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CP-violation in the decay $B^0, \overline{B}^0 \to \pi^+\pi^-\gamma$

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Abstract

The decay $\overline{B}^0 \to \pi^+\pi^-\gamma$ has a bremsstrahlung component determined by the amplitude for $\overline{B}^0 \to \pi^+\pi^-$, as well as a direct component determined by the penguin interaction $V_{tb}V_{td}^* c_7 \mathcal{O}_7$. Interference of these amplitudes produces a photon energy spectrum $d\Gamma/dx = \frac{a}{x} + b + c_1 x + c_2 x^2 + \cdots$ $(x = 2E_\gamma/m_B)$ where the terms $c_{1,2}$ contain a dependence on the phase $\alpha_{\text{eff}} = \pi - \arg[(V_{tb}V_{td}^*)^* \mathcal{A}(\overline{B}^0 \to \pi^+\pi^-)]$. We also examine the angular distribution of these decays, and show that in the presence of strong phases, an untagged $B^0/\overline{B}^0$ beam can exhibit an asymmetry between the π^+ and π^- energy spectra.

1. Introduction

In this Letter, we analyze the reaction $\overline{B}^0(B^0) \to \pi^+\pi^-\gamma$, with the aim of finding new ways of probing CP-violation in the non-leptonic Hamiltonian, and in particular to test current assumptions about the weak and strong phases in the amplitude for $\overline{B}^0 \to \pi^+\pi^-$. We will focus on observables that can be measured in an untagged $\overline{B}^0, B^0$ beam, which are complementary to the time-dependent asymmetries in channels such as $\overline{B}^0, B^0 \to \pi^+\pi^-$ which are currently under study.

The amplitude for the decay of $\overline{B}^0$ into two charged pions and a photon can be written as

$$\mathcal{A}(\overline{B}^0 \to \pi^+\pi^-\gamma) = \mathcal{A}_{\text{brems}} + \mathcal{A}_{\text{dir}}, \tag{1}$$

where $\mathcal{A}_{\text{brems}}$ is the bremsstrahlung amplitude and $\mathcal{A}_{\text{dir}}$ is the direct emission amplitude. Our main interest will be in the continuum region of $\pi^+\pi^-$ invariant masses (large compared to the ρ-mass), and the possible interference of the two terms in Eq. (1).

The bremsstrahlung amplitude is directly proportional to the amplitude for a $\overline{B}^0$ decaying into two pions, the modulus of which is determined by the measured branching ratio [1]. Theoretically, the $\overline{B}^0 \to \pi^+\pi^-$ decay amplitude can be written as [2]

$$\mathcal{A}(\overline{B}^0 \to \pi^+\pi^-) \sim V_{ub}V_{ud}^* T + V_{cb}V_{cd}^* P \sim e^{-i\gamma} + \frac{P_{\pi\pi}}{T_{\pi\pi}}, \tag{2}$$

where $\gamma = \arg(-V_{ud}V_{ub}^*/V_{cd}V_{cb}^*)$, and T and P denote the tree- and penguin-amplitudes, which can possess strong phases. (We follow the notation of Ref. [2], in which the phase of $\frac{P_{\pi\pi}}{T_{\pi\pi}}$ is estimated to be $\leqslant 10°$). The $B^0 \to \pi^+\pi^-$ amplitude is obtained by taking the complex conjugate of the CKM factors in Eq. (2), leaving possible strong phases unchanged. In the experiments [1], one measures the time-dependent

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CP-asymmetry

$$A(t) = -S\sin(\Delta m_B t) + C\cos(\Delta m_B t), \tag{3}$$

where

$$S = \frac{2\,\mathrm{Im}\,\lambda}{1+|\lambda|^2}, \qquad C = \frac{1-|\lambda|^2}{1+|\lambda|^2},$$

$$\lambda = \frac{p}{q}\frac{\mathcal{A}(\overline{B}^0 \to \pi^+\pi^-)}{\mathcal{A}(B^0 \to \pi^+\pi^-)} = e^{-i2\beta}\frac{e^{-i\gamma} + \frac{P_{\pi\pi}}{T_{\pi\pi}}}{e^{i\gamma} + \frac{P_{\pi\pi}}{T_{\pi\pi}}}. \tag{4}$$

In the limit of neglecting the penguin contribution, $P_{\pi\pi} \to 0$, $\lambda = \exp[-i2(\beta+\gamma)] = e^{2i\alpha}$, so that $S = \sin 2\alpha$, $C = 0$. Theoretical considerations [2] suggest $|\frac{P_{\pi\pi}}{T_{\pi\pi}}| \sim 0.28$. Present measurements of S and C are yet inconclusive [1,3].

