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Comparison and analysis of unmodelled errors in GPS and BeiDou signals

Zhetao Zhang, Bofeng Li*, Yunzhong Shen

College of Surveying and Geo-Informatics, Tongji University, Shanghai 200092, China

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ABSTRACT

In Global Navigation Satellite Systems (GNSS) positioning, one often tries to establish a mathematic model to capture the systematic behaviors of observations as much as possible. However, the observation residuals still exhibit, to a great extent, as (somewhat systematic) signals. Nevertheless, those systematic variations are referred to as the unmodelled errors, which are difficult to be further modelled by setting up additional parameters. Different from the random errors, the unmodelled errors are colored and time correlated. In general, the larger the time correlations are, the more significant the unmodelled errors. Hence, understanding the time correlations of unmodelled errors is important to develop the theory for processing the unmodelled errors. In this study, we compare and analyze the time correlations caused by unmodelled errors of Global Positioning System (GPS) and BeiDou signals. The time correlations are estimated based on the residuals of double differenced observations on 11 baselines with different lengths. The results show that the time correlation patterns are different significantly between GPS and BeiDou observations. Besides, the code and phase data from the same satellite system are also not the same. Furthermore, the unmodelled errors are affected by not only the baseline length, but also some other factors. In addition, to make use of the time correlations with more efficiency, we propose to fit the time correlations by using exponent and quadratic models and the fitting coefficients are given. Finally, the sequential adjustment method considering the time correlations is implemented to compute the baseline solutions. The results show that the solutions considering the time correlations can objectively reflect the actual precisions of parameter estimates.

1. Introduction

In Global Navigation Satellite Systems (GNSS) positioning applications, accurately modelling the errors of spatial and temporal variation is crucial for precise positioning [1–4]. However, due to the complicated spatiotemporal characteristics of GNSS disturbing atmosphere and multipath effect as well as the lack of their knowledge, it is rather difficult to further establish the suitable parameterization models that can fully capture all systematic effects. As a result, the residual systematic errors still remain in the observation models, which are referred to as the unmodelled errors. They differ from the random errors since they exhibit as spatiotemporal correlated signals [5,6]. Therefore, it is critical and important to handle the unmodelled errors for further improving the GNSS positioning accuracy [7,8].

In Global Positioning System (GPS) applications, there has been some research on the observation time correlation. Most of them processed this physical correlation as the colored noise. In fact, if all systematic errors are completely modelled and only pure random errors remain, the observations should be time independent. However, there are always some errors cannot be further modelled, the observations are time correlated inevitably in practical applications. Bona [9] and Li et al. [10] discussed the time correlations of GPS observations based on zero baseline with different receivers. El-Rabbany [11] and Han and Rizos [12] presented the time correlations for double differenced (DD) observations on short baselines

* Corresponding author.
E-mail address: bofeng_li@tongji.edu.cn (B. Li).

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with short session. Howind et al. [13] and Jin et al. [14] investigated time correlation for DD observations of long baselines with long session. Odolinski [15] studied time correlation in network real-time kinematic (RTK) solutions. These studies arrive at the conclusion that physical correlations have marginal effects on parameter estimates, but significant effects on the precision measures of parameter estimates.

Although the BeiDou Navigation Satellite System (BDS) is similar to GPS, it is the first GNSS composed of three different satellite orbits, the medium Earth orbit, the geostationary Earth orbit and the inclined geosynchronous satellite orbit. Many studies have recently done to make a better use of the BDS signals [16–19]. In real applications, the unmodelled errors are of course existent in BeiDou. However, there is little research so far addressing the characteristics of BeiDou unmodelled errors and their impacts on precise positioning.

This paper dedicates to initially studying the unmodelled errors of the GPS and BeiDou signals in precise positioning. As mentioned above, the time correlation is an important property for unmodelled errors. To study the theory of processing unmodelled errors, we should first master the characteristics of time correlations and their relations to unmodelled errors. Therefore, we will focus on the time correlations of unmodelled errors in this paper. First, based on the different GPS and BeiDou data sets, the time correlations are compared and analyzed. Second, the time correlations as a function of time lags are fitted by using empirical functions, i.e., the exponent and quadratic functions, respectively. Finally, the time correlations are captured into the sequential adjustment, to show how the time correlations affect the baseline solutions.

2. GNSS DD observation model and its solution

2.1. The single-epoch model

For the short baseline, the atmospheric errors are basically eliminated. After the integer ambiguities are fixed, the single-epoch observation equation reads

$$\boldsymbol{l} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{\varepsilon} \tag{1}$$

where $\boldsymbol{l}$ is the DD code and phase observations; $\boldsymbol{A}$ is the design matrix to baseline $\boldsymbol{x}$; $\boldsymbol{\varepsilon}$ is the errors terms of observations.

