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## Radiative transmission of lepton flavor hierarchies

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## ABSTRACT

We discuss a one loop model for neutrino masses which leads to a seesaw-like formula with the difference that the charged lepton masses replace the unknown Dirac mass matrix present in the usual seesaw case. This is a considerable reduction of parameters in the neutrino sector and predicts a strong hierarchical pattern in the right handed neutrino mass matrix that is easily derived from a  $U(1)_H$  family symmetry. The model is based on the left–right gauge group with an additional  $Z_4$  discrete symmetry which gives vanishing neutrino Dirac masses and finite Majorana masses arising at the one loop level. Furthermore, it is one of the few models that naturally allow for large (but not necessarily maximal) mixing angles in the lepton sector. A generalization of the model to the quark sector requires three iso-spin singlet vector-like down type quarks, as in  $E_6$ . The model predicts an inert doublet type scalar dark matter.

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## 1. Introduction

One of the major puzzles in particle physics beyond the standard model (SM) is to understand the origin of neutrino masses [1]. A simple paradigm is the seesaw mechanism [2] which introduces three right handed (RH) neutrinos with arbitrary Majorana masses additionally to the SM with the resulting seesaw formula for the light neutrino mass matrix given by

$$\mathcal{M}_\nu = -m_D^T M_R^{-1} m_D. \quad (1)$$

The input values of  $m_D$  and  $M_R$  are then required to find the neutrino masses. In the simple seesaw framework, the RH neutrino spectrum can therefore not be determined from neutrino observations. Clearly, the knowledge of the right handed neutrino spectrum would be of great phenomenological interest for testing the model. If seesaw is embedded into grand unified theories it is sometimes possible to predict  $m_D$ , so that one could get some idea about the right handed neutrino masses. In this Letter, we present a bottom-up one loop scheme where we obtain the following seesaw-like formula from a left–right symmetric model even though the Dirac mass matrix vanishes to all orders in perturbation theory:

$$\mathcal{M}_\nu = \frac{\lambda'}{16\pi^2} M_\ell^{\text{diag}} M_N^{-1} M_\ell^{\text{diag}}, \quad (2)$$

where  $M_\ell^{\text{diag}}$  is the diagonal charged lepton mass matrix:  $M_\ell^{\text{diag}} = \text{diag}(m_e, m_\mu, m_\tau)$  and  $\lambda'$  is a Higgs self coupling. As a result, the flavor structure of the RH neutrino mass matrix is completely determined. We find a stronger hierarchy in the RH neutrino sector compared to the charged leptons. Thus the radiative corrections transmit the charged lepton mass hierarchy into the RH neutrino sector (*radiative transmission of hierarchies*). Furthermore the hierarchy in the RH sector is such that it is easily obtainable from a simple  $U(1)_H$  family assignment. This is the main result of the Letter. As an application, we predict  $B(\mu \rightarrow e + \gamma)$  in this model.

We also discuss how the quark sector can be made realistic since the  $Z_4$  symmetry leads to vanishing down quark masses at tree level. Two ways to generate realistic down quark masses and CKM angles are: (i) introduction of color triplet iso-spin singlet fields that give radiative masses to down quarks or (ii) the addition of three iso-spin singlet vector-like down quarks which generate a tree level mass for the down quarks. We only present the second scenario here, which also has the property that it leads to an inert doublet type scalar dark matter.

## 2. The model

Our model is based on the left–right (LR) symmetric group [3]  $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$  supplemented by a discrete symmetry group  $Z_4$ . The quarks and leptons are assigned as in the minimal LR model to left–right symmetric doublets. The symmetry breaking is implemented also as in the minimal LR model by the Higgs fields  $\phi(2, 2, 0)$  and  $\Delta_R(1, 3, +2) \oplus \Delta_L(3, 1, +2)$ .

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**Table 1**

Fields	$Z_4$ charge
$Q_R$	$-i$
$L_R$	$+i$
$\phi$	$+i$
$\tilde{\phi} \equiv \tau_2 \phi^* \tau_2$	$-i$
$\Delta_R$	$-1$

In the leptonic sector of the model, the  $SU(2)_R \times U(1)_{B-L}$  breaking by the right handed triplet with  $B - L = 2$  gives large Majorana masses to the RH neutrinos [4]. Unlike in the usual implementation of the seesaw formula however, in our model, the Dirac mass for neutrinos vanishes to all orders in perturbation theory due to the  $Z_4$  symmetry, whose effect on the various fields is given in Table 1.

