other unilateral models, an irregular evolution of the damage variable since it depends on r. However the variables Yt and Yc are continuously increasing, it is the reason why they are chosen as thermodynamic variables.



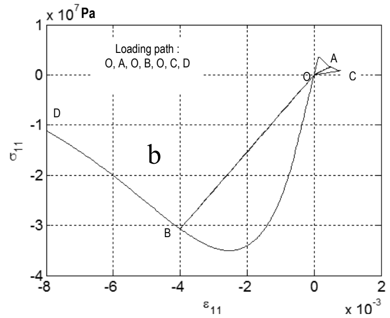


Figure 1. μ unilateral model : a) damage threshold (dashed curve) and failure surface (solid curve), b) response for a tension-compression loading path.

Figure 1a shows, in the plane $\sigma_3 = 0$, the trace of the surface initiation of damage (d = 0) and this of failure (maximum stress envelop) compared to experimental values obtained on biaxial tests performed by Kupfer et al. (1973). On figure 1b the curve shows the unilateral response of the μ model for a tension-compression loading path.

3. Numerical simulations on structures

Even though the μ model can be applied to simulate complex cases, its equations and implementation remain quite simple. Its formulation is explicit, meaning that all variables can be evaluated directly from the strain increment. Firstly its implementation has been done on Matlab and more recently on Cast3M.

3.1. Application on reinforced concrete beams under cyclic loading

The applications covered by the μ model are mainly severe loadings on concrete structures. Among these applications, earthquake is an important issue. Earthquakes generate on structural elements non linear cyclic loadings. LMT at ENS Cachan in France has been performed an experimental campaign on reinforced concrete beams in order to study phenomena that play a major role in the response of RC structures during an earthquake (Ragueneau et al., 2010, 2013). Phenomena such as, damage evolutions during increasing loading, unilateral effects and energy dissipation due to cyclic loads have been analysed. These results serve for the applications that follow.

Beams are tested with a three-point bending set up. The size of the specimens is: 1.65m long, 0.22m high and 0.15m large (Figure 2a). Concrete is a regular one: fc= 35MPa, ft=3.2MPa, E=28GPa.

The loading path is reproduced Figure 2c. It is displacement controlled and includes sets of 3 cycles at progressive intensity (from ±1mm to ±8mm).

The test specimen has been modelled using Q4 elements (four-nodes) under a plane strain assumption and bar elements are used for rebar. The mesh (711 nodes, 776 elements) is uniform in the central part of the beam and boundary conditions are defined to correctly represent the experimental test (Figure 2b).

In order to avoid mesh sensitivity, crack band approach using the fracture energy concept, has been used in this application (Bazant et al., 1983). G_f is defined according to Planas *et al.* (1992) and Bazant (2002). This means that in the central part of the beam (where damage and plasticity are concentrated) the size of elements is consistent with the crack band width:

$$w_c = \frac{2G_f}{f_t^2}\left(\frac{1}{E} - \frac{1}{E_t}\right)^{-1} \tag{12}$$

E_t being the post peak tangent stiffness for an equivalent triangular shape of the σ–ε curve. The selected model parameter values have been adopted in accordance with the data provided from material tests. Regarding rebar, bar elements are used respecting the elasto-plastic hardening model from Menegoto-Pinto (1973).