The least squares (LS) solution of model (1) is given as

$$\widehat{\boldsymbol{x}} = \left(\boldsymbol{A}^{\mathrm{T}}\boldsymbol{Q}^{-1}\boldsymbol{A}\right)^{-1}\boldsymbol{A}^{\mathrm{T}}\boldsymbol{Q}^{-1}\boldsymbol{l},\ \ \boldsymbol{Q}_{\widehat{\boldsymbol{x}}} = \left(\boldsymbol{A}^{\mathrm{T}}\boldsymbol{Q}^{-1}\boldsymbol{A}\right)^{-1} \tag{2}$$

where $\boldsymbol{Q}$ is the covariance matrix of $\boldsymbol{l}$. The corresponding DD observation residuals can be computed as

$$\boldsymbol{v} = \boldsymbol{A}\widehat{\boldsymbol{x}} - \boldsymbol{l} \tag{3}$$

where $\boldsymbol{v}$ is the DD residuals of code and phase observations, containing the random and unmodelled errors mainly specified by the residual atmospheric biases and multipath effects.

2.2. The sequential model

If the time correlations of between-epoch observations are considered, the sequential model for DD code and phase observations is introduced. The DD observation equations can be formed

$$\boldsymbol{L} = \boldsymbol{B}\boldsymbol{X} + \boldsymbol{E} \tag{4}$$

$$\text{with } \boldsymbol{L} = \left[\boldsymbol{l}_{i-1}^{\mathrm{T}}, \boldsymbol{l}_i^{\mathrm{T}}\right]^{\mathrm{T}};\ \ \boldsymbol{B} = \mathrm{blkdiag}([\boldsymbol{A}_{i-1}, \boldsymbol{A}_i]);$$
$$\boldsymbol{X} = \left[\boldsymbol{x}_{i-1}^{\mathrm{T}}, \boldsymbol{x}_i^{\mathrm{T}}\right]^{\mathrm{T}};\ \ \boldsymbol{E} = \left[\boldsymbol{\varepsilon}_{i-1}^{\mathrm{T}}, \boldsymbol{\varepsilon}_i^{\mathrm{T}}\right]^{\mathrm{T}}$$

where the subscripts $i-1$ and i denote epoch numbers. The operator 'blkdiag' denotes block diagonal concatenation of matrices.

In practice, GNSS measurements are usually assumed to have the same covariance matrix in two consecutive epochs. But now, the time correlations are introduced for code and phase observations. The formula for the time correlation at lag k is

$$\rho_k = \frac{c_k}{c_0} \tag{5}$$

where $c_k = \frac{1}{N-k}\sum_{i=1}^{N-k}[v(i) - \overline{v}][v(i+k) - \overline{v}]$, N is the epoch number, $\overline{v}$ is the mean value of $\boldsymbol{v}$. The standard deviation for the time correlation at lag k is

$$\sigma_{\rho_k} = \sqrt{\frac{1}{N}\left(1 + 2\sum_{t=1}^{\tau}\rho_t^2\right)} \tag{6}$$

where τ is the lag beyond which the theoretical time correlation is effectively zero [20,21].

As a result, the covariance matrix for two consecutive epochs is formulated as

$$\boldsymbol{Q}_{\boldsymbol{ll}} = \boldsymbol{R}\otimes\boldsymbol{Q} \tag{7}$$

where the symbol '$\otimes$' denotes the Kronecker product of two matrices; $\boldsymbol{R}$ captures the time correlations of DD code and phase data as follow

$$\boldsymbol{R} = \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}$$

where ρ is the time correlation coefficient. To make the observations of two epochs independent, we make the following transformation. Since the matrix $\boldsymbol{R}$ holds true the equation

$$\boldsymbol{U}\boldsymbol{R}\boldsymbol{U}^{\mathrm{T}} = \boldsymbol{D} \tag{8}$$

with $\boldsymbol{U} = \begin{bmatrix} 1 & 0 \\ -\rho & 1 \end{bmatrix}$ and $\boldsymbol{D} = \begin{bmatrix} 1 & 0 \\ 0 & 1-\rho^2 \end{bmatrix}$. If we multiply both sides of Eq. (4) by $(\boldsymbol{U}\otimes\boldsymbol{I}_m)$, the transformed observation equations read

$$\overline{\boldsymbol{L}} = \overline{\boldsymbol{B}}\boldsymbol{X} + \overline{\boldsymbol{E}} \tag{9}$$

