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Sraffa, Leontief, Lange: The political economy of input–output economics

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Abstract

The fields of Political Economy and Input–Output economics are commonly unrelated due to the discrepancy between the qualitative and quantitative analysis in economic science. With the aim to overcome this separation, this paper relates the basic model of *Production of Commodities by Means of Commodities* to the paradigm of planning as the substitute for the market by sketching the logical-historical development of Input–Output economics. The model of Sraffa, the Leontief matrices and the Political Economy of Marxist tradition, represented by Oskar Lange, are strongly associated so that the typical qualitative concepts of planning, law of value and historical development of the capitalist system can be connected to the quantitative framework of matrix, programming and mathematical formalization.

JEL classifications: B23; B24; P51

Keywords: Political economy; Value theory; History of economic thought; Sraffa; Leontief; Marx

Resumo

Os campos da Economia Política e da Economia do Insumo-Produto não são geralmente relacionados devido à discrepância entre as análises qualitativas e quantitativas na ciência econômica. Com o objetivo de superar essa separação, o artigo relaciona o modelo básico de Produção de Mercadorias por Meio de Mercadorias com o paradigma da planificação enquanto substituto do mercado ao descrever o desenvolvimento lógico-histórico da Economia do Insumo-Produto. O modelo de Sraffa, as matrizes de Leontief e a Economia Política de tradição Marxista, representada por Oskar Lange, são associadas fortemente de modo que os conceitos tipicamente qualitativos de planificação, lei do valor e desenvolvimento histórico do sistema capitalista possam ser conectados ao mundo quantitativo das matrizes, da programação e da formalização matemática.

Palavras-chave: Teoria do Valor; História do Pensamento Econômico; Sraffa; Leontief; Marx

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1. Introduction

Since the pioneer works of Leontief (1928, 1936, 1966) input–output economics became one of the most important branches in economic science. Along with this development, there was an impressive improvement of computers. On one side, this enabled the construction of extremely complex mathematical models. On the other side, it made the presentation of input–output economics and of economic planning very complicated for economists outside this tradition, who primarily dedicate their attention to the historical development of the social relations reflecting the technical base of production.[1]

It does not mean that the mathematical formalization should be controlled or repressed. But we claim that it is essential to constantly give precise and meaningful explanations for the equations if we are interested in enlarging the number of scientists in conditions of communication, as Teixeira's (1984) reflection also indicates. In a certain way, this is the same appeal made by Leontief (1964) to approximate 'technical economists' and 'political economists'. Given that this union may be highly useful for dealing with practical topics of national economic policy, it is reasonable to affirm that Leontief's appeal still applies today.

Another problem is the following. From the historical-qualitative perspective, input–output economics traditionally does not distinguish historical specific economic categories from categories embracing all modes of production. As a consequence, many models of input–output begin with capitalist categories which should be understood as specific forms of the general economic categories. Profits and interest rates are then explained before it is agreed that these forms of income are only expressions of the existing economic surplus in society. If we want to talk about the interest rate, it is necessary to previously develop the notion of profit, which, on its turn, can only be fully comprehended after the concept of growth rate is entirely described. Economic growth, on its turn, gains a concrete meaning for the economist only after the notion of surplus is discussed, and so on.

The monetary exchanges between sectors presented in the matrices of accountability usually introduce the general economic model from the perspective of value. Now, if the reader is familiar to the Critique of Political Economy and to "the pivot on which a clear comprehension of political economy turns" (Marx ([1867] 1887) chapter 1, section 2), i.e., to the difference between use value and value (or concrete and abstract labour), this is not a structural problem. Equipped with this instruments, one can certainly read and translate the input–output tables as to see whether the material flow follows the demand of working people or not. However, this is not the normal case for most economists initiating the studies on input–output economics. Most of them are trained as technicians or economist–engineer with high qualifications for the quantitative methods, statistics and mathematics.[2] In order to comprehend the social utility of input–output economics, it is necessary to add to the techniques of quantitative mapping of economic relations the

[1] For an introduction to the input–output analysis in relation to economic planning from the traditional perspective, see Stone (1984). On the relationship between ideology and the economic structure, see Karl Marx's major conclusions after studying political economy in the *Preface to the Critique of Political Economy* (Marx ([1859] 1977)).

[2] Input–output analysis is part of economists toolbox and it is nowadays taught within the framework of Neoclassical economics. This reinforces the mentioned distance between technical and political economists and indicates that there are two great traditions of the input–output perspective. One of them is the original formulation which goes back to the traditions of Phisiocracy and Classical Political Economy. The other one is the input–output schemes as a part of general equilibrium analysis, which resembles the work of Walras ([1874] 1954). There are profound epistemological differences between the Classical and the Neoclassical approach to input–output analysis. While the Classical approach focus on transmission effects in the long run and relates them to the concept of technical coefficient due to its theory of objective value, the Neoclassical approach is concerned with the changes in the short run and relates them to the structure of preferences of consumers due to its theory of subjective value. The problem of aggregation is rapidly solved by the Neoclassical approach because it abstracts from the concrete movements necessary in production to perform the transformation of one use value into another. The ongoing question whether there exists an equilibrium for the whole system is another picture very present at the Neoclassical board which is not central in the Classical perspective. If equilibrium is considered in the Classical framework, it is dynamic rather than static, as it is usual in the Neoclassical models. Measurement of wealth (as a sum of different use values) is also a non-problem for the Neoclassical view, because it does not bother asking what capital is qualitatively made of. This last discrepancy is, by the way, linked to the Capital Controversy, a debate about the concept of capital that gained popularity during the 1960s and 1970s. Despite the divergences between the Classical and Neoclassical interpretations of input–output analysis being important to direct the reader to this controversy, we believe that both perspectives can be a starting point for becoming closer to the Critique of Political Economy of Karl Marx, a system under which all three authors here studied (Sraffa, Leontief and Lange) located themselves. In that sense, we agree with Akhabbar and Lallement (2011), to whom both Neoclassical and Classical perspectives merge in the models of Leontief, who consciously used a rhetoric of consensus between Classical and Neoclassical economic thought in a way similar to Lange but differently from Sraffa and Samuelson. For a connection between Classical and Neoclassical input–output analysis, see Ten Raa and Mohnen (1994). On the present state of the capital controversy see Garegnani (2012).

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comprehension of the economy as a historical and social process. In that sense, to recover the fundamental theoretical problems of Political Economy contributes to increase the overall level of development not only of input–output economics but of economic science as a whole.

One of the most important achievements of Marx (1993) was his explanation that up until the rise of the capitalist society, all economists confused the economy in general with the specific historical form of the economy under the rule of capital. It means that capitalism is only one form of economic organization that exists due to the presence of certain historical, social and technical conditions. Despite being frequently disregarded in many environments of economics teaching, Marxist economics points out to an utmost important distinction that is also forgotten by many economic scientists. The discovery of the historical specificity of capitalism indicates that all rules and patterns described by economic theory up until the Critique of Political Economy refer exclusively to the capitalist economy, and not to the general mode of production of distribution that can be used as an abstract frame to explain all modes of production in history. It is imperative to differentiate the general laws of the economic system from those which prevail in the capitalist mode of production not only to consolidate economics as a science, but also to correctly develop the instruments of economic planning in order to overcome all problems created by capitalism. The distinction between historical-specific economic categories from categories present in all forms of social organization of production and distribution is equivalent to the distinction made by the polish economist Oskar Lange between the technical laws of balance and the specific laws of a given social formation. This division is an open topic within historical materialism, a new stage of construction in social sciences under which economic science may be allocated according to Marxist economics.[3]

This paper presents the theory of economic planning as the new paradigm after economic liberalism observing these reservations on the basis of the simplest model of Sraffa (1960). Departing from the equations of the first chapter of *Production of Commodities by Means of Commodities*, it will be possible to establish the relationship between Sraffa's model, the Leontief matrices and the Political Economy of Marxist tradition represented by Oskar Lange.[4] The aim of the paper is to maintain the quantitative and the qualitative aspects in a dialectical relationship in its synthesis of Sraffa (1898–1983), Leontief (1906–1999) and Lange (1904–1965). In doing so, we hope to eliminate occasional conflicts between economists of two opposite traditions (the historical-qualitative and the technical-mathematical) and to start organizing the connections between three great contemporary economists of the 20th century who could not directly cooperate while living.