All other fields are assumed to be singlets of  $Z_4$ . The most general potential for the left–right model has been discussed in the literature before [5]. The presence of the  $Z_4$  symmetry in our model forbids terms linear in the invariant  $\text{Tr}(\phi^\dagger \phi)$  in the potential so that the minimum energy configuration corresponds to the following vev for the  $\phi$  field (instead of the general one in [3]):

$$\langle \phi \rangle = \begin{pmatrix} \kappa & 0 \\ 0 & 0 \end{pmatrix}. \quad (3)$$

For the  $\Delta_{L,R}$  fields we have

$$\langle \Delta_R^0 \rangle = \begin{pmatrix} 0 & 0 \\ v_R & 0 \end{pmatrix}, \quad \langle \Delta_L^0 \rangle = 0. \quad (4)$$

The gauge invariant Yukawa couplings of the above  $Z_4$  supplemented LR model are

$$\mathcal{L}_Y = h_q \bar{Q}_L \phi Q_R + h_l \bar{L}_L \tilde{\phi} L_R + [f(L_R^T \Delta_R L_R + L_L^T \Delta_L L_L) + \text{h.c.}]. \quad (5)$$

By an appropriate choice of the basis, we can choose both  $h_{q,l}$  to be diagonal matrices. There is no loss of generality in this. It is easy to see that with the above assignment, we get the Dirac neutrino mass  $m_D = 0$  and the diagonal Yukawa coupling matrix  $h_l$  is given by  $h_l = \text{diag}(m_e, m_\mu, m_\tau)/v_{wk}$ . We also note that there is no type II seesaw [6] contribution to the neutrino masses unlike in usual left–right models.

The stability of the minimum of the potential under radiative corrections can be seen as follows: If we write  $\phi \equiv (H, \eta)$  where  $H$  and  $\eta$  are two  $SU(2)$  Higgs doublets with  $Y = \mp 1$ , then the above vev pattern corresponds to  $\langle H \rangle = v_{wk} \neq 0$  and  $\langle \eta \rangle = 0$ . In the language of the  $\eta$  and  $H$  fields, it is easy to see that the Lagrangian of the model respects a remnant  $Z_2$  symmetry under which  $\eta \rightarrow -\eta$  and  $N_R \rightarrow -N_R$  and all other fields are singlets, so that the zero vev for  $\eta$  is protected by this symmetry. Below the  $SU(2)_R \times U(1)_{B-L}$  breaking scale, the model is a two Higgs extension of the standard model that allows for small neutrino masses, similar to one discussed in Ref. [7].

### 3. Seesaw-like formula for neutrino masses

As noted, at tree level, both neutrino Dirac masses and the down quark masses vanish. We will address the question of down quark masses in the next section. As far as neutrinos are concerned, at one loop level they pick up mass from the left diagram in Fig. 1 with the neutrino mass matrix given by the one loop formula

$$\mathcal{M}_{\nu,ij} = \frac{1}{16\pi^2} m_{l,i} \Lambda_{ij}(\lambda', M_{N,ij}) m_{l,j}, \quad (6)$$

where  $\Lambda_{ij}$  is given by

$$\frac{M_{N,ij}}{16\pi^2} \left[ \frac{m^2(\sqrt{2}\Im\eta^0)}{m^2(\sqrt{2}\Re\eta^0) - M_{N,ij}^2} \log\left(\frac{m^2(\sqrt{2}\Re\eta^0)}{M_{N,ij}^2}\right) - \frac{m^2(\sqrt{2}\Im\eta^0)}{m^2(\sqrt{2}\Re\eta^0) - M_{N,ij}^2} \log\left(\frac{m^2(\sqrt{2}\Im\eta^0)}{M_{N,ij}^2}\right) \right]. \quad (7)$$

The Higgs masses are given by

$$\begin{aligned} m^2(\sqrt{2}\Re\phi^0) &= 2\lambda_1 \kappa^2, \\ M^2(\sqrt{2}\Re\eta^0) &= M_2^2 + (\lambda_3 + \lambda_4 + \lambda_5) \kappa^2, \\ m^2(\sqrt{2}\Im\eta^0) &= M_2^2 + (\lambda_3 + \lambda_4 - \lambda_5) \kappa^2, \quad \text{and} \\ m^2(\eta^\pm) &= M_2^2 + \lambda_3 \kappa^2. \end{aligned} \quad (8)$$

Note that these couplings  $\lambda_i$  are the effective couplings which we get at low energies when the left–right symmetry is broken.