with

$$\overline{\boldsymbol{L}} = \left[\boldsymbol{l}_{i-1}^{\mathrm{T}}, \overline{\boldsymbol{l}}_i^{\mathrm{T}}\right]^{\mathrm{T}},\ \ \overline{\boldsymbol{l}}_i = \boldsymbol{l}_i - \rho\boldsymbol{l}_{i-1};\ \ \overline{\boldsymbol{B}} = \mathrm{blkdiag}(\left[\boldsymbol{A}_{i-1}, \overline{\boldsymbol{A}}_i\right]),$$
$$\overline{\boldsymbol{A}}_i = \boldsymbol{A}_i - \rho\boldsymbol{A}_{i-1};\ \ \overline{\boldsymbol{E}} = \left[\boldsymbol{\varepsilon}_{i-1}^{\mathrm{T}}, \overline{\boldsymbol{\varepsilon}}_i^{\mathrm{T}}\right]^{\mathrm{T}},\ \ \overline{\boldsymbol{\varepsilon}}_i = \boldsymbol{\varepsilon}_i - \rho\boldsymbol{\varepsilon}_{i-1}$$

where m is the number of observations in a single epoch. $\boldsymbol{I}_m$ denotes the $(m \times m)$ identity matrix. In terms of error propagation law, the covariance matrix of DD observations for two epochs follows

$$\boldsymbol{Q}_{\overline{\boldsymbol{ll}}} = (\boldsymbol{U}\otimes\boldsymbol{I}_m)(\boldsymbol{R}\otimes\boldsymbol{Q})\left(\boldsymbol{U}^{\mathrm{T}}\otimes\boldsymbol{I}_m\right) = \boldsymbol{D}\otimes\boldsymbol{Q} \tag{10}$$

Obviously, the transformed observations in Eq. (7) are independent between epochs. So now the LS solution of the ith epoch can be easily derived in terms of the sequential adjustment as

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$$\widehat{\bar{\boldsymbol{x}}}_i = \left(\boldsymbol{A}_i^{\mathrm{T}}\boldsymbol{Q}_{\bar{\boldsymbol{l}}_i|\widehat{\boldsymbol{x}}_{i-1}}^{-1}\boldsymbol{A}_i\right)^{-1}\boldsymbol{A}_i^{\mathrm{T}}\boldsymbol{Q}_{\bar{\boldsymbol{l}}_i|\widehat{\boldsymbol{x}}_{i-1}}^{-1}(\bar{\boldsymbol{l}}_i + \rho\boldsymbol{A}_{i-1}\widehat{\boldsymbol{x}}_{i-1}),\quad \boldsymbol{Q}_{\widehat{\boldsymbol{x}}_i\widehat{\boldsymbol{x}}_i} = \left(\boldsymbol{A}_i^{\mathrm{T}}\boldsymbol{Q}_{\bar{\boldsymbol{l}}_i|\widehat{\boldsymbol{x}}_{i-1}}^{-1}\boldsymbol{A}_i\right)^{-1} \tag{11}$$

with $\boldsymbol{Q}_{\bar{\boldsymbol{l}}_i|\widehat{\boldsymbol{x}}_{i-1}} = (1-\rho^2)\boldsymbol{Q} + \rho^2\boldsymbol{A}_{i-1}\boldsymbol{Q}_{\widehat{\boldsymbol{x}}_{i-1}\widehat{\boldsymbol{x}}_{i-1}}\boldsymbol{A}_{i-1}^{\mathrm{T}}$

3. Comparison of GPS and BeiDou unmodelled errors

The data used in this comparison were obtained from Hong Kong Satellite Positioning Reference Station Network (SatRef). Total 11 baselines evenly spaced from about 5–50 km were chosen. Dual-frequency GPS/BeiDou data of 1 h were collected for each baseline with sampling interval of 1 s. In computations, the cut-off elevation was set to 10°, and all data was corrected by using the Hopfield model. The ambiguities have been correctly fixed with LAMBDA method [22] by postprocessing. Only the baseline parameters are assumed unknown, which means that the ionospheric biases, the residual tropospheric biases and the other systematic errors remained. The DD residuals were computed with Eq. (3), where the time correlations are ignored.

First, the results for the shortest baseline HKPC-HKMW with the length of 4.8 km are analyzed. Figs. 1 and 2 show the DD residuals of GPS and BeiDou observations, respectively. Each color denotes one pair of DD satellites. It can be clearly seen that for most of the satellite pairs, the residuals contain the significant unmodelled errors. This is further confirmed by the histograms of GPS and BeiDou DD residuals, as shown in Fig. 3. Based on the residual histograms, the skewnesses and kurtosises are computed. Obviously, the results differ from those of zero-mean normal distribution for this shortest baseline. Besides, the results also indicate that the influences of unmodelled errors do not share the same pattern for GPS and BeiDou. The main reason is probably due to the different satellite constellations and signal qualities, thus resulting different effects of multipath and atmosphere.