2. Sraffa's basic model in *Production of Commodities by Means of Commodities*

In order to present the concept of economic programming within the specificities outlined here, it is necessary to start with the simplest model of *Production of Commodities by Means of Commodities*. It is necessary to begin with the basic economy in order to advance gradually to more complex economies, as it was the case with history itself. In doing so, it will be easier to check which terms in the exchange value perspective refer to which general economic concept in its use value sphere. Sraffa (1960) adopts a similar procedure, beginning with a simple society which produces only the necessary for its reproduction. He adds then complexity in the system along the book.[5]

[3] See Lange ([1959] 1966), especially chapter 3, and Lange (1945–1946). In this paper, square brackets indicate the work's original year of publication.

[4] The connection of the results of these economists (and we could include here Neumann ([1938] 1945–1946 and others) is still uncompleted. That line of theoretical construction goes back to the Classical Political Economy and, even earlier, to the Physiocracy. Clark (1984) and Kurz and Salvadori (2000) make a historical analysis of this tradition. Bêrni (1995) presents the input–output analysis with focus on the labor theory of value. Pasinetti (1977) and Kurz and Salvadori (2006) present the analysis of the matrices on the basis of the similarities between Leontief and Sraffa. For the current stage of this research line, see Kurz (2011).

[5] When dealing with the general economic model, Sraffa (1960) uses a terminology which does not distinguish the historical from the general categories. His main work should, strictly speaking, be called 'Production of Use Values by Means of Use Values', so that the commodity form as product could be developed logically and historically. It does not mean that Sraffa was unaware of the overlap of categories, but that it was not essential for his supreme objective of criticizing the marginalist theory of value. On Sraffa using historical categories in order to refer to general categories, see Bellofiore (2008), p. 89. The interpretation of Sraffa (1960) adopted in this paper is similar to that of Tolipan (1979), who believes that Sraffa makes a 'last return to Ricardo' in order to put economic science on the right track towards the Critique of Political Economy advanced by Marx.

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2.1. Wheat, iron and pigs

In the first chapter of *Production of Commodities by Means of Commodities* (PCC) Sraffa envisages a system of production for subsistence. In the first section he considers a method of production[6] consisting of only two products (wheat and iron), which is presented as following[7]:

$$280\,\text{qr. wheat} + 12\,\text{t iron} \rightarrow 400\,\text{qr. wheat}$$

$$120\,\text{qr. wheat} + 8\,\text{t iron} \rightarrow 20\,\text{t iron}$$

This representation means that 'an extremely simple society', by employing a specific combination of the use values wheat and iron, produces the use values wheat and iron in quantities just enough to maintain itself. We call this reproduction scheme method of production.

Each line of the scheme represents a sector of economic activity of this society. The first line is the production sector of wheat, while the second one is a representation for the sector producing iron. In that example, the sector producing wheat uses 280 quarters of wheat and 12 t of iron to produce 400 quarters of wheat. Each sector uses as input wheat as well as iron in the indicated quantities. We can say that from the combination of inputs (left side of the scheme) outputs (right side of the scheme) are created. Sraffa points out that after the production process the two products of this economy, which were previously distributed in a certain way among the sectors, are now all concentrated in their respective sectors. That is, all wheat is in the wheat sector and all iron is in the iron sector.

As society has to allocate these products again as inputs in the sectors in the next production period, Sraffa explains that there is a unique set of exchange values which restores the original displacement of the use values wheat and iron. In this first example, such set is expressed in the relation of equivalence between 10 quarters of wheat and one ton of iron.[8] The determination of the quantitative relations between the use values which puts the products back in the original combination can be mathematically modeled with help of a system of equations.

In Section 2 of the first chapter of PCC Sraffa includes a third product, pigs. When the system is expanded, it becomes increasingly difficult to find the mentioned set of exchange values without the support of calculating machines. Let us analyze the case for three products in a production for subsistence. The method of production imagined by Sraffa in this case is represented as following:

$$240\,\text{qr. wheat} + 12\,\text{t iron} + 18\text{ pigs} \rightarrow 450\,\text{qr. wheat}$$

$$90\,\text{qr. wheat} + 6\,\text{t iron} + 12\text{ pigs} \rightarrow 21\,\text{t iron}$$

$$120\,\text{qr. wheat} + 3\,\text{t iron} + 30\text{ pigs} \rightarrow 60\text{ pigs}$$

How is it possible to write this scheme as a system of equations so that the model presented by Sraffa can be inserted in the computer? There are two necessary steps to convert Sraffa's model into an adequate representation in order to make the computational simulation and consequently, to have a proper representation of the economy in the input-output matrices.

[6] Sraffa calls the relations of this scheme the 'methods of production'. Here, we will use the term in the singular, i.e., 'method of production', in order to characterize a specific scheme.

[7] Sraffa uses the words 'commodity' and 'product' without distinction in his presentation. In this paper, as long as the social relations for material reproduction are not mercantile relations, we will use the word 'product' or 'output' to make clear that we are treating the thing as use value. Product, output and use value are used as synonyms. So, the reproduction here represented refers only to the material reproduction. About Sraffa's treatment of commodity circulation in material terms, see Kurz and Salvadori (2005). According to these authors, modeling the circulation of commodities as a process of material reproduction is extremely difficult when fixed capital exists. One of Sraffa's objectives was to show it is possible to construct such model, even in the case of fixed capital.

[8] It is possible to say that '10 quarters of wheat are one ton of iron worth' or that 'the price of one ton of iron is ten times higher than the price of one quarter of wheat'. Although these expressions make sense only when the use values allocation is made through the market, they can be used as a means to refer to the 'quantitative relations between use values which put the system in reproduction' according to the original position within the input matrix. In the same sense, the word 'market' in Sraffa has this broader meaning.

2.2. *Sraffa's basic model in the computer*

First we write the method of production in the form of a system of equations, noting the prices of each product as unknowns. For the example under consideration, the equation system is:

$$240p_a + 12p_b + 18p_c = 450p_a$$
$$90p_a + 6p_b + 12p_c = 21p_b$$
$$120p_a + 3p_b + 30p_c = 60p_c$$

where p_a is the price of one quarter of wheat, p_b the price of one ton of iron and p_c the price of one pig. How did this presentation change the meaning of the previous scheme of reproduction? Notice that now in the place of the symbol '→', which represented a qualitative transformation, we have the equal sign. So we are considering that the use values of the reproduction scheme are quantified in terms of value in the equation system and that they can, therefore, be added. This is the reason for writing the method of production as a system of equations and it was this passage that logically solved the basic economic problem of aggregation.[9]

The second step is writing the system in matrix notation and manipulating the equation in order to have a single matrix containing all unknowns, which can be then thought as the general unknown. For the example here presented, we write:

$$\begin{bmatrix} 240 & 12 & 18 \\ 90 & 6 & 12 \\ 120 & 3 & 30 \end{bmatrix} \times \begin{bmatrix} p_a \\ p_b \\ p_c \end{bmatrix} = \begin{bmatrix} 450p_a \\ 21p_b \\ 60p_c \end{bmatrix}$$

Because we still do not have all unknowns in one single matrix, we need to rewrite the system as:

$$(240p_a - 450p_a) + 12p_b + 18p_c = 0$$
$$90p_a + (6p_b - 21p_b) + 12p_c = 0$$
$$120p_a + 3p_b + (30p_c - 60p_c) = 0$$

Which is equivalent to:
(4)

$$\begin{bmatrix} (240-450) & 12 & 18 \\ 90 & (6-21) & 12 \\ 120 & 3 & (30-60) \end{bmatrix} \times \begin{bmatrix} p_a \\ p_b \\ p_c \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

From this matrix form it is possible to use the computer in order to calculate the exchange values p_a, p_b e p_c which recreate the original distribution of use values, with $X = \begin{bmatrix} p_a \\ p_b \\ p_c \end{bmatrix}$. It is necessary to choose one use value as standard of measure. Here, we take the use value wheat and write $p_a = 1$ or anything different from zero. This also avoids the trivial solution $p_a = p_b = p_c = 0$.