The direct emission amplitude $\mathcal{A}_{\mathrm{dir}}$ is determined by the Hamiltonian

$$H_{\mathrm{peng}} = V_{td}^* V_{tb} c_7 \mathcal{O}_7. \tag{5}$$

This is the interaction which also leads to the exclusive decay $B \to \rho\gamma$ [5]. Here, however, we will be interested in the decay $\overline{B}^0 \to \pi^+\pi^-\gamma$ for $\pi^+\pi^-$ masses in the continuum region, especially for large s (or low photon energy). The Hamiltonian H_{peng} leads to

$$\begin{aligned}\mathcal{A}_{\mathrm{dir}} = {} & \overline{E}_{\mathrm{dir}}(\omega,\cos\theta)[\epsilon\cdot p_+ k\cdot p_- - \epsilon\cdot p_- k\cdot p_+] \\ & + i\overline{M}_{\mathrm{dir}}(\omega,\cos\theta)\epsilon_{\mu\nu\rho\sigma}\epsilon^\mu k^\nu p_+^\rho p_-^\sigma,\end{aligned} \tag{6}$$

where the electric ($\overline{E}_{\mathrm{dir}}$) and magnetic ($\overline{M}_{\mathrm{dir}}$) amplitudes depend on the two Dalitz plot coordinates: the photon energy ω in the $\overline{B}^0$-meson rest frame and θ, the angle of the π^+ relative to the photon in the $\pi^+\pi^-$ c.m. frame.

As long as the photon polarization is not observed, only the electric component of the direct amplitude interferes with the bremsstrahlung amplitude. This interference is in principle sensitive to the relative phase of $T\lambda_u + P\lambda_c$ and λ_t ($\lambda_i = V_{ib}V_{id}^*$) and, therefore, could serve as a probe of the phase of $\mathcal{A}(\overline{B}^0 \to \pi^+\pi^-)$.

2. Differential decay rate

In accordance with the Low theorem the bremsstrahlung matrix element is directly proportional to the amplitude of $\overline{B}^0 \to \pi^+\pi^-$ on the mass shell:

$$\mathcal{A}_{\mathrm{brems}} = \frac{e\mathcal{A}(\overline{B}^0 \to \pi^+\pi^-)}{(k\cdot p_+)(k\cdot p_-)}$$

$$\times\left[(\epsilon\cdot p_+)(k\cdot p_-) - (\epsilon\cdot p_-)(k\cdot p_+)\right]. \tag{7}$$

To obtain the direct amplitude $\mathcal{A}_{\mathrm{dir}}$, we observe first that the operator $\mathcal{O}_7 \sim \bar{d}\sigma_{\mu\nu}(1+\gamma_5)F^{\mu\nu}b$, and the identity $\sigma_{\mu\nu} = \frac{i}{2}\epsilon_{\mu\nu\alpha\beta}\sigma^{\alpha\beta}\gamma_5$ enables one to write $\overline{E}_{\mathrm{dir}}(\omega,\cos\theta) = \overline{M}_{\mathrm{dir}}(\omega,\cos\theta)$. One can write a multipole expansion for these direct amplitudes in the form [4]

$\overline{B}^0$:

$$\overline{E}_{\mathrm{dir}}(\omega,\cos\theta) = E^{(1)}(\omega) + \cos\theta\frac{\beta\omega}{m_B}E^{(2)}(\omega) + \cdots,$$

B^0:

$$E_{\mathrm{dir}}(\omega,\cos\theta) = E^{(1)}(\omega) - \cos\theta\frac{\beta\omega}{m_B}E^{(2)}(\omega) + \cdots. \tag{8}$$

The simplest assumption is the dipole approximation

$$\overline{E}_{\mathrm{dir}}(\omega,\cos\theta) = E^{(1)}(\omega) \tag{9}$$

in which $\overline{E}_{\mathrm{dir}}$ is independent of $\cos\theta$. In Section 4 we will consider also consequences arising from a quadrupole term.