We computed the DD residuals for all 11 baselines though they are not shown here. Based on these residuals, the time correlation coefficients with time lag of 1 s are computed, and their mean values are shown in Fig. 4 for all baselines as a function of baseline length. In general, the time correlations increase as the baseline length becomes longer. It makes sense since the larger unmodelled errors are associated to the longer baselines. The time correlations are different between GPS and BeiDou observations, which indicate that the effects of unmodelled errors on GPS and BeiDou are different. Besides, the time correlations of phase are generally smaller than those of code. The reason could be that the code observations are more easily influenced by the unmodelled errors. For a given satellite system, the time correlations of two frequencies are quite similar for code and phase, respectively, which is possibly attributed to the similar unmodelled errors introduced by the same propagation path.

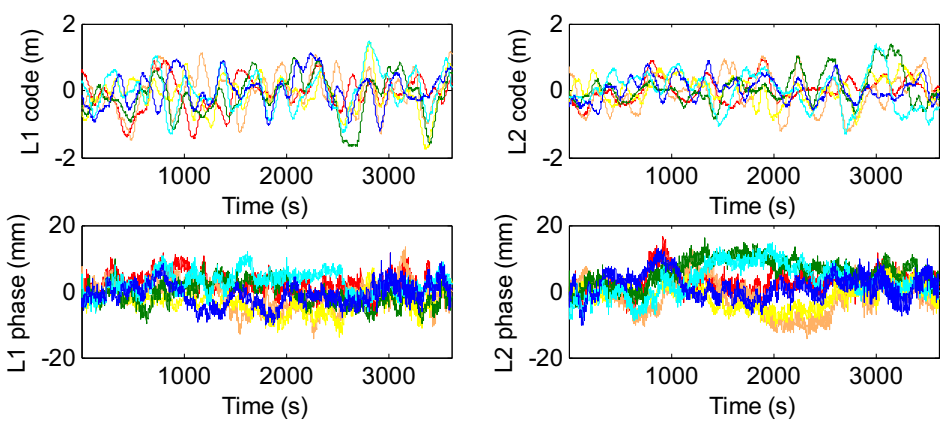

Fig. 1. GPS DD residuals obtained from the baseline HKPC-HKMW for various satellite pairs.

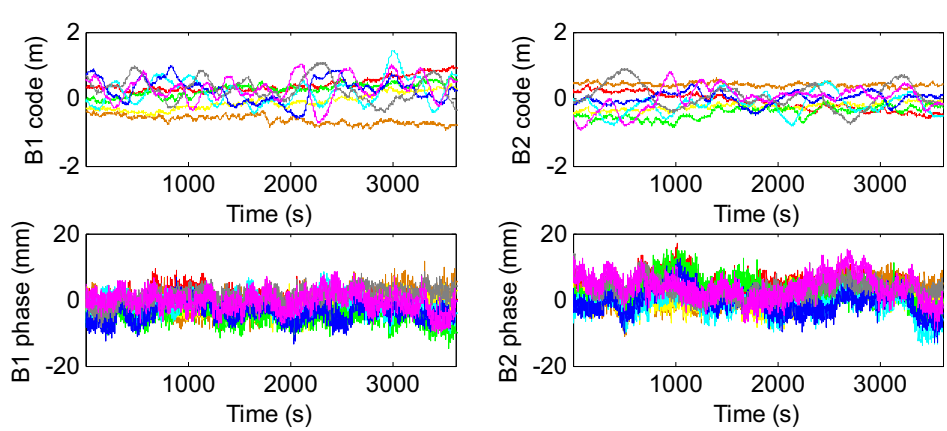

Fig. 2. BeiDou DD residuals obtained from the baseline HKPC-HKMW for various satellite pairs.

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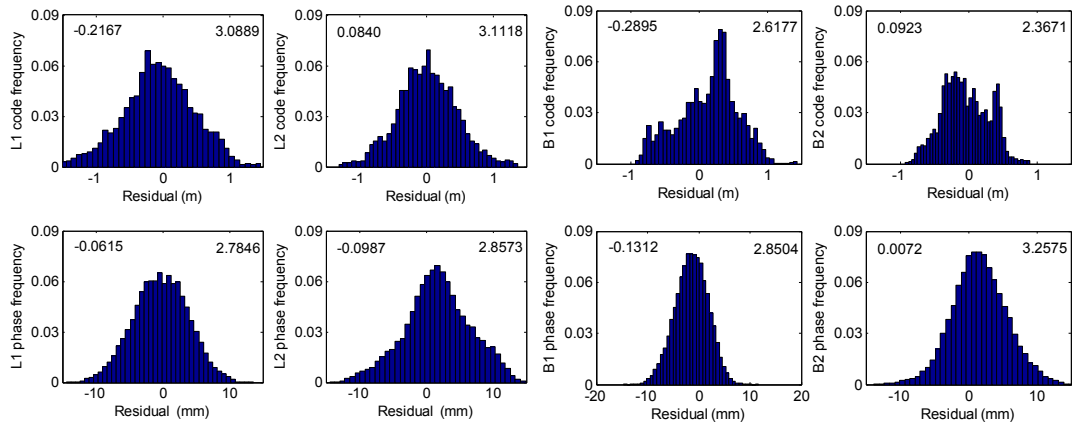

Fig. 3. Frequency histograms of GPS (left) and BeiDou (right) DD residuals obtained from the baseline HKPC-HKMW. The numbers in the top left and top right corners indicate the corresponding skewnesses and kurtosises, respectively.