Thus, departing from the method of production, which is given by the objective circumstances of production, it is possible to determine the quantitative relations between the use values that compose the total social product so that the original input matrix is rebuilt. These quantitative relations, expressed here in the notation 'p_i', are what Marx calls production prices. The classical economists also referred to them as 'natural prices'.

[9] On the problem of aggregation in relation to the input–output model, see Bêrni (1995). The notation sign change from '→' to '=' here is another way of explaining the difference between the signs '⊕' and '+' as employed by Kurz (2006), p. 10, who also indicates how Sraffa's notation changed in his manuscripts.

In the third and last section of the first chapter of PCC, Sraffa presents the model for a subsistence economy for the general case, that is, for a system with k products. Now, the presented procedure for arranging the Sraffa's system in a proper form for the computational simulation can be written as:

$$A_a + B_a + \ldots + K_a \rightarrow A$$
$$A_b + B_b + \ldots + K_b \rightarrow B$$
$$\ldots$$
$$A_k + B_k + \ldots + K_k \rightarrow K$$

where each use value is represented by a letter. In that sense, the sector producing the use value 'a' uses the quantities $A_a, B_a, \ldots K_a$ of the use values $a, b, \ldots, k$; the sector producing the use value 'b' uses the quantities $A_b, B_b, \ldots K_b$ of the use values $a, b, \ldots, k$; etc. We call 'A' the existing quantity of use value 'a' after the production process, 'B' the existing quantity of use value 'b' after the production process, etc. and 'A_a' the quantity of use value 'a' used in the sector producing the use value 'a', 'A_b' the quantity of use value 'a' used in the sector producing the use value 'b', etc. After solving the aggregation problem logically, the reproduction scheme of use values can be written as the following system of equations:

$$\begin{aligned} &A_a p_a + B_a p_b + \ldots + K_a p_k = A p_a \\ &A_b p_a + B_b p_b + \ldots + K_b p_k = B p_b \\ &\ldots \\ &A_k p_a + B_k p_b + \ldots + K_k p_k = K p_k \end{aligned} \tag{1}$$

Or:

$$\begin{aligned} &(A_a - A) p_a + B_a p_b + \ldots + K_a p_k = 0 \\ &A_b p_a + (B_b - B) p_b + \ldots + K_b p_k = 0 \\ &\ldots \\ &A_k p_a + B_k p_b + \ldots + (K_k - K) p_k = 0 \end{aligned} \tag{2}$$

Which can be written in the matrix form:

$$\begin{bmatrix} (A_a - A) & B_a & \ldots & K_a \\ A_b & (B_b - B) & \ldots & K_b \\ \ldots & \ldots & \ldots & \ldots \\ A_k & B_k & \ldots & (K_k - K) \end{bmatrix} \times \begin{bmatrix} p_a \\ p_b \\ \ldots \\ p_k \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \ldots \\ 0 \end{bmatrix} \tag{3}$$

This is a direct representation for the insertion of data in any programming system which is ought to be employed as instrument for modeling an economy using matrices. The solution for this system will show the values of the matrix $\begin{bmatrix} p_a \\ p_b \\ \vdots \\ p_k \end{bmatrix}$, which indicate what is the quantitative relation between the use values that restores the input matrix from which the process departed.[10]

It is appropriate to emphasize that this system is homogeneous. Therefore, it only allows a solution (besides the trivial solution) in the case of nullity of the determinant of the first matrix which is on the left side of Eq. (3). In other words, the parts $A_a, A_b, \ldots, A_k$ must attend this restriction so we can have a non-trivial solution (different from zero)

[10] In the case of surplus, the equivalent reposition reconstructs the balance equations in proportion, so that the expansion of the system is in equilibrium. The complications of the surplus will not be investigated in this paper. For instructions on using Stata to engage with this system, see Feiveson et al., 2009.

for the price system. The system of equations will have an infinity set of solutions in the case that the determinant of the coefficient matrix is zero. This means that there is linear dependence between the parts of input used in the production of each industry. This condition seems to be an exceptional case, not the regular or natural one. Another way to express the idea is to say that, in order to have an economic solution to the price system (non-trivial solution in the previous system) in a reproduction system without surplus, a very special condition must be fulfilled. Notice that this condition is not the same as that established by the balance equations.

3. Leontief's contribution: pictures of an economy

According to Lange ([1961] 1978), the theory of economic programming has two parts: the first one deals with the 'internal coherence of the program' and the second one with the 'optimal level of the program'. These two parts may also be thought as two levels of analysis of the input–output tables: while the first refers to the static analysis, the second corresponds to the dynamic study of the matrices. Section 3 presents the static and Section 4, the dynamic analysis of input–output.[11]

3.1. The contemporary form of the Tableu Économique

In the presentation of the method of production, we saw that at the end of the production period, each use value of the economy is concentrated in its respective sector. Therefore, if the productive cycle is to be repeated, the society needs to reallocate these products through the various sectors of production. Considering that the input matrix becomes the output matrix successively, the method of production over time can be written as:

$$\begin{bmatrix} A_a & B_a & \dots & K_a \\ A_b & B_b & \dots & K_b \\ \vdots & \vdots & \vdots & \vdots \\ A_k & B_k & \dots & K_k \end{bmatrix} \rightarrow \begin{bmatrix} A \\ B \\ \vdots \\ K \end{bmatrix} \rightarrow \begin{bmatrix} A_a & B_a & \dots & K_a \\ A_b & B_b & \dots & K_b \\ \vdots & \vdots & \vdots & \vdots \\ A_k & B_k & \dots & K_k \end{bmatrix} \rightarrow \begin{bmatrix} A \\ B \\ \vdots \\ K \end{bmatrix} \rightarrow \dots$$

Or:

$$q \rightarrow Q \rightarrow q \rightarrow Q \rightarrow \dots$$

This representation concatenates the input matrix in physical units (q) and the output matrix also in physical units (Q) over time, connecting them with the sign '→'. The transition from the input matrix to the output matrix represents the sphere of production, or the qualitative transformation of the use values. The subsequent passage from the output matrix A, B, ..., K to the input matrix A_a, B_a, ..., K_k represents the sphere of circulation or distribution of the products, or, still, the quantitative transformation of the produced use values, which are merely allocated to form the input matrix for the next production period.

What is the logic of distribution in Sraffa's model? Price determination in Sraffa (the determination of the production prices, which he calls values or prices) refers to this second passage, which in this model of PCC *is always subordinated to a very specific logic*. As already said, the distribution of the output is done in such a way that the initial display of use values is restored. That is the determinant of pricing in this model and that is the criterion for the distribution of the product here. All reproduction exercises since the scheme of flows between different sectors of the Physiocracy follow this pattern of continuous material replacement at a higher level, since the economy they observe expands according to the logic of capital accumulation. According to Rubin (1979) the French authors preceding Adam Smith were modeling the in- and outflows of products as it is imposed by the capitalist system, that is, as a body of material goods which increases quantitatively. In spite of considering the ruptures of this growth typical of capitalist crisis, they maintain

[11] The standard presentations of the theory of linear programming normally do not distinguish these two parts because the first part alone does not represent the formalization of a problem to be solved. It corresponds to the restriction equations only. Including the objective function to the conditions of restriction converts the static analysis into the dynamic analysis. It is relevant to present the two parts separately because the maximization of efciency in production is not the unique objective possible of society, as many textbooks seem to assert. Dorfman et al., 1958 make the distinction when they notice that the plain input–output matrices of Leontief do not denote a problem of choice or optimization.

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the system at an expanding state of equilibrium. This thought exercise was acceptable at a time when the crisis were not as important as they would become in the 19th and 20th century and when the harmony of natural equilibrium was the inspiring core for every branch of science, from physics to mechanics and biology. The logic of equilibrated restoration of the system is common to many fields of research during the illuminist epoch and its specific form in the beginnings of economics is this pattern recreated by Sraffa which was conceived at the peak of Political Economy as the law of value.