To get a dimensionless decay distribution we introduce

$$x = \frac{2\omega}{m_B}, \quad 0 < x < 1 - \frac{4m_\pi^2}{m_B^2}. \tag{10}$$

Then the differential branching ratio for the process $\overline{B}^0 \to \pi^+\pi^-\gamma$ is:

$$\begin{aligned}\frac{dBr}{dx\,d\cos\theta} = {} & \frac{1}{512\pi^3}\left(\frac{m_B}{2}\right)\left(\frac{x}{2}\right)^3\beta_\pi^3(1-x)\sin^2\theta \\ & \times\left[2|\overline{E}_{\mathrm{dir}}|^2 + |\overline{E}_{\mathrm{brems}}|^2 \right.\\ & \left. + 2\,\mathrm{Re}\left(\overline{E}_{\mathrm{dir}}^*\overline{E}_{\mathrm{brems}}\right)\right],\end{aligned} \tag{11}$$

where

$$\begin{aligned}|\overline{E}_{\mathrm{brems}}| = {} & e\sqrt{\frac{\pi}{m_B}}\frac{64}{x^2(1-\beta_\pi^2\cos^2\theta)} \\ & \times\sqrt{Br\left(\overline{B}^0 \to \pi^+\pi^-\right)},\end{aligned} \tag{12}$$

$$\arg(\overline{E}_{\mathrm{brems}}) = -\gamma_{\mathrm{eff}} = \arg\left(e^{-i\gamma} + \frac{P_{\pi\pi}}{T_{\pi\pi}}\right), \tag{13}$$

$$\arg(\overline{E}_{\mathrm{dir}}) = \arg\left(V_{tb}V_{td}^*\right) = \beta, \tag{14}$$

$$\beta_\pi^2 = 1 - \frac{4m_\pi^2}{s}, \tag{15}$$

$$s = m_B^2(1-x), \tag{16}$$

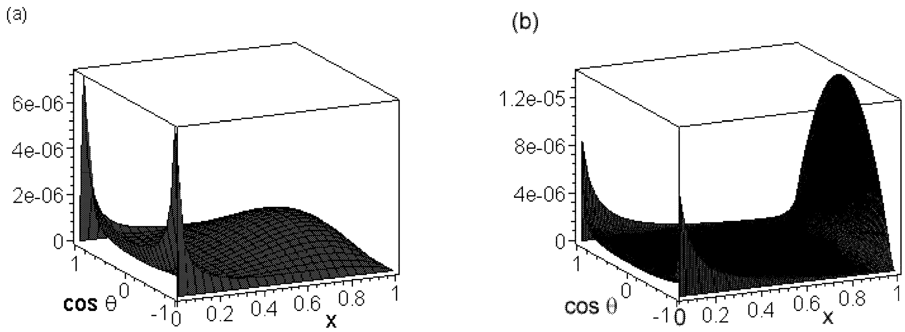

Fig. 1. The differential branching ratio $\frac{dBr}{dx\,d\cos\theta}$ for (a) $E^{(1)} = \text{const}$ and (b) $E^{(1)} \sim 1/s$. Parameters in both plots: $\alpha = 80^\circ$ and $Br_{\text{dir}} = 10^{-6}$.

Table 1
The expansion parameters a, b, c_1, c_2 in the photon energy spectrum (Eq. (17)) for two form factor models. The parameters a and b depend only on bremsstrahlung and are, therefore, the same in both models

	$E^{(1)} = \text{const}$	$E^{(1)} \sim 1/s$
a	1.45×10^{-7}	1.45×10^{-7}
b	-1.69×10^{-7}	-1.69×10^{-7}
c_1	$1.16 \times 10^{-8} + 1.67 \times 10^{-6} \sqrt{\frac{Br_{\text{dir}}}{10^{-6}}} \cos\alpha_{\text{eff}}$	$1.16 \times 10^{-8} + 2.19 \times 10^{-7} \sqrt{\frac{Br_{\text{dir}}}{10^{-6}}} \cos\alpha_{\text{eff}}$
c_2	$3.88 \times 10^{-9} - 1.69 \times 10^{-6} \sqrt{\frac{Br_{\text{dir}}}{10^{-6}}} \cos\alpha_{\text{eff}}$	$3.88 \times 10^{-9} - 2.27 \times 10^{-9} \sqrt{\frac{Br_{\text{dir}}}{10^{-6}}} \cos\alpha_{\text{eff}}$

where $Br(\overline{B}^0 \to \pi^+\pi^-) = 5.1 \times 10^{-6}$ [1], $\beta = 24^\circ$ [1,3] and $P_{\pi\pi}/T_{\pi\pi} = 0.28$, with a negligible strong phase [2].

