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Multiple Nonholonomic Wheeled Mobile Robots Trajectory Tracking While Maintaining Time-Varying Formation via Synchronous Controller

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Abstract

A new synchronization control method is developed for multiple nonholonomic wheeled mobile robot path tracking while maintaining time-varying formations. Every robot is controlled to track its desired path while its movement is synchronized with nearby robots to maintain the desired time-varying formation. A new derivation for dynamic model of the nonholonomic wheeled mobile robot (WMR) is proposed based on the Lagrange methods. The robot model is divided into translational and rotational models, such that, each model will be controlled individually. Furthermore, synchronous controller for each robot’s translation is developed to guarantee the asymptotic stability of both position and synchronization errors. In addition, an orientation controller is proposed to ensure that the robot is always oriented towards its desired position. The simulation results validate the effectiveness of the proposed synchronous controller in the formation control tasks.

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1. Introduction

The importance of cooperative control research of a multi-agent system has been raised in latest decades. This is motivated by the technological advancements and the growth of affordable communication, computation and sensing apparatuses. In order to operate efficiently and fulfill good execution, multi-robot system has to be correctly organized. Multiple robots collaboration is one of the most important topics in robotics.

Formation control has received a lot of attention from the researcher for its many applications such as surveillance, search and rescue, transportation, formation, etc. One of the important cooperative tasks is multi-robot formation control, where a team of robots can maintain the desired formation shape along their path or change the formation shape when required. Several control approaches have been proposed to solve the formation control problem. The leader-following, virtual structure and behavior-based approaches are classified as the main methods for the formation control. In the behavior-based approach [1-3] numerous desired behaviors are advocated for each robot, and the effective formation control is derived from a weighted summation of individual behavioral output. The advantage of this method is its suitability for generation of control behavior in the appearance of multiple competing goals, and the explicit feedback through communication between adjacent robots. The disadvantage is that the group behavior cannot be clearly described, and there is a difficulty in describe the group dynamics. In addition, it is difficult to evaluate mathematically the stability of the whole system. In the virtual structure approach [4-6], the entire formation is managed as a one unit. The desired posture for every

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robot is given to the virtual structure that traces out the path for each robot in the formation to be tracked. Its advantages are easy to prescribe formation strategy, guaranteed stability and more robust to formation by using group dynamics. On the contrary, the disadvantages are difficult to control multi-robot formation in a decentralized approach and unsuitable for time-varying formation. In the leader-following approach [7-11], each follower robot had been controlled to track one leader with $l - \phi$ controller, or two leader with $l - l$ controller. The leader-following method is easy to implement by just two controllers, it has simple structure and reliance on local sensor data only. However, there are no feedback errors from the follower to the leader; moreover, the leader is a single entity for failure and it is hard to consider the performance skills of different robots.

Furthermore, a recent approach to formation control is the synchronization method [12]. One of the most effective approaches used for synchronization is the cross-coupling control established by Koren [13], which can minimize the formation error efficiently. In the synchronization approach, the control goal is derived according to the desired formation, which based on the synchronization error defined as the differential position errors between every pair of two adjacent robots. The motion control for the individual robot has been divided into two parts: one is to force the robot to move along the desired path to accomplish the tracking control goal. The second one is to synchronize the motion of each robot with the two neighbors’ robots. Each robot will synchronize its motion with the two nearby robot. As a result, all the robots in the system will be synchronized, where it will reduce the system complexity in achieving the synchronization goal. In this approach, synchronization error is used to measure the synchronicity of the multi-robots formation. The synchronization approach can be designed in a decentralized manner; in addition, the controller is flexible to work with any number of robots. However, the synchronization approach still needs more research to be conducted in order for different type of mobile robot dynamics to perform formation efficiently.

In this paper, a new synchronous controller that extends Sun’s work [12] is proposed, where the robot’s dynamics can handle the nonholonomic constraints of the wheeled mobile robot (WMR). In this work, a new translational synchronous controller is proposed that can effectively control the nonholonomic WMR to track its desired trajectory while synchronizing its movement with the neighboring robots to achieve a desired time-varying formation. The dynamical model has been derived based on Lagrange’s method, where, a novel derivation approach is proposed to solve for the Lagrange multipliers. Based on our knowledge, we are the first to use this type of model derivation with the Lagrange multipliers to control the nonholonomic WMR. Furthermore, the model is divided into a translational and rotational model to control the robot translation and the rotation separately.