Possas (1983) had suggested that this unilateral treatment of the distribution resulted from the static analysis in which the economic problem of Sraffa was formulated. According to the division of the theory of programming here proposed, Sraffa's model is restricted to the first part. In other words, it is constrained to the static study of input–output economics. Indeed, the condition of production prices, which Ricardo also calls 'absolute values' according to Kurz and Salvadori (2006), is the imposition that the process repeats itself identically. That is the same restriction we find in the analysis of the balance equations of production made by Lange ([1961] 1978) and which he calls also study about the internal coherence of the program. This study is equal to the static analysis of the theory of programming.[12]

Similarly, based on the accumulated statistics up until then, the balance of the American economy in the year 1947 is presented in a table of transactions between various sector by Leontief (1951). Given that it does not link the input and output matrices over time, but shows only the sector interconnections captured in a certain time period, it is a study restricted to the 'internal coherence of the program'. As Leontief (1951) described it: '(. . .) a static model, an instant in time'. For that reason, the main concern of input–output economics was initially focused on the relation between the industries, that is, on the flow of exchange between the various sectors of the economy. Restricted to the static perspective, it was merely an advanced form of the reproduction schemes since the 18th century.

It is important to notice that the matrix presented by Leontief is an empirical data of the economy. Therefore, it differs from Sraffa's model representing an abstract situation in which all technical data of the whole economy are already known. In other words, in the model of Sraffa, all technical coefficients are available information. In fact, the construction of Sraffa's tables is the practical process of obtaining real economic data from the input and output matrices that the national statistical offices build. The other manner of revealing the technical coefficients is through the method of engineering, that is, through the direct knowledge about the technological processes related to the production of a certain use value.[13] The balance scheme for the national economy presented by Leontief on the table of monetary flow between the sectors is used by Lange ([1961] 1978) in order to derive the balance equations. These can be thought as the Sraffa's equation for determining the prices of production.

In that case, the transformation of the type $q \to Q \to q$ is modelled by the equations:

$$\begin{aligned} & A_a p_a + B_a p_b + \ldots K_a p_k = A p_a \\ & A_b p_a + B_b p_b + \ldots K_b p_k = B p_b \\ & \ldots \\ & A_k p_a + B_k p_b + \ldots K_k p_k = A p_k \end{aligned} \tag{4}$$

Which are the balance equations in the model of Sraffa. If we want to write the same system in physical terms, it is possible to change lines with columns taking the prices away. The balance equations become then:

$$\begin{aligned} & A_a + A_b + \ldots + A_k = A \\ & B_a + B_b + \ldots + B_k = B \\ & \ldots \\ & K_a + K_b + \ldots + K_k = K \end{aligned} \tag{5}$$

[12] In general, the balance equations correspond to the macroeconomic identities that are presented in social accountability. Richard Stone, winner of the Sveriges Riksbank Prize in Economic Sciences awarded by the Royal Swedish Academy of Sciences 1984, is the international reference here. He developed and organized the system of national income accounting as a system of double entry accounting. On the relation between the national accounting system and the input–output tables, see Ten Raa (2010).

[13] On the difference between the two methods of obtaining the technical coefficients, see Lange ([1961] 1978). For a practical procedure of how to obtain the technical coefficients from the input-output tables, see Ten Raa (2007).

As a result, we conclude that the balance equations of the production can be expressed both in physical terms and in terms of value. At this moment it seems to be correct to assert that in PCC the quantitative relations of exchange between the use values are given by the technical structure of the economy itself, since the systems are equivalent. However, as already pointed out, the allocation of use values by the end of the production process is in fact free from the determination imposed by the technical coefficients. The problem is that in PCC (as in any analysis that sees the economy as a circular flow)[14] it is supposed that the process must be repeated infinitely or, as mentioned, according to the logic of equilibrated restoration. This forces the quantitative relations between the use values to be correspondent to the technical coefficients of production derived from the original method of production. The result is that the quantitative relations between the use values, become a direct reflection of the technical coefficients. It is interesting to notice that this attachment of empirical prices to the physical structure of production is what allows both the previous dissolution of the transformation problem of values into production prices and the conception of incorporated labour, that is, the labour theory of value of Smith and Ricardo.

Treating the economy as a circular flow is, by the way, the only way to fit economics in the process of material reproduction, as determined by the conditions progressively discovered by the natural sciences. Sraffa himself recalls that this approximation in economics goes back to Quesnay's *Tableau Économique* from 1759 (Quesnay, 1996). In fact, Leontief (1936) starts his presentation by pointing that his study may well be defined as an attempt to build such *tableau* for the United States of 1919. Historically, one should notice that every effort made by Leontief dates back to his first work of 1928, which begins his theoretical contribution to the notion that the economy is a reproduction system.[15] Indeed, economics, understood in the framework of the materialist conception of history, is precisely the study of the social reproduction conditioned by this material reproduction that the Physiocrats modeled and which reached its development peak with David Ricardo. At this point, the differences between natural and social determinations had to be clarified by Karl Marx. If input–output economics represents the "modern" version of the modeling of material reproduction of societies, then it is urgent to relate Marxist Political Economy to the developments of all advancements in this field. Describing the material transformation that occurs in the constant exchange between production and distribution is not sufficient to develop economic science, because it would be reduced to the framework of analysis of the natural sciences.

For example, we can make an analogy. On the basis of the subsistence model of Sraffa, and with the support of the interpretative approach of Kurz and Salvadori (2005), we write the chemical equations that correspond to the biological carbon cycle and that keep the total energy of the system constant. Now, we can think of CO_2 and H_2O as the 'use values' as being inputs in the photosynthesis process and outputs in the respiration process, while the 'use values' $C_6H_{12}O_6$ e O_2, on the contrary, are outputs in the photosynthesis process and inputs in the respiration process. A perfect identification to Sraffa's basic model is not achieved because in the subsistence model of PCC each sector produces a single use value, while the transformations of the biological cycle of carbon produce in one process two different use values.[16] But no matter how detailed is the model of the economy integrating all physical and chemical modification of matter by mankind, this type of analysis does not discover how humans relate to each other. So, it is not enough to conceive economic reproduction as mere material reproduction. The reproduction of the pattern of human relations that conduct production and distribution must also be stored. Only then can we speak of the reproduction of a certain mode of production, as capitalism, feudalism and socialism.

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[14] The consideration of the economy as a circular flow does not correspond to the dynamic analysis, which must also include the case where the matrices are not simply restored, but modified over time.

[15] For a brief evaluation on the importance of the *Tableau Économique* in the history of economic thought and its relation to Leontief's model, see Phillips (1955). For a broader context of Quesnay's contribution in the history of economic thought, see Hunt (2002). The English version of Leontief's work of 1928, *The economy as a circular flow* (Leontief, 1991), is a shortened translation from the original *Die Wirtschaft als Kreislauf* (Leontief, 1928).

[16] On the identification of the chemical–biological cycles to Sraffa's model, see Schefold (1989), p. 338 and Schefold (2004), where the subsistence economic model with joint production is constructed and compared to the global chemical cycles. The model for two sectors ('industries') which corresponds to the process of photosynthesis and respiration can be written as: $6CO_2 + 6H_2O \rightarrow C_6H_{12}O_6 + 6O_2$; $C_6H_{12}O_6 + 6O_2 \rightarrow 6CO_2 + 6H_2O$, which is the formalized form for the carbon cycle and the symbols for the substances are equivalent to the specification of a certain use value.