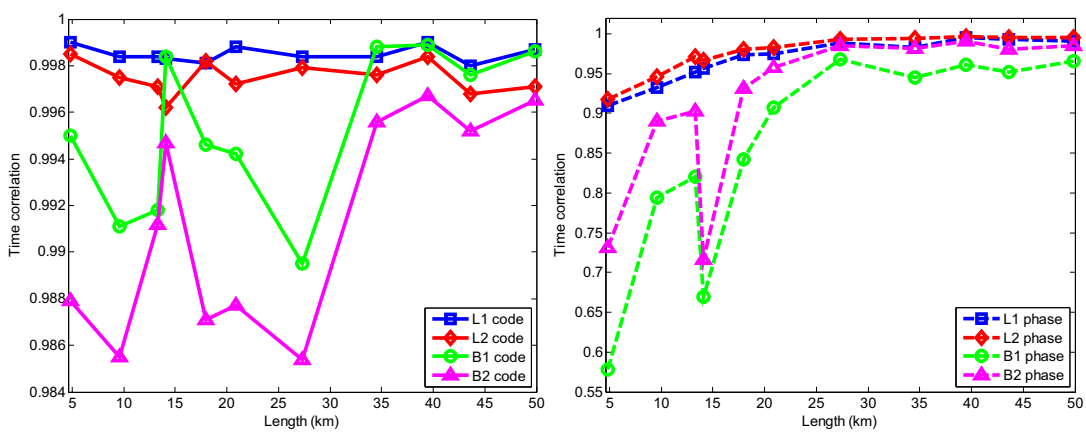

Fig. 4. Mean time correlation coefficients of 1 s time lag for GPS/BeiDou code (left) and phase (right) observations obtained from 11 different baselines.

4. Time correlation analysis of GPS and BeiDou unmodelled errors

As mentioned above, the time correlation coefficients will have some variations due to the unmodelled error uncertainty. To be more accurate, the time correlation coefficients of any time lag should be updated at every epoch. However, in real application, the bottleneck problem is the huge computation burden of time correlation coefficients, particularly when the observations are often discontinued. For simplicity of computation, an efficient estimation method is proposed by introducing general empirical models. The empirical functions are applied to fit the time correlation coefficients as a function of time lag. In this study, two different empirical functions have been tested. The first one is an exponential function given by

$$f(k) = exp(-|k|/T) \tag{12}$$

where k is the time lag in seconds and T is the unknown correlation time to be determined (the 1/e point). The second one is a quadratic form given by

$$f(k) = a + bk + ck^2 \tag{13}$$

where a, b and c are the unknown parameters.

The LS technique is applied to test whether these two functions can fit the time correlation coefficients and which one is better. In Figs. 5 and 6, the time correlation coefficients of each satellite pair and their two best fit functions are presented, where Exp. and Quad. denote the exponential and quadratic function respectively. It is clearly evident that the time correlation coefficients for different baseline lengths indicate little variations. Therefore, a general empirical time-correlation function which is valid for the range up to 50 km can be developed. The results for the general empirical time-correlation functions are shown in Table 1. It can be seen that the GPS and BeiDou DD observations are positively correlated over a time period of about 5 and 3.5 min, respectively. The results of GPS agree with the analyses of El-Rabbany [11] and Howind et al. [13]. Also, comparing with the root mean square errors (RMSE) from these two empirical functions, it is shown that the empirical quadratic function given by Eq. (13) gives the better fit for the estimated time correlation coefficients, especially for the BeiDou data. In conclusion, the quadratic function is advised to fit the time correlations in BeiDou applications.

5. Impact analysis of time correlation on precise positioning

To investigate the impact of time correlation on precise positioning, L1, L2, B1 and B2 phase observations from baseline HKPC-HKMW were used again. There were a total of 10800 epochs