2. Formation Control via Synchronization

Fig. 1 illustrates a nonholonomic WMR, where $q = [x, y, \phi]^T$ denotes as the coordinate of the center of the mass of the robot in the $x$-$y$ plane, and $\phi$ denotes the robot heading angle measured from the positive $x$-axis, $x$-$y$ represents the world coordinate system, $X$-$Y$ the coordinate system of the WMR, $b$ the distance between each driving wheel and the axis of symmetry, $r$ the radius of each driving wheel, $m_e$ the mass of the WMR without the driving wheel and the rotors of the motors, $m_e$ the mass of each driving wheel with the rotor, $I_w$ the moment of inertia of the WMR without the driving wheels and the rotors of the motors about a vertical axis through $p$, $I_m$ the moment of inertia of each wheel and the rotor of the motor about the wheel axis. In order to simplify the analysis, an assumption that the centre of the mass is located at the geometrical center of the robot is taken place. In this manner, the centripetal and Coriolis effects are not considered in the robot dynamics. The nonholonomic WMR dynamics for each robot is given by:

![Fig. 1. Wheeled Mobile Robot](image)
\[
M_i \ddot{q}_i = \tau_{\varphi i} - A^T(q_i) \lambda_i \\
I_i \ddot{\phi}_i = \tau_{\phi i}
\]

where \(M_i\) and \(I_i\) represent the inertia of the robot with constant terms, \(\tau_{\varphi i}\) and \(\tau_{\phi i}\) are torque control inputs with respect to \(q_i\) and \(\phi_i\), respectively, \(A^T(q_i)\) is the nonholonomic constraints matrix given by:

\[
A^T(q_i) = \begin{bmatrix}
-\sin \phi & -\cos \phi & -\cos \phi \\
\cos \phi & -\sin \phi & -\sin \phi
\end{bmatrix}
\]

and \(\lambda_i = [\lambda_{i1}, \lambda_{i2}, \lambda_{i3}]^T\) are the Lagrange Multipliers, which constraints the \(i\)th robot to move in the lateral direction. The value for \(\lambda_i\), for the \(i\)th robot at instant time can be found in terms of \(\tau_{\varphi i}\) and \(\tau_{\phi i}\) as:

\[
\lambda_{i1} = -m \left( \dot{x}_i \cos \phi_i + \dot{y}_i \sin \phi_i \right) \phi_i
\]

\[
\lambda_{i2} + \lambda_{i3} = \frac{4r^2mb^2(2b^2I_m + r^2I)}{\left(4b^2I_m + r^2I + r^2mb^2\right)^2 - \left(r^2mb^2 - r^2I\right)^2} \cos \phi_i \tau_{\phi i}
\]

\[
\lambda_{i2} - \lambda_{i3} = \frac{4r^2b}{\left(4b^2I_m + r^2I + r^2mb^2\right)^2 - \left(r^2mb^2 - r^2I\right)^2} \tau_{\phi i}
\]

where \(m = m_i + 2m_{m_i}\), and \(I = I_i + 2I_m + 2m_i b^2\).

Similar to [12], a time-varying desired shape is introduced for each robot, denoted by \(S(p, t)\), where \(p\) denotes 2-D position vector and \(t\) the time. The target position \(q_i^d\) for the \(i\)th robot must be located in the curve as \(\partial S(q_i^d, t) = 0\). The goal is to design the control inputs for the dynamics (1) and (2), in such a way that the robot converges to its desired position \(q_i^d\) while maintaining its position in the desired shape \(S(p, t)\). The desired orientation \(\phi_i^d\) of the \(i\)th robot is defined such that the robot heading is always facing the robot desired position \(q_i^d\).

The position and orientation errors of each robot are defined as: \(e_i = q_i^d - q_i\) and \(\Delta \phi_i = \phi_i^d - \phi_i\), respectively. The robot has to achieve a translational control goal of \(e_i \to 0\) and \(\Delta \phi_i \to 0\) as \(t \to \infty\), as well as to achieve a formation control goal for maintaining the robots on the desired curve.

The following example shows how the synchronization control goal is determined based on the formation goal \(\partial S(q_i, t) = 0\).