We assume that  $m^2(\sqrt{2}\Re\eta^0) \ll M_{N,ij}^2$ ,  $\Lambda_{ij}(\lambda', M_{N,ij}) \simeq 2 \frac{\lambda'}{M_{N,ij}^2} \times \log\left(\frac{M_{N,ij}^2}{m^2(\sqrt{2}\Re\eta^0)}\right)$ , where  $\lambda'$  is equivalent to  $\lambda_5$  in the Ma-model [7]. Then, the light neutrino mass matrix can then be written as

$$\mathcal{M}_{\nu,ij} = \frac{2\lambda'}{16\pi^2} m_{l,i} (M_N^{-1})_{ij} m_{l,j} \log(M_{N,ij}^2/m^2(\sqrt{2}\Re\eta^0)). \quad (9)$$

Note that we can absorb  $\log\left(\frac{M_{N,ij}^2}{m^2(\sqrt{2}\Re\eta^0)}\right)$  into  $(M_N^{-1})_{ij}$  without loss of generality.

Since we have a rough idea about the form of the neutrino mass matrix in the limit of zero CP phase and small reactor angle  $\theta_{13}$ , we can use it to get an idea about the elements of the RH neutrino mass matrix. It is interesting that all elements of this mass matrix can be determined.

#### 3.1. Normal hierarchy

The neutrino mixing observables [8] we use are:

$$\begin{aligned} \Delta m_{21}^2 &= 7.65 \times 10^{-5} \text{ eV}^2, & |\Delta m_{31}^2| &= 2.40 \times 10^{-3} \text{ eV}^2, \\ \sin^2 \theta_{12} &= 0.304, & \sin^2 \theta_{23} &= 0.5, \quad \text{and} \quad \sin^2 \theta_{13} = 0.01. \end{aligned} \quad (10)$$

The charged lepton masses we take from Ref. [9]:

$$m_e = 0.511, \quad m_\mu = 105.658, \quad \text{and} \quad m_\tau = 1776.84 \text{ MeV}. \quad (11)$$

To fit the neutrino oscillation data, we can use

$$M_N = \frac{2\lambda'}{16\pi^2} \begin{pmatrix} 1.83 \times 10^6 & -1.76 \times 10^8 & 2.87 \times 10^9 \\ \times & 1.80 \times 10^{10} & -2.91 \times 10^{11} \\ \times & \times & 4.81 \times 10^{12} \end{pmatrix} \text{ GeV}, \quad (12)$$

where the mass eigenvalues are given by  $(M_{N1}, M_{N2}, M_{N3}) = \frac{2\lambda'}{16\pi^2} (9.55 \times 10^4, 4.65 \times 10^8, 4.83 \times 10^{12})$  GeV. Note that, in order to avoid the  $N$  detection in the  $Z$ -boson decay width,  $\lambda'$  has to be larger than 0.0037.

The neutrino masses are given by

$$m_1 = 0.0001, \quad m_2 = 0.0087, \quad \text{and} \quad m_3 = 0.049 \text{ eV}. \quad (13)$$

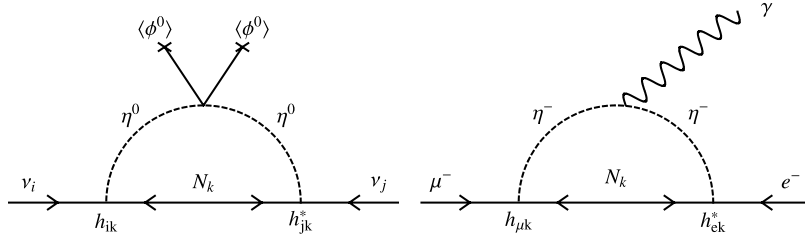


Fig. 1. The diagrams responsible for neutrino masses and for the rare decay  $\mu \rightarrow e\gamma$  in a Ma-like model, cf. Ref. [7].

### 3.2. Inverted hierarchy

To fit the neutrino oscillation data, we can use

$$M_N = \frac{2\lambda'}{16\pi^2} \begin{pmatrix} 5.57 \times 10^3 & -3.80 \times 10^6 & 6.42 \times 10^7 \\ \times & 5.59 \times 10^{10} & 9.37 \times 10^{11} \\ \times & \times & 1.58 \times 10^{13} \end{pmatrix} \text{ GeV}, \quad (14)$$

where the mass eigenvalues are given by  $(M_{N1}, M_{N2}, M_{N3}) = \frac{2\lambda'}{16\pi^2} (5.31 \times 10^3, 4.48 \times 10^8, 1.59 \times 10^{13})$  GeV, where  $\lambda'$  now has to be larger than 0.67.