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3.2. The general equation of internal coherence

But if the technical coefficients and the prices of production are two sides of the same coin, how can the relationship between the technical coefficients and the prices of production be directly shown? In order to explicit this connection let us write the balance equations in terms of value with help of this notation for the technical coefficients:

$$\begin{bmatrix} a_a & b_a & \dots & k_a \\ a_b & b_b & \dots & k_b \\ \dots & \dots & \dots & \dots \\ a_k & b_k & \dots & k_k \end{bmatrix} = \begin{bmatrix} {}^{A_a}\!/_{A} & {}^{B_a}\!/_{A} & \dots & {}^{K_a}\!/_{A} \\ {}^{A_b}\!/_{B} & {}^{B_b}\!/_{B} & \dots & {}^{K_b}\!/_{B} \\ \dots & \dots & \dots & \dots \\ {}^{A_k}\!/_{K} & {}^{B_k}\!/_{K} & \dots & {}^{K_k}\!/_{K} \end{bmatrix} \tag{6}$$

So that the balance equations in terms of value can be written as:

$$\begin{aligned} & a_a A p_a + a_b B p_a + \dots + a_k K p_a = A p_a \\ & b_a A p_b + b_b B p_b + \dots + b_k K p_b = B p_b \\ & \dots \\ & k_a A p_k + k_b B p_k + \dots + k_k K p_k = A p_k \end{aligned} \tag{7}$$

where $a_a A$ is the physical quantity of use value ‘a’ applied in the production of use value ‘a’ (that is, $A_a = a_a A$), $b_a A$ is the physical quantity of use value ‘b’ applied in the production of use value ‘a’ (that is $B_a = b_a A$), and so on.

What is the meaning of this notation for the technical coefficients? Shortly, a_a is the technical coefficient of production of use value ‘a’ in relation to the production of use value ‘a’ (a_a units of use value ‘a’ are needed to produce one unit of use value ‘a’), b_a is the technical coefficient of production of use value ‘b’ in relation to the production of use value ‘a’ (b_a units of use value ‘b’ are needed to produce one unit of use value ‘a’), and so on. Generally, i_j is the physical quantity of use value ‘i’ needed to produce one unit of use value ‘j’.

The System (1) can also be written with the notation of the technical coefficients of production when we divide each equation respectively by $A, B, \dots, K$:

$$\begin{aligned} & a_a p_a + b_a p_b + \dots + k_a p_k = p_a \\ & a_b p_a + b_b p_b + \dots + k_b p_k = p_b \\ & \dots \\ & a_k p_a + b_k p_b + \dots + k_k p_k = p_k \end{aligned} \tag{8}$$

For this system, we consider as price of reference $p_a = 1$ and eliminate the first equation of the system, so that we can write the other prices as a function of the technical coefficients of the remaining sectors. The system is reduced to:

$$\begin{aligned} & a_b 1 + b_b p_b + \dots + k_b p_k = p_b \\ & a_c 1 + b_c p_b + \dots + k_c p_k = p_c \\ & \dots \\ & a_k 1 + b_k p_b + \dots + k_k p_k = p_k \end{aligned} \tag{9}$$

Rearranging, it can be written as:

$$\begin{aligned} & (b_b - 1) p_b + \qquad + k_b p_k = -a_b \\ & b_c p_b + (c_c - 1) p_c + \dots + k_c p_k = -a_c \\ & \dots \\ & b_k p_b + \dots + (k_k - 1) p_k = -a_k \end{aligned} \tag{10}$$

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So, if the determinant Δ of the coefficient matrix of System (9) is different from zero, we have a unique solution that links the production prices to the set of technical coefficients of production. That is:

$$\Delta = \left| \begin{matrix} (b_b - 1) & c_b & \ldots & k_b \\ b_c & (c_c - 1) & \ldots & k_c \\ \vdots & \vdots & \vdots & \vdots \\ b_k & c_k & \ldots & (k_k - 1) \end{matrix} \right| \quad (11)$$

With:

$$\Delta_b = \left| \begin{matrix} -a_b & c_b & \ldots & k_b \\ -a_c & (c_c - 1) & \ldots & k_c \\ \vdots & \vdots & \vdots & \vdots \\ -a_k & c_k & \ldots & (k_k - 1) \end{matrix} \right|; \Delta_c = \left| \begin{matrix} (b_b - 1) & -a_b & \ldots & k_b \\ b_c & -a_c & \ldots & k_c \\ \vdots & \vdots & \vdots & \vdots \\ b_k & -a_k & \ldots & (k_k - 1) \end{matrix} \right|; \ldots; \Delta_k = \left| \begin{matrix} (b_b - 1) & c_b & \ldots & -a_b \\ b_c & (c_c - 1) & \ldots & -a_c \\ \vdots & \vdots & \vdots & \vdots \\ b_k & c_k & \ldots & -a_k \end{matrix} \right|$$

So that we can explicit the production prices as functions of the technical coefficients like this:

$$p_a = 1; p_b = \frac{\Delta_b}{\Delta}; p_c = \frac{\Delta_c}{\Delta}; \ldots; p_k = \frac{\Delta_k}{\Delta}$$

This demonstrates that the technical coefficients and the production prices have such a correspondence that the static analysis of input–output can be done either in terms of value or in physical terms. This is the same conclusion reported by Oskar Lange when he interprets the coefficients of production costs (which are the technical coefficients in value terms) in the light of the Marxist theory of value (Lange ([1961] 1978), pp. 207).[17]

What is the economic meaning of this result? Shortly, it means that if the empirical prices were equal to the prices of production, the table of monetary flow of the Leontief type would reflect exactly the technical conditions of production. In other words, the matrix of value transfer between the sectors would provide the correct information about all technical coefficients of the productive structure under consideration. In such a situation, the value system and the price system would coincide and the deviation between values and prices would disappear, as well as the theoretical problem of transforming values into prices of production. That would be a state of equilibrated restoration.

However, as it has been pointed in theory since the classical economists and the concrete analysis for planning confirm, the quantitative relations of exchange that take place in practice on the market are only approximations of those that correspond to the production prices. Another way to say this is to emphasize that 'the prices deviate systematically from the values'. This happens, on one side, because the negotiations at the market do not follow strictly the logic that would put the system in identical reproduction, and, on the other, because markets always have some degree of concentration, that is, of monopolization. In this sense we affirm that *the allocation of use values in the end of the production process is in fact free from the determinations imposed by the technical coefficients*.

On the other hand (and that is the remark to which Marx calls attention in book 3 of Capital) this liberty has limits which are put by the conditions of reproducibility of the system. That is the meaning of the statement that the empirical prices are regulated and dominated by the law of value. Lange ([1959] 1966) points to these restrictions of empirical

[17] Mathematically, it is possible to notice that we can consider the same kind of solution without assuming $p_a = 1$. In other words, it is logically viable to treat a system that has 'absolute' prices which correspond to the technical coefficients. The System (7) may be converted to a system similar to Eq. (9) by adding one more equation. But, in that case, we fall back in a system of homogeneous equations similar to System (2). If it is supposed to have a non-trivial solution, the determinant of the coefficient matrix must be zero. This is a very restrictive condition for a solution of a system of complete (and absolute) prices in terms of technical coefficients. From the perspective of pure mathematics, it seems that the difference of procedure matters: in the first case (adopting a use value as reference for the price system) we reach a unique solution. In the second case (using a system of 'absolute' prices) we reach an infinite number of distinct solutions, but conditioned to very narrow restrictions. The relationship between this dilemma (choosing a standard use value or constructing a system of absolute prices from the coefficients) and the problem of the invariable standard of measure and the standard commodity must be explored in another occasion, because it is treated by Sraffa only in chapter 4. This line of investigation can start from Bellino (2004) and Roberts (2009).