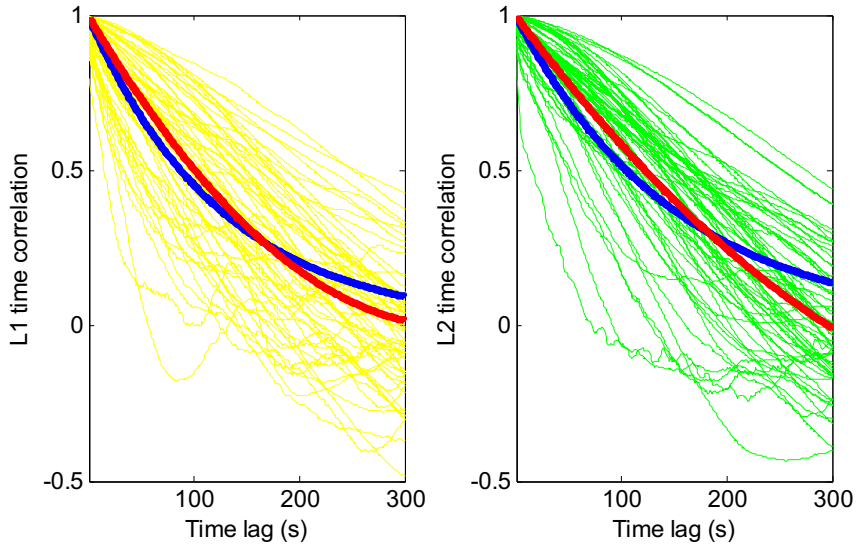

Fig. 5. The time correlations of GPS DD observations. The blue line denotes the fit result of the exponential function, and the red line denotes the fit result of the quadratic function.

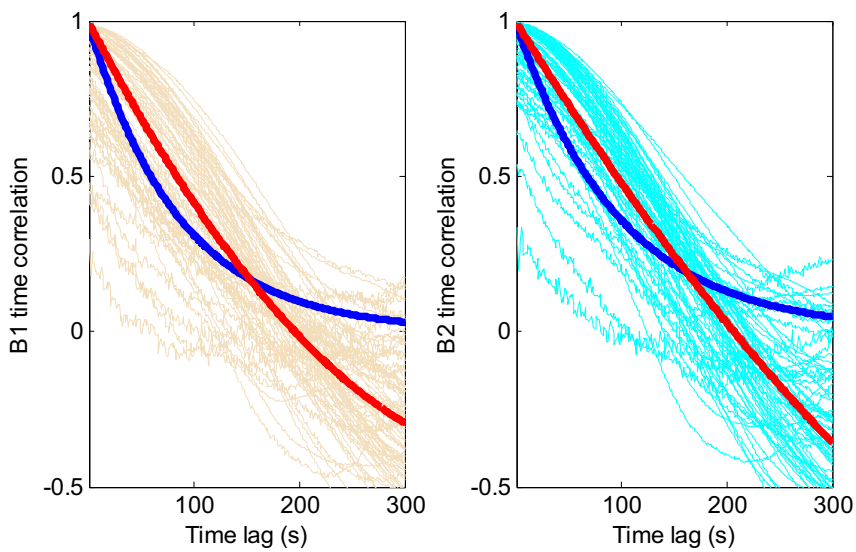

Fig. 6. The time correlations of BeiDou DD observations. The blue line denotes the fit result of the exponential function, and the red line denotes the fit result of the quadratic function.

collected with elevation mask set to 5°.The other settings are also the same as those defined in Section 3. Because the results solved by GPS and BeiDou data showed similar conclusions, for simplicity of discussion only the results obtained from BeiDou observations are analyzed.

Two methods are examined, identified by whether the time correlation estimated by the empirical quadratic function is taken into account or not. For these two methods, the single-epoch model and the sequential model considering the time correlation are referred to the empirical and the realistic method respectively. Similar to Li [19] and El-Rabbany and Kleusberg [23], the baseline solutions of both two methods are very close with each other, as shown in Fig. 7. Then the DD residuals with the two methods are calculated respectively. The empirical DD residuals of the *i*th epoch

Table 1
Results for general empirical time-correlation functions.

Obs. types	Zero crossing	Exponential model	RMSE	Quadratic model			RMSE
	s	T		a	b	c	
L1	295	127	0.20	1	-5.77×10^{-3}	8.31×10^{-6}	0.20
L2	316	152	0.20	1	-4.55×10^{-3}	3.98×10^{-6}	0.19
B1	202	86	0.23	1	-6.59×10^{-3}	7.59×10^{-6}	0.18
B2	219	98	0.24	1	-5.52×10^{-3}	3.32×10^{-6}	0.16