The neutrino masses are

$$m_1 = 0.049, \quad m_2 = 0.050, \quad \text{and} \quad m_3 = 0.0001 \text{ eV}. \quad (15)$$

Note that in both cases, there is a strong hierarchy in the RH neutrino sector in a way similar to the charged lepton sector. This is what we label as the *radiative transmission of hierarchy* from charged leptons to the RH neutrinos. Note that this mechanism, given a certain form of  $M_N$  (with small mixings), naturally allows for large mixing angles in the SM lepton sector, that are not necessarily maximal. This is different from many other models, where in most cases only zero or maximal mixing is predicted. Note however, that there are also exceptions to this: E.g., the size of the mixing angle could be determined by underlying discrete symmetries [10], or it could arise from an anarchical pattern of the neutrino mass matrix [11].

To see analytically why this happens, let us try to reconstruct  $M_N$  from the tri-bimaximal form for the PMNS-matrix [12],

$$U_{\text{PMNS}} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}. \quad (16)$$

Using this and Eq. (6), we can write down  $M_N$  as function of  $\lambda'$  and of the light neutrino mass eigenvalues  $m_{1,2,3}$ . It is given by  $\frac{\lambda'}{6m_1 m_2 m_3}$  times

$$\begin{pmatrix} 2(m_1 + 2m_2)m_3 m_e^2 & 2(m_1 - m_2)m_3 m_e m_\mu & 2(m_1 - m_2)m_3 m_e m_\tau \\ 2(m_1 - m_2)m_3 m_e m_\mu & (3m_1 m_2 + m_2 m_3 + 2m_1 m_3)m_\mu^2 & (-3m_1 m_2 + m_2 m_3 + 2m_1 m_3)m_\mu m_\tau \\ 2(m_1 - m_2)m_3 m_e m_\tau & (-3m_1 m_2 + m_2 m_3 + 2m_1 m_3)m_\mu m_\tau & (3m_1 m_2 + m_2 m_3 + 2m_1 m_3)m_\tau^2 \end{pmatrix}. \quad (17)$$

If we assume normal ( $m_1 = p^2 m_0$ ,  $m_2 = p m_0$ , and  $m_3 = m_0$ , with small  $p$ ) or inverted hierarchy ( $m_1 = m_0$ ,  $m_2 = m_0$ , and  $m_3 = p m_0$ ), the corresponding matrices will roughly look like

$$(M_N)_{\text{NH}} = \frac{\lambda'}{6p^2 m_0} \begin{pmatrix} 4m_e^2 & -2m_e m_\mu & -2m_e m_\tau \\ -2m_e m_\mu & m_\mu^2 & m_\mu m_\tau \\ -2m_e m_\tau & m_\mu m_\tau & m_\tau^2 \end{pmatrix} \quad (18)$$

and

$$(M_N)_{\text{IH}} = \frac{\lambda'}{2p m_0} \begin{pmatrix} 2p m_e^2 & 0 & 0 \\ 0 & m_\mu^2 & -m_\mu m_\tau \\ 0 & -m_\mu m_\tau & m_\tau^2 \end{pmatrix}. \quad (19)$$

Note that the reconstruction of all matrices (Eqs. (18) and (19)) has led us to heavy neutrino mass matrices which are hierarchical and stiff. In all cases, having a light neutrino mass close to zero ( $p \rightarrow 0$  in Eqs. (18) and (19)) can only increase this hierarchy, but not destroy it. Especially in Eq. (19) the 11-entry is fixed, which means that we will generically have one fixed RH neutrino mass that is not too heavy. A similar situation happens for the quasi-degenerate case.

These mass matrices for RH neutrinos have a structure that is easily obtainable from the Froggatt-Nielsen (FN) mechanism [13] with a  $U(1)_H$  family symmetry with  $H$  charges  $(0, 1, 2)$  for the third, second, and the first generation right handed lepton doublets. The left-right and  $U(1)_H$  invariant Yukawa couplings in this case can be written as

$$\mathcal{L}_{Y,H} = h_3^l \bar{L}_{3,L} \tilde{\phi} L_{3,R} + h_2^l \bar{L}_{2,L} \tilde{\phi} L_{2,R} + h_1^l \bar{L}_{1,L} \tilde{\phi} L_{1,R} \left( \frac{\sigma}{M} \right)^2 + \left[ \sum_{a,b=1,2,3} f_{ab} L_{a,R}^T \tilde{\Delta} L_{b,R} \left( \frac{\sigma}{M} \right)^{6-(a+b)} + \text{h.c.} \right]. \quad (20)$$

For an appropriate choice of  $\frac{\langle \sigma \rangle}{M}$  (roughly  $1/20$  in the normal hierarchy case), we get the desired hierarchy in both the charged lepton masses as well as in the RH neutrino sector. This hierarchy then translates into a structure of the light neutrino mass matrix that naturally yields large mixing angles, although no values are excluded a priori.