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prices as a means to emphasize that the technical laws (or the balance equations, or still, the 'vertical' proportions) delimit the range of freedom of the concrete exchanges.[18]

Within this interpretation the system of balance can be written in terms of value and in physical units in one single equation which shows the formal counterpart between the technical conditions of the economy and the quantitative relations of exchange between use values that put the system in the state of equilibrated restoration. To this end, we write the balance equations in physical units in a matrix form:

$$\begin{bmatrix} a_a & a_b & \dots & a_k \\ b_a & b_b & \dots & b_k \\ \vdots & \vdots & \vdots & \vdots \\ k_a & b_b & \dots & k_k \end{bmatrix} \times \begin{bmatrix} A \\ B \\ \vdots \\ K \end{bmatrix} = \begin{bmatrix} A \\ B \\ \vdots \\ K \end{bmatrix}$$

By introducing a price vector to this system we can rewrite the System (3) in the following form, after lines and columns are inverted:

$$\begin{bmatrix} a_a & a_b & \dots & a_k \\ b_a & b_b & \dots & b_k \\ \vdots & \vdots & \vdots & \vdots \\ k_a & b_b & \dots & k_k \end{bmatrix} \times \begin{bmatrix} A \\ B \\ \vdots \\ K \end{bmatrix} \circ \begin{bmatrix} p_a \\ p_b \\ \vdots \\ p_k \end{bmatrix} = \begin{bmatrix} A \\ B \\ \vdots \\ K \end{bmatrix} \circ \begin{bmatrix} p_a \\ p_b \\ \vdots \\ p_k \end{bmatrix}$$

Or:

$$\mathrm{T} \times \mathrm{Q} \circ \mathrm{P} = \mathrm{Q} \circ \mathrm{P}$$

where the sign '$\bigcirc$' indicates the Hadamard product of the resulting matrix of the product of T and Q by P. The equation shows simultaneously the conditions of balance formulated in terms of technical coefficients, of the total produced quantities and of prices expressed in terms of the technical coefficients of production.[19]

We call this equation 'general equation of internal coherence'. It expresses the identity between the technical balance equations and the balance equation in value terms. One should note that no additional information was gained when we passed from the physical system to the system in terms of value. This would be another form of showing the correspondence between the technical coefficients and the production prices. Here, the equation reveals that the sphere of value accountability has a material basis which explains and delimits it, as Lange ([1959] 1966) argues.

From this also follows that Sraffa's equations indicate only the technical balance of the economy in terms of value, since the prices in PCC are always production prices. But what is wrong with that? Apart from dismantling the famous transformation problem of values into production prices, as mentioned, the problem is that the liberty for allocation of use values completely disappears if the process is to be repeated identically. The logic of equilibrated restoration impedes us to detach the distribution from a given production structure, forcing the theoreticians to conceive the reproduction as something out of the influence of the acting of humans that consciously shape and transform nature. The fetish of the law of value reveals itself shockingly as if humans under the economic organization of capitalism

[18] The empirical studies indicate that the deviation between empirical prices and the production prices are not large, although the size of the average deviation depends on the estimation method employed. Some references on these studies are Shaikh (1998), Petrović (1987), Ochoa (1989), Cockshott et al. (1995), Cockshott and Cottrell (1997), Steedman and Tomkins (1998), Tsoulfidis and Maniatis (2002), Zachariah (2006), Tsoulfidis and Mariolis (2007), Tsoulfidis (2008) and Mariolis and Soklis (2009). On the notion of equilibrium in Classical Political Economy and Neoclassics and on the difference between 'vertical' and 'horizontal' proportions, see Lange (1957). The empirical studies were made in the context of the 'Sraffa schock' over the labour theory of value and for that reason there is no unique methodology for the study about the relationship between the system of technical coefficients (the value system, in Marxist language) and the empirical prices. For an introduction to this topic within Marxist economics, see the debates around the transformation of values into production prices condensed in Farjoun and Machover (1983). For a mathematical model of the labour theory of value, see Bródy (1974). For a presentation of the role of prices of production in structural change, see Duménil and Lévy (1995).

[19] For our model, any combination of prices, technical coefficients and physical quantities that do not satisfy this equivalence do not represent a subsistence economy.

could not change the purpose of their economic system, which, when dominated by the logic of valorization, is the infinite reproduction of capital.

Here, we can overcome such limitation by analyzing the model in a dynamic environment. In this case, the society uses the technical knowledge about the production to alter its input matrix and so to change its output matrix. The use of science for consciously reformulating the input matrix in order to achieve some determined aim, that is, a reformulation which does not follow the automatic logic of equilibrated restoration (apparently obvious for Sraffa and the classic economists) is equivalent to what is called economic planning.[20] Transposing the economic system from the law of value to planning, however, does not mean directly the shift from capitalism to socialism, because the technique of controlling the law of value can well be used as a means to serve the aim of total valorization again. The economic history of the 20th century is a myriad of different subtypes of this form of economic organization. In terms of the equation systems, it is the process of transposition of the economy from a certain internal coherence to a different internal coherence. When we take the static model of PCC to a dynamic scenario, we go from the static to the dynamic analysis of input–output economics.

4. Lange's discovery of the usefulness of input–output economics for socialism

The dynamic analysis of input–output is equivalent to the analysis of the optimal level of the program. In this case, different from the static perspective, the problem becomes concrete, because now it is needed to make a choice aiming at some determined objective. This aim, on its turn, may be different from case to case. While in the static analysis we were not concerned with the program or internal coherence as a means to achieve some aim, this is exactly the case in the dynamic analysis of input–output. The economic planning as a process of combining means to reach certain states can only be completely formalized from this dynamic perspective, as various presentations on the topic had already noted (for example Matus, 1991; Rosseti, 1993; CEPAL, 2000; Lanzer, 1982 and Ferreira ([1978] 1997)).

Although the presentation here follows from an abstract model of the economy, passing through the input–output matrices as they are empirically obtained, in order to finally formalize the problem, the theory of linear programming was already reasonably mature when they started to use it in combination with the data obtained with help of the Leontief tables. In accordance to the methodology of research and exposition (*Forschung und Darstellung*) developed by Marx ([1857] 1976), the sequence of presentation goes from the abstract to the concrete. But, historically, the theory was constructed from practical problems of optimization in restricted environments. As these issues began to be related to the total productive structure of the society, the problem to be solved started to contemplate the whole economy. Here, in the paper, we departed from this broad perspective, as if we had already mapped all interconnections of all sectors of the economy. In practice, it is the opposite movement that occurs: the use of the theory for economic planning begins at the private sphere (for individual capitalists) and only afterwards it embraces the coordination of different units of capital. Extending the technique to the collective is difficult not only because of lacking information and organization of the available data in a central office, but also due to political and ideological obstacles. After all, it comes down to the transition from the capitalist mode of production to the socialist/communist mode of production.

The theory of programming as a technique for using resources intended to achieve certain objective was formalized independently in the Soviet Union by Kantorovich ([1939] 1960) and in Europe and in the United States by Koopmans ([1942] 1970). It was erected as the optimal allocation problem became clear in the concrete cases they were working on. They share the Sveriges Riksbank Prize in Economic Sciences awarded by the Royal Swedish Academy of Sciences of 1975 for their contributions to the theory of optimal resources allocation, while Leontief had won the prize two years earlier, in 1973. The definitive impulse for application of the theory was the Second World War, which required the solution for a series of interrelated problems of military supply. After the war, the theory has been applied directly by major industries as a way of planning their production. Since the tables of the Leontief type were showing the connections of the economy in its entirety, the theory of programming could be used in combination to it in order to make the economic planning not restricted to a single capital, enterprise or branch. This opened the possibility for the coordination of the entire economy.

[20] It is possible to consider that the automatic logic is the law of value. However, a more rigorous specification relating the law of value with the use values is necessary. A study in this direction is Bryceson (1983).

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According to Lange ([1961] 1978), this possibility converts the theory of programming in alliance to input–output economics into an extremely useful tool for the socialist society (see, for example, Lange, 1949). However, it becomes also a powerful weapon for the construction of other structures of society that can reinforce the phenomenon of alienation. In that sense, the terms programming and planning become very close to each other and start to penetrate the political field. The real experiments of planning, both in socialist economies and in capitalist countries (both in their democratic and facist fashion) were only partially based on the theory of programming because in practice there are many obstacles for a full and detailed application of the economic plan. Problems usually cited are lack of statistical data, lack of theoretical knowledge and difficulties related to political issues attached to the organization of production and distribution. Lange (1945), for example, in his commentary about the teaching of economics in the USSR in the 1940s, is of the opinion that the soviet planners had only a vague notion about the deviation of the prices around the values. This would explain why a more systematic use of the technique of controlling the price mechanism did not prevail there.

4.1. Optimal level of the program or dynamic analysis of input–output

Back to theory, in the first chapter of PCC, the system is already in a state of self-reposition, that is, $A_a + A_b + \ldots + A_k = A$; $B_a + B_b + \ldots + B_k = B$; etc. Sraffa explains that systems that do not attend this requirement can be driven to such situation through the variation in the proportions of the entry equations. Those systems which are incapable of achieving this self-reposition state are considered economic unviable and are not considered. The self-reposition state, however, does not require the input and output matrices to be identical over time. In order to verify this, we may make some computer simulations changing the data of the input and output tables without violating the condition of self-reposition. In that case, we would be changing the method of production, and, consequently, the prices.