The bremsstrahlung part of the decay distribution populates preferentially the region of small x and $|\cos\theta| \sim 1$. Far from this region, the spectrum is determined by the direct term $|\overline{E}_{\text{dir}}|^2$. In principle, a fit to the Dalitz plot can determine the scale and the shape of the direct amplitude.

Fig. 1 shows the two-dimensional decay distribution, for an assumed direct branching ratio $Br_{\text{dir}} = 10^{-6}$ and two choices of form factor $E^{(1)} = \text{const}$ and $E^{(1)} \sim 1/s$. In the dipole approximation the distribution is symmetric with respect to $\cos\theta$ and identical for $\overline{B}^0$ and B^0 decay.

3. Photon energy spectrum

Integrating over the variable $\cos\theta$, the branching ratio, for small energies x, can be written in the form:

$$\frac{dBr}{dx} = \frac{a}{x} + b + c_1 x + c_2 x^2 + \cdots. \tag{17}$$

In agreement with the Low theorem, the coefficients a and b are determined entirely by bremsstrahlung whereas c_1, c_2 depend on the interference of bremsstrahlung and direct emission and, therefore, contain information about the relative phase of $T\lambda_u + P\lambda_c$ and λ_t given by $\beta + \gamma_{\text{eff}} \equiv \pi - \alpha_{\text{eff}}$. Higher order terms in x (c_i, $i = 3, 4, \ldots$) contain pure direct emission, in addition to interference terms involving higher multipoles. Numerical estimates of the expansion parameters are given in Table 1.

In Fig. 2 we show typical photon energy spectra, for direct branching ratios 10^{-6} or 10^{-7}, and two choices of the form factor in $\overline{E}_{\text{dir}}$. The phase α is allowed to vary between 60° and 120°. As expected, the sensitivity to α depends on the degree of overlap between the bremsstrahlung and direct amplitudes. The expansion in Eq. (17) turns out to be a good description in the region $x \leqslant 0.25$. (Note that the resonant contribution $\overline{B}^0 \to \rho\gamma \to \pi^+\pi^-\gamma$ would appear as a spike in $\frac{dBr}{dx}$ at $x = 1 - \frac{m_\rho^2}{m_B^2}$.)

In the absence of strong phases, the photon energy spectrum (17) holds for B^0 as well as $\overline{B}^0$ decay, and the results in Table 1 and Fig. 2 are, in that limit, valid for an untagged B^0, $\overline{B}^0$ beam as well.

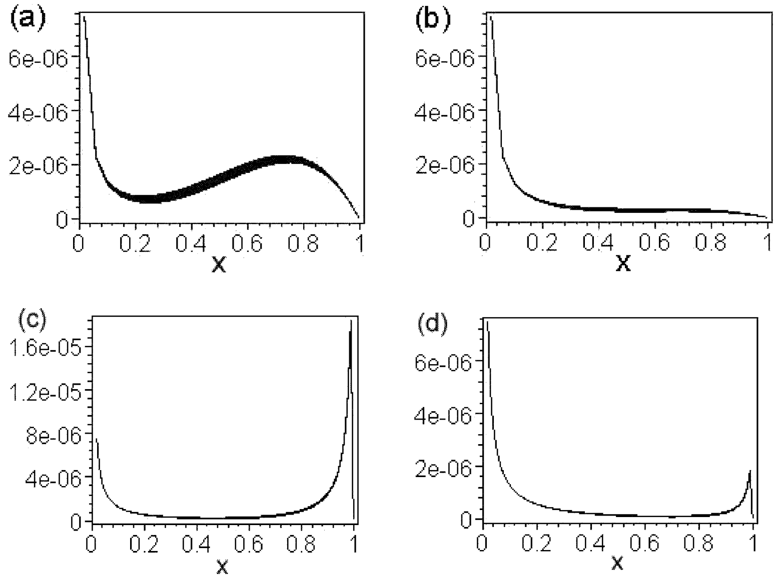

Fig. 2. $\frac{dBr}{dx}$ for $E^{(1)} = \text{const}$: (a) $Br_{\text{dir}} = 10^{-6}$, (b) $Br_{\text{dir}} = 10^{-7}$ and $E^{(1)} \sim 1/s$, (c) $Br_{\text{dir}} = 10^{-6}$, (d) $Br_{\text{dir}} = 10^{-7}$. The thickness of the lines represents the variation of α from 60° to 120°.