**Example:** Consider that \(n\) robots are required to maintain in an ellipse curve during the motions. The coordinates \(q_i\) of the \(i\)th robot are required to meet the following restrictions:

\[
q_i(t) = \begin{bmatrix}
x_i(t) \\
y_i(t)
\end{bmatrix} = \begin{bmatrix}
\cos \varphi_i(t) \\
\sin \varphi_i(t)
\end{bmatrix} \begin{bmatrix}
a(t) \\
b(t)
\end{bmatrix} = A_i(t) \begin{bmatrix}
a(t) \\
b(t)
\end{bmatrix}
\]

where \(a\) and \(b\) denote the longest and the shortest radii of the ellipse, respectively, \(\varphi_i = \tanh(b \sin \alpha_i / a \cos \alpha_i)\), with \(\alpha_i = \tanh \left(\frac{y_i}{x_i}\right)\), denotes the angle of the robot lying on the ellipse with respect to the center of the ellipse. Assume that the robots are not located in the longest or the shortest axis of the ellipse so that the inverse of \(A_i\) exists. The synchronization constrains for \(q_i\) is derived as:

\[
A_i^{-1}q_1 = A_i^{-1}q_2 = \ldots = A_i^{-1}q_n = \begin{bmatrix} a \\ b \end{bmatrix}^T
\]

Based on the above example, the synchronization constraint can be represented as:

\[
c_i q_i = c_i q_2 = \ldots = c_i q_n
\]

where \(c_i\) denotes the coupling parameter for the \(i\)th robot, and its inverse exists based on (8). Moreover, (9) can be hold at all the desired coordinates \(q_i^d\),

\[
c_i q_i^d = c_i q_2^d = \ldots = c_i q_n^d
\]

Subtracting (9) from (10) yields the synchronization goal as follows:

\[
c_1 e_1 = c_2 e_2 = \ldots = c_n e_n
\]
The synchronization control goal represented by (11) implicitly, where it can be divided into \( n \) sub goals of \( e_i = c_i e_i - c_j e_j \). Notice that, if \( i = n \) , \( n+1 \) is donated as 1. Then, the position synchronization errors can be defined as a subset of all possible pairs of two neighboring robots as:

\[
\begin{align*}
\varepsilon_1 & = c_1 e_1 - c_2 e_2, \\
\varepsilon_2 & = c_2 e_2 - c_3 e_3, \\
& \vdots \\
\varepsilon_n & = c_n e_n - c_1 e_1 \\
\end{align*}
\]

(12)

where \( \varepsilon_i \) represents the synchronization errors of the \( i \)-th robot. Notice that, if the synchronization error \( \varepsilon_i = 0 \) for all \( i = 1, \ldots, n \) , the synchronization goal (11) can be achieved automatically.

3. Synchronous Control Design

In order for both position errors and synchronization errors to converge to zero, a coupled position error \( E_i \) that links the position and synchronization errors is defined as:

\[
E_i = c_i e_i + \beta \int_0^t (\varepsilon_j - \varepsilon_{i-1}) d\zeta
\]

(13)

where \( \beta \) is a diagonal positive gain matrix. From (12) and (13) it is clearly that this coupled position error for the \( i \)-th robot feeds back the information of the two neighboring robots \( i-1 \) and \( i+1 \). Note that when \( i = 1 \), then \( i-1 \) is denoted as \( n \).

Differentiating (13) with respect to time, yields:

\[
\dot{E}_i = \dot{c}_i e_i + c_i \dot{e}_i + \beta (\varepsilon_j - \varepsilon_{i-1})
\]

(14)

In order to accomplish \( \dot{E}_i \to 0 \) and \( E_i \to 0 \), a control vector \( u_i \) that leads to a combined position and velocity error is introduced as:

\[
u_i = c_i \ddot{q}_i + \dot{c}_i e_i + \beta (\varepsilon_j - \varepsilon_{i-1}) + \Lambda E_i
\]

(15)

where \( \Lambda \) is a diagonal positive gain matrix. The definition of \( u_i \) lead to the following position/velocity vectors as:

\[
r_i = u_i - c_j \dot{q}_j = c_i \dot{e}_i + c_i \dot{e}_i + \beta (\varepsilon_j - \varepsilon_{i-1}) + \Lambda E_i = \dot{E}_i + \Lambda E_i
\]

(16)

Then, a controller is designed to drives \( r_i \) to zero, in such a way the coupled errors \( E_i \) and \( \dot{E}_i \) tend to zero as well.

A torque input that controlling the robot translation (1) is designed as follows:

\[
\tau_{\omega} = M_i c_i^{-1} (\dot{u}_i - c_i \dot{q}_j) + K_n c_i^{-1} r_i + c_i^T K_e (\varepsilon_j - \varepsilon_{i-1}) + A^T (q_j) \Lambda_i
\]

(17)

where \( k_n, k_e \) are positive feedback control gains, and the last term in (17) is used to compensate for nonholonomic constraints of the WMR with.

By substituting the proposed controller (17) into the dynamic model of the nonholonomic WMR translational (1), yields the closed-loop dynamics of the system as follows:

\[
M_i c_i^{-1} r_i + K_n c_i^{-1} r_i + c_i^T K_e (\varepsilon_j - \varepsilon_{i-1}) = 0
\]

(18)

In order to proof the asymptotic stability of this closed-loop system, reader can refer to our previous work [14].