Thus, in the case of the dynamic analysis of the input–output for a subsistence economy, the representation of the input and output matrices over time would be done like this:

$$\begin{bmatrix} A_a & B_a & \ldots & K_a \\ A_b & B_b & \ldots & K_b \\ \vdots & \vdots & \vdots & \vdots \\ A_k & B_k & \ldots & K_k \end{bmatrix} \rightarrow \begin{bmatrix} A \\ B \\ \vdots \\ K \end{bmatrix} \rightarrow \begin{bmatrix} A'_a & B'_a & \ldots & K'_a \\ A'_b & B'_b & \ldots & K'_b \\ \vdots & \vdots & \vdots & \vdots \\ A'_k & B'_k & \ldots & K'_k \end{bmatrix} \rightarrow \begin{bmatrix} A' \\ B' \\ \vdots \\ K' \end{bmatrix} \rightarrow \ldots$$

Or shortly:

$$q \rightarrow Q \rightarrow q' \rightarrow Q' \rightarrow \ldots$$

where the sign $(')$ indicates that there was an alteration in the disposal of use values. Accordingly, the pair of matrices $q \rightarrow Q$ represents the internal coherence #1 and the pair of matrices $q' \rightarrow Q'$ represents the internal coherence #2, so that the passage from one to another is limited by the objective conditions of matter transformation. Within which bounds is this transformation possible?

Let us return to the example of Sraffa (1960) exposed in the second section in order to show how the theory of programming is used in the economic planning. Suppose we have an initial state (A), where a simple society undergoes the following reproduction of use values:

$$(A): \begin{bmatrix} 240 & 12 & 18 \\ 90 & 6 & 12 \\ 120 & 3 & 30 \end{bmatrix} \rightarrow \begin{bmatrix} 450 \\ 21 \\ 60 \end{bmatrix} \rightarrow \begin{bmatrix} 240 & 12 & 18 \\ 90 & 6 & 12 \\ 120 & 3 & 30 \end{bmatrix} \rightarrow \begin{bmatrix} 450 \\ 21 \\ 60 \end{bmatrix} \rightarrow \ldots$$

As it was exposed, the quantitative relations between use values that put the system in reproduction can be expressed in the relative prices $p_a = 1$, $p_b = 10$ and $p_c = 5$. This relation establishes the internal coherence of this system. According to the principle of macroeconomic aggregation, we can calculate the gross output trough the addition

$Y = 450 \times p_a + 21 \times p_b + 60 \times p_c = 960$.[21] This is the abstract value referring to the available wealth of this society. It is concretely composed of 450 quarters of wheat, 21 t of iron and 60 pigs.[22]

4.2. Determining the objective function

Now, in order to leave the static analysis, we need to explicit the two elements of every programming problem: the objective function and the conditions of restriction. Let us begin assuming (as it is the normal case when the theoretician follows the logic of valorization) that the aim of the society is to maximize its abstract wealth (that is, regardless of its composition in different use values) and that the society discovers, besides the method of production described in (A) two further possibilities that allow arranging the input and output matrices in two distinct internal coherences. The programming problem becomes then:

$$Max\ Y = A \times p_a + B \times p_b + C \times p_c\,(objective\ \ function)$$

$$subjet\ to\ available\ methods\ of\ \ production\,(A)\,,(B)\ \text{ and }\ (C)\,(constraints)$$

In the first new method, the society would distribute the output in the following input matrix, which would on its turn generate the sequence (B):

$$(B): \begin{bmatrix} 200 & 10 & 12 \\ 130 & 9 & 20 \\ 120 & 2 & 28 \end{bmatrix} \rightarrow \begin{bmatrix} 400 \\ 38 \\ 58 \end{bmatrix} \rightarrow \begin{bmatrix} 200 & 13 & 12 \\ 100 & 14 & 20 \\ 100 & 11 & 26 \end{bmatrix} \rightarrow \begin{bmatrix} 400 \\ 38 \\ 58 \end{bmatrix} \rightarrow \ldots$$

In the second new method, the technological rearrangement would lead to process (C):

$$(C): \begin{bmatrix} 220 & 10 & 18 \\ 120 & 9 & 14 \\ 110 & 2 & 28 \end{bmatrix} \rightarrow \begin{bmatrix} 420 \\ 30 \\ 59 \end{bmatrix} \rightarrow \begin{bmatrix} 200 & 14 & 17 \\ 110 & 12 & 14 \\ 110 & 4 & 28 \end{bmatrix} \rightarrow \begin{bmatrix} 420 \\ 30 \\ 59 \end{bmatrix} \rightarrow \ldots$$

In both cases, the first pair of input–output matrices represents a link in the chain between two distinct internal coherences. This link is not a stable situation. It does not represent a circular reproduction, even though it is responsible for creating the possibility to change from one internal coherence to another. If we observe this intermediary stadium carefully, we see that it reveals how the original output matrix was altered. In case (B), 50 quarters of wheat and 2 pigs were permanently transformed in 17 t of iron. In case (C), 30 quarters of wheat and 1 pig were permanently transformed in 9 t of iron. The interrelation between these two possibilities is generally described by the economics textbooks with the concept of production function.[23]

The second pair of input–output matrices, differently, represents the new internal coherence achieved. It is the new 'static' system of reproduction. The process repeats itself from then on identically.[24] We can determine what the production prices are for these two new reproduction scenarios in the same way as in the initial section (or through the general equation of internal coherence).

For case (B), the production prices are $p_a = 1$, $p_b = 9489$ e $p_c = 6,387$. As the output is composed of 400 quarters of wheat, 38 t of iron and 58 pigs, the output value in this situation is $Y_{(B)} = 400 \times p_a + 38 \times p_b + 58 \times p_c = 1,131,022$.[25]

[21] The model does not differentiate gross output from net output because there is no fixed capital. More accurately, we would have to say that, in this case, the means of production are restored immediately period from period, because the model is not restricted to the capitalist mode of production. The model is in fact the simplest possible to conceive, for every use value are at the same time final and intermediary goods in the same extension.

[22] We could measure the wealth in terms of one use value. It would be of 960 quarters of wheat, or 96 t of iron, or still 192 pigs.

[23] The notation would be, in this case, F(wheat, pigs) = iron, with $F_{(B)}(50,2) = 17$ and $F_{(C)}(30,1) = 9$.

[24] There are possible problems that may origin from this example. Passing from the initial internal coherence to another internal coherence resulted in two different input matrices generating the same output matrix. Let us suppose this is in accordance with the conditions of qualitative transformation of matter, that is, in harmony with our exogenous production function.

[25] In other words, the total wealth in situation (B) becomes 1,131,022 quarters of wheat, or 119,193 t of iron, or 17,708 pigs.

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For case (C), the production prices are $p_a = 1$, $p_b = 98,606$ e $p_c = 48,207$. As the output is composed of 420 quarters of wheat, 30 t of iron and 59 pigs, the output value in this situation is $Y_{(C)} = 420 \times p_a + 30 \times p_b + 59 \times p_c = 10,004,013$.[26]

In this sense, we can compare situations (A), (B) and (C) and conclude that, for the aim of maximizing the aggregate output *Y*, the allocation of use values of current output must follow the display as pointed by (B). In other words, the objective *Max Y* is achieved through the economic plan (B). As a result, the economic planning must have the function to allocate the initial use values in the input matrix of option (B).

We must remember that this simple example is only an intuitively presentation of the theory of programming in economics. We tried to link the traditional language of input-output economics to the field of Political Economy in the Marxist tradition and there are alternative paths to this topic which depart from more formal and mathematical grounds. The basic difference, as already mentioned, is that the usual study of the tables does not think about capitalism as a system of economic organization within the history of civilization but tends only to precisely determine the quantitative relations of equivalence between all the use values in the economy. For the discussion on the operative methods of input-output related to the topic of economic planning at the formal level, see, for example Ghosh (1958) and the subsequent debate on the relationship between Ghosh's and Leontief's models in Davar (1989), Dietzenbacher (1997), Oosterhaven (1988), Guerra and Sancho (2011) and Oosterhaven (2012).