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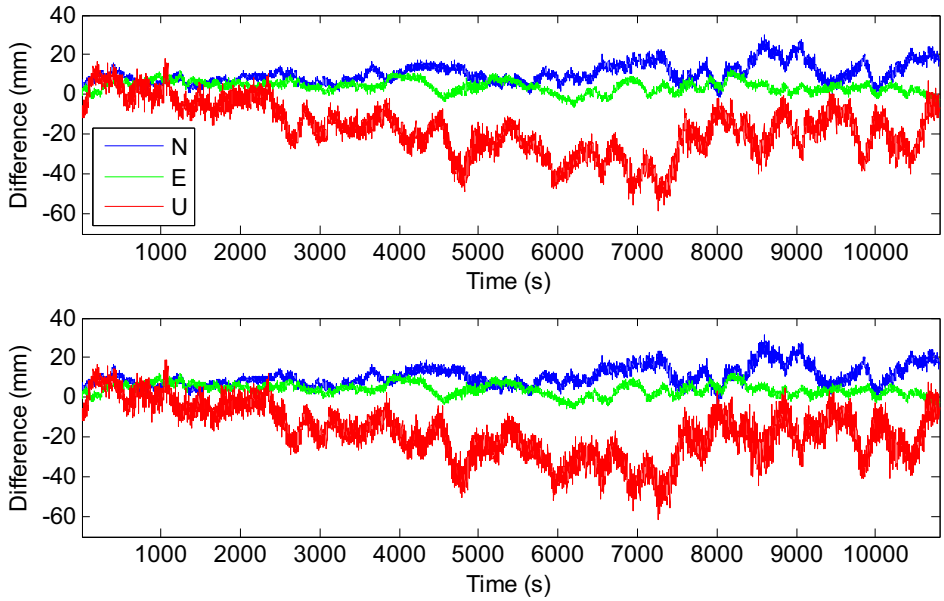

Fig. 7. Coordinate differences relative to the reference solved by ignoring (top) and considering (bottom) the time correlations with BeiDou data.

can be obtained with Eq. (3). According to Eq. (11), the realistic DD residuals are obtained by

$$v_i = A_i\widehat{\bar{\boldsymbol{x}}}_i - \widehat{\bar{l}}_i \tag{14}$$

with $\widehat{\bar{\boldsymbol{x}}}_i = (\boldsymbol{A}_i^T\boldsymbol{Q}^{-1}_{\bar{\boldsymbol{l}}_i|\widehat{\bar{\boldsymbol{x}}}_{i-1}}\boldsymbol{A}_i)^{-1}\boldsymbol{A}_i^T\boldsymbol{Q}^{-1}_{\bar{\boldsymbol{l}}_i|\widehat{\bar{\boldsymbol{x}}}_{i-1}}\widehat{\bar{l}}_i$ and $\widehat{\bar{\boldsymbol{l}}}_i = \bar{\boldsymbol{l}}_i + \rho\boldsymbol{A}_{i-1}\widehat{\bar{\boldsymbol{x}}}_{i-1}$. The other terms are the same as those defined previously. It can be clearly seen from Fig. 8 that the residuals of the transformed observations are much stable than those of the original observations. Each color from Fig. 8 denotes one pair of DD satellites. This can be further confirmed by Fig. 9 that the residuals for the transformed observations are more random judging from the time correlation coefficients with Eq. (5). It means that the method considering time correlation can truly obtain the baseline solutions.

To demonstrate the impact of time correlation caused by unmodelled errors on precise positioning, the baseline precisions are used to compare these two methods. The variance matrix of baseline solutions over k epochs can be derived as

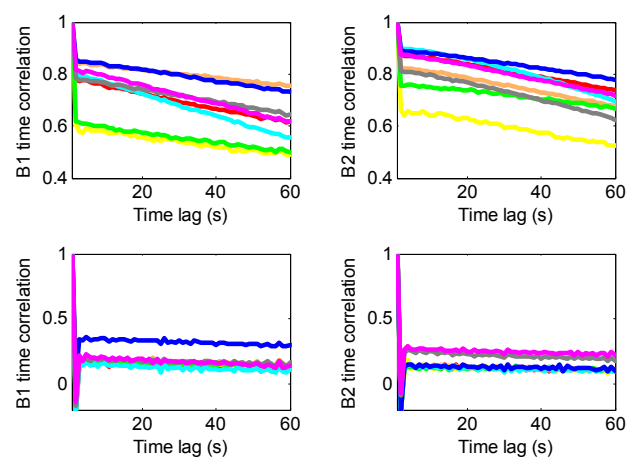

Fig. 9. The time correlations of B1 (left) and B2 (right) phase observations for various satellite pairs solved by ignoring (top) and considering (bottom) the time correlations.