To control the robot’s orientation, a general computed torque method is utilized as;

\[
r_{\omega} = I_i (\ddot{\phi}_i + k_{\omega} \Delta \dot{\phi}_i + k_{\mu} \Delta \phi_i)
\]

(19)

where \( k_{\omega} \) and \( k_{\mu} \) are the computed torque control gains. And the desired orientation \( \phi_i^d \) is defined in order the robot to be always facing its desired position.
4. Simulation Results

Simulations are carried out to validate the effectiveness of the proposed synchronous controller. All the desired formation shapes are assumed to be regular, closed, smooth, and simple planar curves. In this study, a generalized super ellipse with varying parameters is used to represent different types of formation curves

\[
\begin{align*}
  x_i &= \pm a \cos^m \phi_i \\
  y_i &= \pm b \sin^m \phi_i
\end{align*}
\]

(20)

where \( m \) represents the exponent index, and \( a, b, \) and \( \phi_i \) are as defined in (7). Throughout the simulation, the value of \( \phi_i \) is fixed, and can be known during the start of the simulation, where each robot can be indexed relying on its value. The exponent \( m, a, \) and \( b \) can be time-varying parameters.

Through varying the exponent \( m \), (20) and (21) represent a number of shapes including rectangles, ovals, ellipses, and diamonds, that belongs to the categories of hyper ellipses for \( m < 1 \) and hypo ellipses for \( m > 1 \). This simulation performs a switch from an ellipse \( m_0 = 1 \) to a pinched diamond shape \( m_f = 3 \), as it appeared in Fig. 2, where \( a \) and \( b \) are fixed value in this case.

Four homogenous nonholonomic mobile robots are simulated in this case, where their initial positions are located in the ellipse curve as represented by little square in Fig. 2. During the switching between an ellipse to a pinched diamond shape, the four robots are required to maintain in a desired time-varying hypo ellipse curves, in which the exponent index changed as follows:

\[
m(t) = m_0 + (m_f - m_0) \left( 1 - e^{-t} \right)
\]

(21)

The desired trajectory for each robot according to the formation task is given as:

\[
\begin{bmatrix}
  x_i^d(t) \\
  y_i^d(t)
\end{bmatrix} =
\begin{bmatrix}
  \cos^{m(t)} \phi_i \\
  \sin^{m(t)} \phi_i
\end{bmatrix} = A_i(t) \begin{bmatrix}
  a \\
  b
\end{bmatrix}
\]

(22)

The coupled parameter matrix is given by:

\[
c_i(t) = A_i^{-1}(t) =
\begin{bmatrix}
  \cos^{m(t)} \phi_i \\
  \sin^{m(t)} \phi_i
\end{bmatrix}^{-1}
\]

(23)

In this simulation, the four robots have similar parameters, where the parameters for each robot are given in our previous paper [14].

The simulation sampling period was set to 0.005 s. the synchronous controller parameters for each robot were chosen as: \( \beta = \text{diag} \{ 200, 250 \} \), \( \Lambda = \text{diag} \{ 45, 45 \} \), \( K_e = \text{diag} \{ 100, 100 \} \), \( K_a = \text{diag} \{ 10, 10 \} \), \( K_v = 10 \), and \( K_d = 15 \). The initial orientation for the four robots is taken as zero degrees.

Fig. 3. (a) illustrates the position errors in the \( x \)- and \( y \)-directions for the four nonholonomic robots under the proposed synchronous controller. Fig. 3. (b) shows the synchronization errors in the \( x \)- and \( y \)-directions. Note that, the position and synchronization errors start from zero to nonzero values and, successively, their values decreased and converge to zero through reaching the final desired formation. From the simulation results, it is proven that the synchronization errors significantly reduced to zero, which means a better formation is accomplished.

5. Conclusion

The proposed controller guarantees asymptotic convergence to zero of both position and synchronization errors. A new approach for the dynamic model of the nonholonomic WMR is derived based on Lagrange’s method, which utilized Lagrange multipliers to achieve the nonholonomic constraint of the robots. The Lagrange multipliers are successfully determined based on the input torques. The orientation controller let the robot heading always facing its desired position. Simulation is performed to demonstrate the effectiveness of the proposed controller. Our future work will focus on parameters estimation of the synchronous controller to achieve optimal formation results. Further, we will concentrate on path planning to make the synchronous controller more valuable in real world.
Fig. 2 switching between an ellipse to a pinched diamond

Fig. 3. (a) Position Errors; (b) Synchronization Errors
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