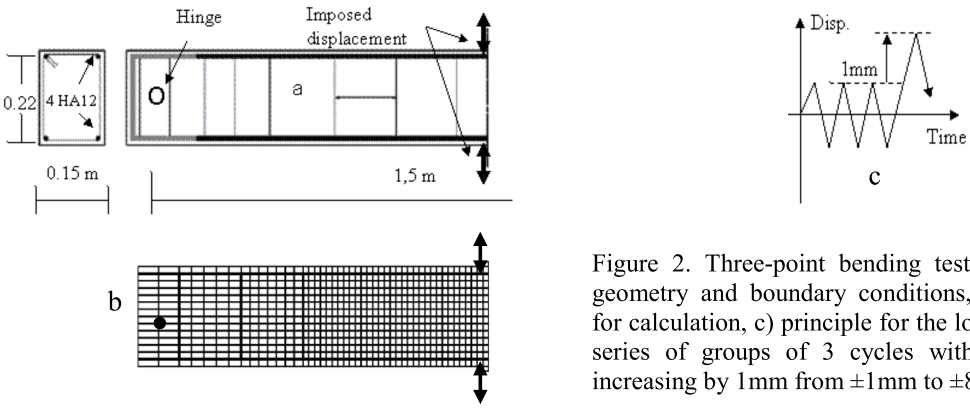
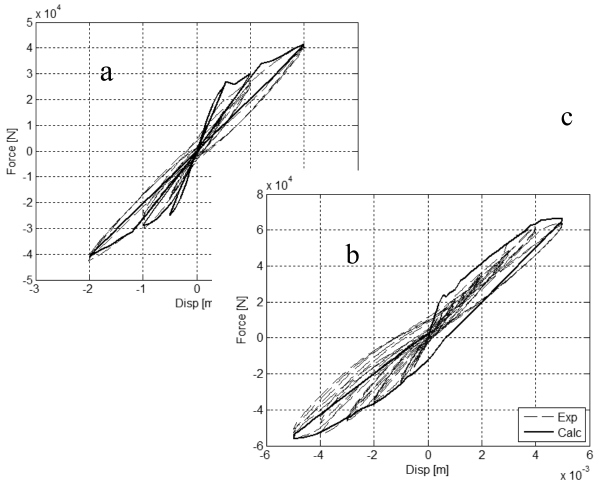


Figure 2. Three-point bending tests: a) specimen geometry and boundary conditions, b) mesh used for calculation, c) principle for the loading program, series of groups of 3 cycles with an amplitude increasing by 1mm from ±1mm to ±8mm.

Various situations have been modelled. In Figure 3, the load-displacement curves resulting from the simulations are compared to the experimental points, i) for a same loading path up to ± 2mm the comparison shows very good agreement (figure 3a), ii) for a global cyclic loading ± 5mm, including plasticity of rebar (figure 3b), the results are also very good.

From these results one can conclude that the stiffness recovery as modelled by the μ model reproduces very well the experimental results and the choice done to have no representation of the permanent concrete strain does not penalize the results.

Such a modelling approach serves to access local information indicating what happens inside the damaged areas both in concrete and in rebar. Figure 3c shows, at a given stage of the loading (±2 mm), the damage field on the lateral beam surface. This figure highlights where damage is localized and then, where cracks are predicted by calculation. More precisely, the figure provides the situation for two different stages (+2mm and -2mm) after a series of cycles extending to ±2 mm (the coloured marks indicate where $0.99 < d < 1$). During a cyclic loading, the effective damage d evolves until reaching a maximum value in one direction; due to changes in the triaxial factor r (7) from 1 to 0, this maximum value vanishes once the local stress is reversed. One can observed that the effective damage d can be used as an indicator to represent a crack opening stage. A good level of agreement has been observed with digital image correlation analysis during experiments (Ragueneau *et al,* 2010).



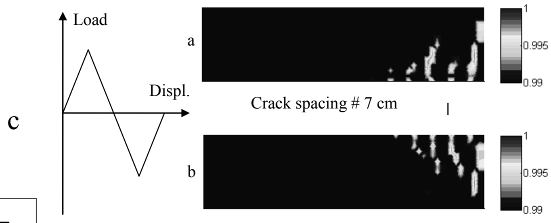


Figure 3. Bending test, experiment-calculation comparison, a) total path up to ±2 mm, b) cycle at ±5 mm, c) Effective damage used as a crack opening indicator ($d > 0.99$), after a displacement up to +2 mm (a) followed by a displacement up to -2mm (b).

3.2. Strain rates effects

Among the various severe loading situations for concrete structures there are blast and impact. To simulate these effects modelling must include strain rate effects. It is well known that concrete strength is strain rate dependent, particularly under traction for which inertia effects cannot explain the phenomenon. As presented in (Pontiroli et al, 2010), this effect is accounted for using a dynamic threshold ($\varepsilon_{0t}{}^{d}$) instead of a static one ($\varepsilon_{0t}{}^{s}$). Dynamic threshold is deduced from the static one through a dynamic increase factor which depends on the strain rate $\dot{\varepsilon}$ (*=dε/dt*):

$$R_t = \dot{\varepsilon}_{0t}^{d} / \dot{\varepsilon}_{0t}^{s} = min\,[max\,(1.0 + a_t \dot{\varepsilon}^{bt}, c_t\, \dot{\varepsilon}^{dt}),\ 5] \tag{13}$$

a_t , b_t , c_t , d_t are material coefficients defined by the user, from experimental results.