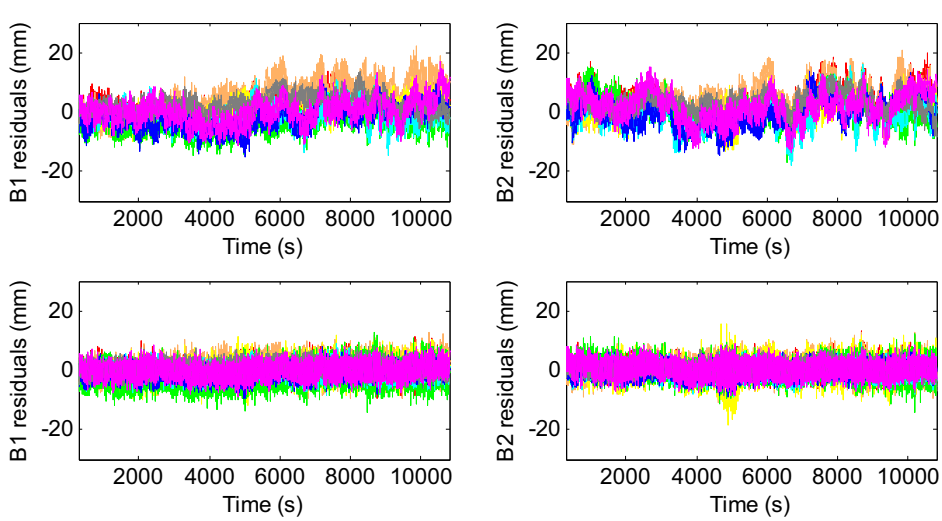

Fig. 8. BeiDou DD residuals for various satellite pairs solved by ignoring (top) and considering (bottom) the time correlations.

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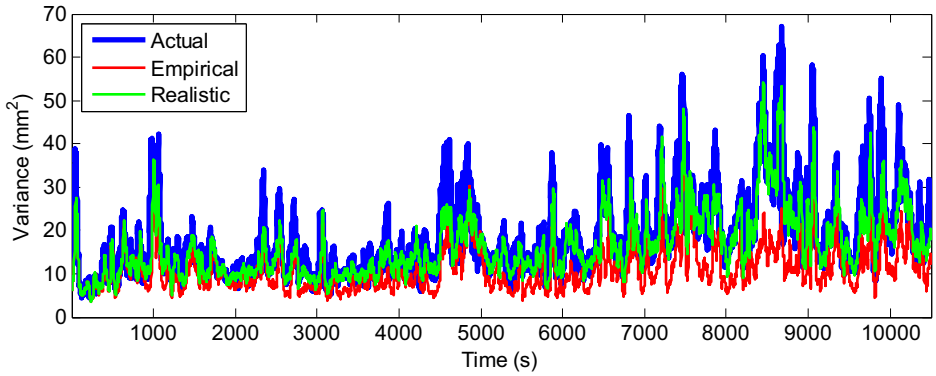

Fig. 10. Impacts of time correlation on baseline precisions in U direction.

$\widehat{\mathbf{Q}}_{\widehat{x}} = \mathbf{X}[\mathbf{I}_k - \mathbf{e}_k\mathbf{e}_k^{\mathrm{T}}/k]\mathbf{X}^{\mathrm{T}}/(k-1)$ [19]. The matrix $\boldsymbol{e}_k$ denotes the f-column vector with all elements of ones. In order to weaken the time correlation, we use a new series data sampled with the interval of 5 s from the original series. Given the data window $k = 60$ epochs, the baseline precisions are obtained as the actual values. After that, the realistic and empirical baseline precisions are computed by considering and ignoring the time correlations, respectively. Fig. 10 shows the relations among the actual, empirical and realistic variances of baseline solutions in up (U) direction with BeiDou data. As expected, the baseline precisions obtained with the realistic method are more similar to the actual ones. In addition, the empirical baseline precisions are smaller than the actual ones. That is to say, ignoring the time correlations will result in baseline precisions that are both too optimistic and unrealistic.

6. Conclusions

In this paper, we compare and analyze the GPS and BeiDou unmodelled errors. The corresponding physical correlations of a temporal nature are systematically studied. Based on the results from 11 baselines of different lengths, the main conclusions can be summarized as follows:

(1) The unmodelled errors indeed exist in GPS and BeiDou signals. In addition, there are systematic discrepancies between GPS and BeiDou, and the unmodelled error patterns are also not the same between code and phase observations from the same satellite system. Although the unmodelled errors are generally positive correlated with the baseline length, there are still other impact factors.
(2) The GPS and BeiDou DD phase observations are both positively correlated over a time period of at least 3.5 min. Then the time correlation coefficients could be well fitted by the exponential and quadratic function, which are valid for the range up to at least 50 km. Then the advised unknown parameters of empirical functions for dual-frequency GPS and BeiDou phase data are all given.
(3) The LS solution considering time correlation based on sequential adjustment is derived. With this method, one can achieve more reliable baseline solutions than the empirical method. Since it is proved that the residuals are more random and the precisions of baseline solutions can objectively reflect the actual precisions of baseline solutions. Without taken into account the time correlations caused by the unmodelled errors, the baseline precisions are too small and not realistic.

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Zhetao Zhang, a Ph.D candidate in College of Surveying and Geo-Informatics, Tongji University, majored in geodesy and geophysics. His research is the development and testing of precise GNSS positioning techniques with a focus on functional, stochastic and error mitigation models, GNSS data quality control and surveying data processing.

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