4. Asymmetry in the angular distribution

In the dipole approximation, the angular distribution is the same for B^0 and $\overline{B}^0$ decay and is symmetric in $\cos\theta$ (cf. Eq. (11)). In the presence of an additional quadrupole term in the direct emission amplitude the multipole expansion for $\overline{B}^0$ and B^0 differs by a sign, as shown in Eq. (8). For a numerical estimate of $E^{(2)}/E^{(1)}$, we take [6]

$$\frac{Br(B \to \pi^+\pi^-\gamma;\, E^{(2)})}{Br(B \to \pi^+\pi^-\gamma;\, E^{(1)})} \approx \frac{Br(B \to K_2\gamma)}{Br(B \to K^*\gamma)} \approx 0.25. \tag{18}$$

The result is plotted in Fig. 3, which is obtained from Eq. (11) after integrating over photon energies $\omega > 50$ MeV. While the distribution $\frac{dBr}{d\cos\theta}$ is symmetric in a dipole approximation, it develops a forward–backward asymmetry in the presence of a quadrupole term.

Comparing $\overline{B}^0$ to B^0 we see that as long as strong phases are absent,

$$\frac{dBr(B^0 \to \pi^+\pi^-\gamma)}{d\cos\theta} = \frac{dBr(\overline{B}^0 \to \pi^+\pi^-\gamma)}{d\cos\theta}(\cos\theta \to -\cos\theta), \tag{19}$$

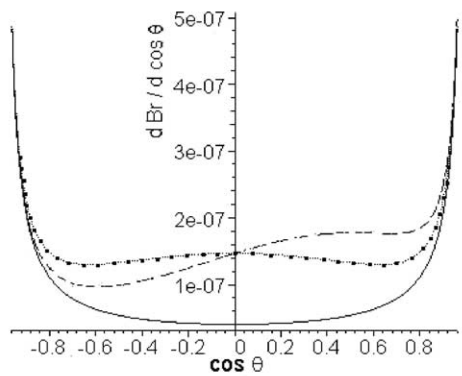

Fig. 3. $\frac{dBr}{d\cos\theta}$ for $\overline{B}^0 \to \pi^+\pi^-\gamma$, assuming $E^{(i)} = \text{const}$ and a lower limit $\omega_{\text{min}} = 50$ MeV for photon energy. The solid line represents pure bremsstrahlung, the dotted line an additional dipole contribution and the dashed line combines bremsstrahlung, dipole and quadrupole contributions.

so that the decay of an untagged beam would be symmetric in $\cos\theta$.

This conclusion changes, however, if strong phases are not negligible. Let us associate strong phases δ_0, $\delta_1(s)$ and $\delta_2(s)$ with the bremsstrahlung, direct dipole, and direct quadrupole terms. Notice that the bremsstrahlung phase δ_0 is the phase of the $\overline{B}^0 \to \pi^+\pi^-$ amplitude, and therefore describes a 2π state with $L = 0$ and invariant mass m_B. By contrast, the