An experimental campaign has been recently conducted at LEM3 laboratory in Metz-France (Erzar and Forquin, 2011). It includes quasi-static tests and dynamic tests conducted on dry and wet concrete specimens. A high speed hydraulic press has been used for intermediate strain rate and an experimental Hopkinson bar device, based on the spalling technique, was used for high rates of strain. From these tests, original identification techniques have been developed to deduce the tensile strength of the material. For an ordinary concrete (fc#30MPa) the whole results has been given figure 4a.

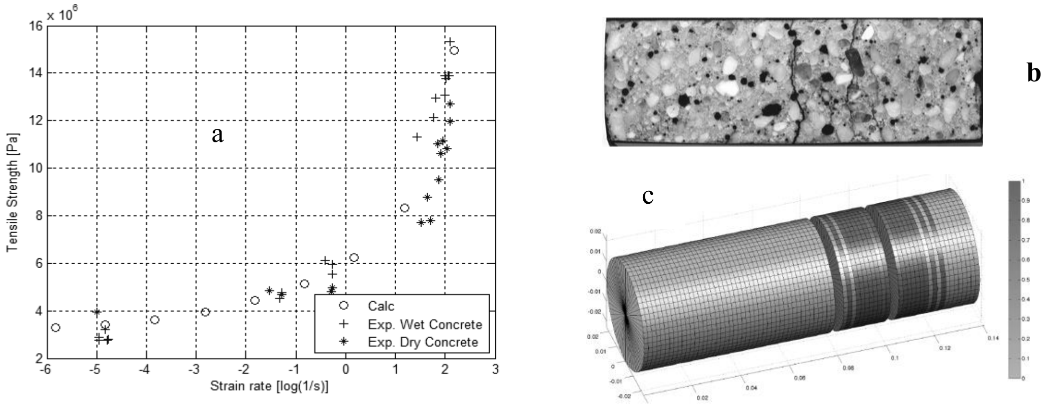


Figure 4. Strain rate effect: a) Tensile strength vs strain rate, experiments (Erzar, Forquin 2011) and model, b) spalling post-test cracking on the concrete specimen, c) computed damage zone exhibiting two main cracks.

In the μ model the dynamic tensile strength is assumed to be the peak of the stress strain curve $f_t = E\, \varepsilon_{0t}^{\ d}$. From (13), $f_t = f(\dot{\varepsilon})$ can be obtained. These results are also plotted on figure 4a, showing that calculations provide very good results.
To highlight the ability of the μ model to describe high velocity problems, a simulation of the spalling test performed at LEM3 has been done. The specimen is a cylinder (L=140mm, Φ=45mm) in contact on left-edge with the Hopkinson bar from which a compressive wave comes. The transmitted compressive pulse propagates along the specimen until the free end where it is reflected as a tensile pulse that travels in the opposite direction. When the reflected pulse exceeds in amplitude the incident one, a dynamic tensile loading spreads out the specimen leading to a possible fracture. Figure 4b gives the experimental result obtained for a given pulse leading to a multi-fracture of the specimen. On figure 4c, the calculation performed on a fibre beam model shows a similar result.

4. Conclusions

This paper presents a new damage model (μ model), it is based on the principles of isotropic damage mechanics. Two thermodynamic variables have been defined to describe, within a 3D formulation, the unilateral behaviour of concrete (crack opening and closure), which is essential for cyclic loadings. Strain rate effects have been also introduced.

In conclusion, this model can provide a useful tool for engineering applications, as was initially expected, and moreover is able to cover a wide diversity of monotonic or cyclic problems from quasi-static to high-velocity loadings (i.e. earthquakes, blasts, soft impacts). Some improvements are in progress related, I) to the ability to account for high confinement (hard impact) and, ii) to simplified approaches (fibre beam elements) based on the use of a 1D enriched version of the μ damage model including hysteretic loops and permanent strains (Mazars et al. 2014).

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