We can also give a prediction for  $\mu \rightarrow e\gamma$  [14], which is transmitted by the heavy neutrinos (cf. right diagram of Fig. 1): The Yukawa coupling in the basis where the heavy neutrino mass matrix is diagonal is given by  $h = U^{-1} \text{diag}(m_e, m_\mu, m_\tau)/v_{wk}$ , where  $U$  is the matrix that diagonalizes  $M_N$ . For a charged Higgs mass of 100 GeV and  $\lambda' = 0.7$ , the prediction for  $\text{Br}(\mu \rightarrow e\gamma)$  is  $6 \times 10^{-16}$  for normal and  $8 \times 10^{-16}$  for inverted ordering, where we have used Eqs. (12) and (14). If we go to smaller values for  $\lambda'$ , the branching ratio increases ( $3 \times 10^{-12}$  for  $\lambda' = 0.01$  and normal ordering), which might be very interesting in light of the upcoming MEG experiment [15].

### 4. Extension to quark sector

It is clear from Eq. (4) that at the tree level in our model, only the up quarks are massive. We present two ways to make the quark sector realistic by giving mass to the down quarks, (i) one where the  $Z_2$  symmetry, that keeps Dirac mass of the neutrino to be zero, is softly broken and (ii) another one by adding three vector-like down quarks, where we can keep the  $Z_2$  symmetry exact. We only discuss the second option here.

For (ii), we extend the model by adding three  $SU(2)_{L,R}$  singlet, color triplet,  $B - L = 2/3$  quarks (denoted by  $D_{L,R}$ ) and two Higgs doublets under the  $SU(2)_{L,R}$  groups with  $B - L = 1$  (denoted by  $\chi_{L,R}$ ). Under the  $Z_4$  symmetry, the  $\chi_{L,R}$  and  $D_R$  are invariant, whereas  $D_L \rightarrow -iD_L$ . It is easy to write down a potential for  $\chi_{L,R}$  with asymmetric mass terms for them so that they have nonzero

VEVs. Since the discrete symmetry does not allow the term  $\chi_L^\dagger \phi \chi_R$  term in the potential, the additional fields do not destabilize the  $\phi$  vev pattern assumed in the bulk of the Letter. The new Yukawa interaction that is invariant under  $Z_4$  and gauge symmetry is given by

$$\mathcal{L}_{new} = f_D (\bar{Q}_L \chi_L D_R + \bar{Q}_R \chi_R D_L) + \text{h.c.} \quad (21)$$

After spontaneous symmetry breaking the down quarks now have masses where they pair with the new down quarks (rather than the usual ones of the SM). As a result, the  $SU(2)_R$  partner of the up quark is a heavy down quark unlike in the minimal left-right model [3]. In fact, after symmetry breaking, one could write the left and right doublets as follows:  $Q_L = (u_L, d_L)$  and  $Q_R = (u_R, D_R)$  ( $D_R$  and  $d_R$  swap roles), where the mass of  $D$  is in the 10 to 100 TeV range. We emphasize that there is no direct mass term between  $D_L$  and  $D_R$ .

To fit the down quark masses and the CKM matrix, the Yukawa coupling need to be (all matrix elements of Eq. (22) are in MeVs):

$$f_D = \begin{pmatrix} 0.89 & 24.7 & 14.1 \\ \times & 106.5 & 169.9 \\ \times & \times & 4192.9 \end{pmatrix} \frac{1}{v_L}, \quad (22)$$

where  $v_L$  is the vev of  $\chi_L$ . This appears to be a completely viable way to generate down quark masses. An interesting feature of this model is that the surviving  $Z_2$  remains an exact symmetry, and as result the neutral member of the second doublet in  $\phi$  can act as dark matter [16], since it couples to quarks as  $\phi_2^0 \bar{d}_L D_R$ , and as long as  $m_{\phi_2^0} \ll M_D$ , the  $\phi_2^0$  is stable with stability guaranteed by the  $Z_2$  symmetry [17].

## 5. Conclusion

In summary, we have shown that a radiative one loop model for neutrino masses proposed in [7] arises as a low energy limit of a left–right model which then provides a natural explanation of the two elements of the [7] proposal: (a) the reason for the extra doublet with its particular discrete symmetry property and (b) the origin of the right handed neutrino mass. Furthermore, the radiative transmission of hierarchies makes large but non-maximal mixing angles in the leptonic sector plausible. Left–right embedding also reduces the number of parameters in the model, making it predictive in the hadronic and leptonic flavor sectors.

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