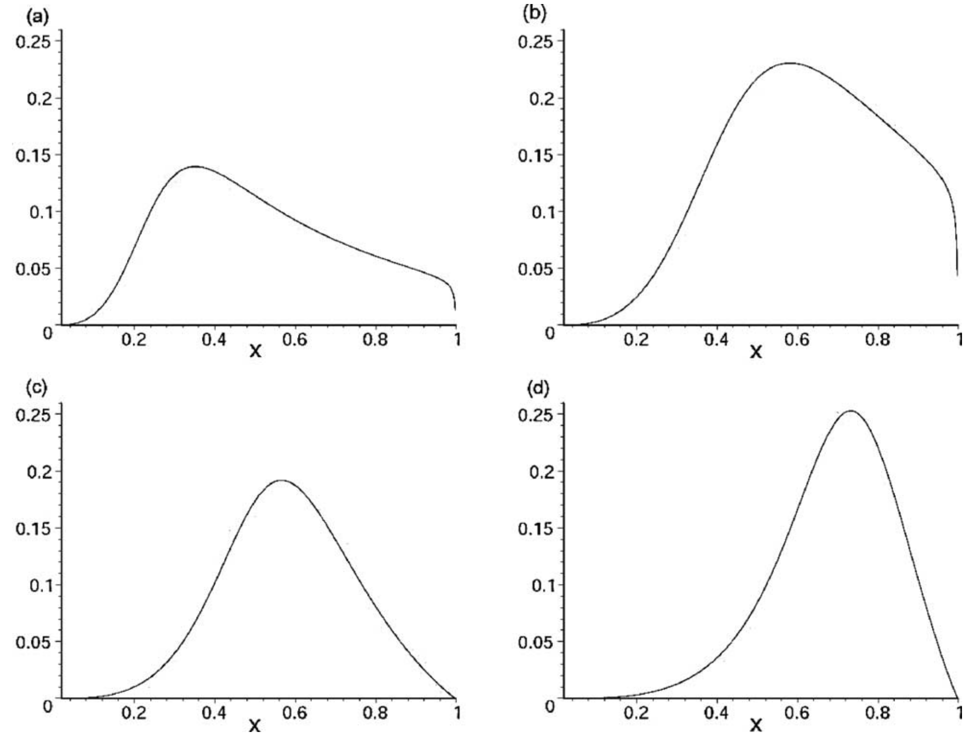

Fig. 4. The function $f(x)$ (Eq. (21)) describing the forward–backward asymmetry in an untagged $B^0/\overline{B}^0$ beam. (a) $Br_{\rm dir} = 10^{-6}$, $E^{(i)} = \text{const}$, (b) $Br_{\rm dir} = 10^{-7}$, $E^{(i)} = \text{const}$, (c) $Br_{\rm dir} = 10^{-6}$, $E^{(i)} \sim 1/s$, (d) $Br_{\rm dir} = 10^{-7}$, $E^{(i)} \sim 1/s$.

phases $\delta_1(s)$ and $\delta_2(s)$ describe $L = 1$ and $L = 2$ states of a 2π system with invariant mass s. As a net result, the relative phase of the bremsstrahlung and direct amplitudes in Eq. (11) may be written as $\pi - \alpha_{\rm eff} + \delta_{\rm str}$ in $\overline{B}^0$ decay, and $-\pi + \alpha_{\rm eff} + \delta_{\rm str}$ in B^0 decay, where $\delta_{\rm str}$ denotes some effective combination of δ_0, δ_1 and δ_2.

Defining

$$\mathcal{F}(x) = \int_0^1 \frac{dBr}{dx\, d\cos\theta}\, d\cos\theta,$$

$$\mathcal{B}(x) = \int_{-1}^0 \frac{dBr}{dx\, d\cos\theta}\, d\cos\theta \qquad (20)$$

we obtain the forward–backward asymmetry in a mixture of B^0 and $\overline{B}^0$:

$$\text{Asy}(x) = \frac{\mathcal{F} - \mathcal{B}}{\mathcal{F} + \mathcal{B}} = f(x) \sin\delta_{\rm str} \sin\alpha_{\rm eff}. \qquad (21)$$

Note, that the forward–backward asymmetry in (untagged) $\overline{B}^0, B^0 \to \pi^+\pi^-\gamma$ decay is equivalent to an asymmetry in the energy spectrum of π^+ and π^- in the B meson rest frame. The function $f(x)$ is plotted in Fig. 4. Thus $\text{Asy}(x)$ is a signal of CP-violation, that is present even in an untagged $\overline{B}^0/B^0$ mixture, and requires $\alpha_{\rm eff} \neq 0$ and $\delta_{\rm str} \neq 0$, in addition to a quadrupole term in the direct electric amplitude.[1]

5. Summary

We have studied observables in the decay

$$B^0, \overline{B}^0 \to \pi^+\pi^-\gamma,$$

[1] Eq. (21) holds when $P_{\pi\pi}/T_{\pi\pi}$ is real. More generally, the asymmetry is proportional to $(\sin\alpha \sin\Delta_T + |P_{\pi\pi}/T_{\pi\pi}| \times \sin\beta \sin\Delta_P)$, where Δ_T and Δ_P denote the strong phase differences $\Delta_T \equiv \delta_{T_{\pi\pi}} - \delta_2$, $\Delta_P \equiv \delta_{P_{\pi\pi}} - \delta_2$.

that do not require tagging or measurement of time-dependence, but which nevertheless probe weak and strong phases appearing in the decay amplitude. Interference of the bremsstrahlung and direct components affects the linear and quadratic terms in the photon energy spectrum Eq. (17), with potential sensitivity to the phase $\arg[(V_{tb}V_{td}^*)^* \mathcal{A}(\overline{B}^0 \to \pi^+\pi^-)]$. In the presence of non-trivial strong phases, there is a difference in the π^+ and π^- energy spectra even for an untagged $B^0/\overline{B}^0$ beam, or, equivalently, a forward–backward asymmetry of the π^+ relative to the photon direction.

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