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When the objective function and the technical constraints are clearly defined it is possible to determine the option that solves the allocation problem. However, the definition of the objective function itself is a matter of political struggle over the pattern of economic reproduction. In the pure capitalist mode of production the law of value determines directly the objective function, which assumes the form of maximization of profits for the unit of capital. This is the reason of why the basic optimization problem appears as something evident in all presentations of maximization of utility or value under conditions of restriction.[27] But, when the allocation of the output follows some constructed plan by the clear definition of specific aims and of the limits, we leave the automatic operation of distribution and production and we move towards economic planning. This distribution of factors which is subordinated to any objective different from the principle of simple maximization of abstract wealth is viable trough the use of the technique of programming. This process goes beyond the logic of the law of value, which is the theoretical expression in Marx for the allocation mechanism imposed by the market.

Nevertheless, and that is the special care of Oskar Lange, this extrapolation can not occur freely. The passage from an internal coherence to another must meet the technical limitations of transformation of matter. For that reason, the leap from one state to another perpetual circular reproduction is limited by these parameters of change between two known technological situations. One great challenge of planning is to conduct these leaps without violating these limits. From historical experience it is known that failure to this was frequent and appeared in forms of shortage of basic items, low product quality, queues and unequal development.

In the capitalist mode of production, these leaps are made by the imposition of the logic of valorization, and for that reason, the social planning was not a precondition for the increase of the aggregate output. Another way to capture this movement is to recognize that the process of capitalist competition forces the development of the productive forces, which is merely another expression for the improvement of the technical coefficients of production. The concept of productivity is generally used to measure this advance. In practice, the expansion of the system through increasing productivity does not follow a stable pattern. It develops through ruptures and crises. Formally, the programming problem can always be written as some objective function Y = income (or profit, etc.) which is dependent on the produced use values and their empirical prices (defined at the time of value realization).

[26] In other words, the total wealth in situation (C) becomes 10,004,013 quarters of wheat, or 1,014,544 t of iron, or 207,522 pigs. Attention: the comparison with the other situations on the perspective of use values can be done directly only on the basis of wheat, for it is the unique commodity maintaining the same price in all three situations. On the problem of the *numéraire* for comparing two systems see Cockshott and Sinha (2008).

[27] The development of the neoclassical school led it at first to model planning at the individual level. That is the reason why it confuses human action over nature with the economic process of capitalism. One of the consequences is that the neoclassical school formalizes the logic of valorization as the aim of the individual, turning the optimization problem a problem of utility maximization. The dynamic analysis of input-output at the individual level in the context of the capitalist mode of production creates the *Homo economicus*.

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The basic problem in capitalism is described by pointing that the aim is to maximize Y, so that the knowledge about the technical constraints will allow punctual actions to avoid the crisis and allow the system to expand in a growth trajectory. All growth models of the steady state type are illustrations of this mechanism which is desired by the capitalist States since the end of *laissez-faire*.[28] The counterpart of the expansion of value without the disruption of reproduction is that the system in its use value structure should expand proportionally. This is the main effort of the capitalist State acting as a 'referee' uniting and coordinating individual capitals.

Consequently, the maximization of the aggregate income would be converted into the aim 'maximize the sum of use values that compound the output in a stable way'. In other words, it would be a situation where the system grows as fast as possible without violating the balance proportions. That is the same idea explained by Amin ([1977] 1981) when he describes a dynamic and expansive equilibrium. In fact, since the generalization of commodity relations, the economic problem of shortage has always been solved by capital. That is why Keynes (1920) says that capitalism is justifiable for the accumulation of wealth (as use values) it provides. Similarly, Marx indicates that the solution to the problem of the low level of development of the productive forces, and consequently of insufficient material wealth, is exactly one of the historical tasks of the capitalist mode of production.

However, the basic model here presented does not need to have the aim 'maximize income'. If we created a problem in which the society demands the three use values under consideration in the proportion given by situation (A), then the allocation that solves the programming problem would be that already in use. It is very important to make this distinction, because in general, it is supposed that the sole purpose of the system is the expansion of production, when in fact this situation is the means by which capital reaches its target of expanding itself. Another way of capturing this idea would be to think of a situation where we had some use value that is clearly anti-social, for example, missiles, rocket-explosives and guns that are not developed to serve the modification of soil and construction of infrastructure, but merely to kill human beings, but which at the same time, acquires the overall power to conduct the expansion of abstract wealth. Every effort goes to the construction of a use value structure that is not desired by the majority of members of the system.

The constant confusion in the use of the techniques of planning between the capitalist economy and the socialist society can be traced back to the lack of clarity about the difference between the expansion of value and the expansion of use value. In the capitalist society, the technique is used to allow a material expansion which constantly pushes the limits of valorization upwards (trying to avoid the so-called detachment). This is reflected in the desire for continuous economic growth, the central topic of macroeconomics. It makes also clear which form planning in capitalism assumes and it is easy to see how the capitalist State of the 20th century acts to fulfill a role that did not exist during the era of classical liberalism.

On the other side, in the socialist society, the technique is used to achieve aims that are socially determined. And these aims can also be the expansion of material wealth, as it collaterally happens under capitalism. Indeed, due to the fact that all socialist revolutions took place in countries with a comparatively low level of productive forces, the aim of economic coordination was always de increase of material wealth trough industrialization. Because of this, at a formal level, the capitalist and socialist economies can be modeled by the dynamics of input-output identically when the objectives of the systems are the same (or at least, when they coincide regarding some aspects).[29] But as it is comprehensible from the ideological debates, these two kinds of social organization must have different objectives.

It follows that the techniques of planning and programming can be used in different political economic systems. In this sense, the tool of economic planning, despite being a neutral body, curls up in the political-ideological debate. At the same time, the growing impositions for control within capitalism show increasingly that the automatic logic dictated by the law of value is being progressively substituted by a broad coordination process of all sectors of the economy.

The application of the theory of economic programming is vast and a more accurate development remains to be done. In particular, it is important to extend this study to a model that explains the surplus. Then, the normal categories

[28] See for example a Marxian optimal growth model in Onishi (2011) developed within the formal framework initiated by Solow (1956).

[29] On the debate about the economic calculation in the socialist economy, which revealed the common framework of the different economic systems (and which can be thought as the input–output modeling), see the collection of contributions in Boettke (2000). For the contemporary developments of the techniques of planning advanced by Oskar Lange, see Cockshott and Cottrell (1989) and their following works. On the methodology for economic analysis here employed regarding the economic schools of thought, see Lange (1945–1946). For a critical introduction to the historical problems of central planning of the soviet system, see Kornai (1986).

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of political economy (capital, profit, interest, etc.) can be adequately addressed within the framework of input–output economics. This will allow us, for example, to raise questions about the qualitative aspect of the theory of value, such as: 'why did the labor theory of value was erected exactly during the consolidation of Classical Political Economy?' and 'how does the value theory relates to the theory of economic planning?'

Above all, this line of research may contribute to distinguish with greater precision the economic systems of capitalism and socialism and to help diminish the strong dichotomy between the qualitative and quantitative analysis within economic science. One of the most important achievements to be pursued along the lines of this research is to interpret the socialist economic calculation problem as organized by Mises ([1920] 1935) on the basis of the entire historical development of input–output economics for purposes of economic planning.

Here, there still remains a chapter to be written explaining how the concrete solution to the transformation problem of values into prices means the realization of planning and the overcoming of market and capitalist organization. All we can point to at this stage is that the political determination of the objective function of the whole economic system signifies the control over the law of value and the shift of paradigm for economic science from liberalism to planning; and that the knowledge of technical modification of matter, together with the knowledge of demand at the individual level are sufficient for calculating economically in an economic system without private property. All this is achievable if there is sufficient training for working people to use and control the instruments of input-output economics. The organization and connection of the contributions of Piero Sraffa, Wassily Leontief and Oskar Lange are crucial activities for the fulfillment of this fundamental technical-political task.

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