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THE ORIGIN OF "ZORN'S LEMMA"

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SUMMARIES

The paper investigates the claim that "Zorn's Lemma" is not named after its first discoverer, by carefully tracing the origins of several related maximal principles and of the name "Zorn's Lemma." Previously unpublished information supplied by Zorn is included.

Cet article porte sur l'affirmation que le "lemme de Zorn" ne porte pas le nom de celui qui le trouva le premier. Pour ce faire, nous retraçons soigneusement l'origine de plusieurs principes de maximalité en rapport avec cette question, et du terme "lemme de Zorn". Nous avons inséré aussi des informations fournies par Zorn et demeurées inédites.

1. INTRODUCTION

A recent letter to the *Notices* of the American Mathematical Society [Minty 1976] inquired after substantiation of the oft-heard claim that "Zorn's Lemma" is not named after the first discoverer of that result.

A few authors have tried to trace the origins of the several formulations of equivalent maximal principles that pass in common parlance as "Zorn's Lemma." (For purposes of this paper, we will not distinguish between a maximal principle and its corresponding minimal principle.) Unfortunately, the entire story is not to be found in any one source. None of the authors of [Rosser 1953, 493-495, 507; Cuesta 1955; Fraenkel and Bar-Hillel 1958, 68-69; Rubin and Rubin 1963, 10-13; Beth 1964, 376-378; Semadeni 1968; Jech 1973, 29] seems to have taken note of the attempts of predecessors to unravel the threads. A capsule summary of the history of "Zorn's Lemma" was given succinctly by Suppes:

> *This maximal principle is baptized after Zorn [1935], but the history of it and some closely related maximal principles is very tangled. Certainly Zorn was essentially anticipated by F. Hausdorff, C. Kuratowski, and R. L. Moore at the least.*
> [Suppes 1960, 245]

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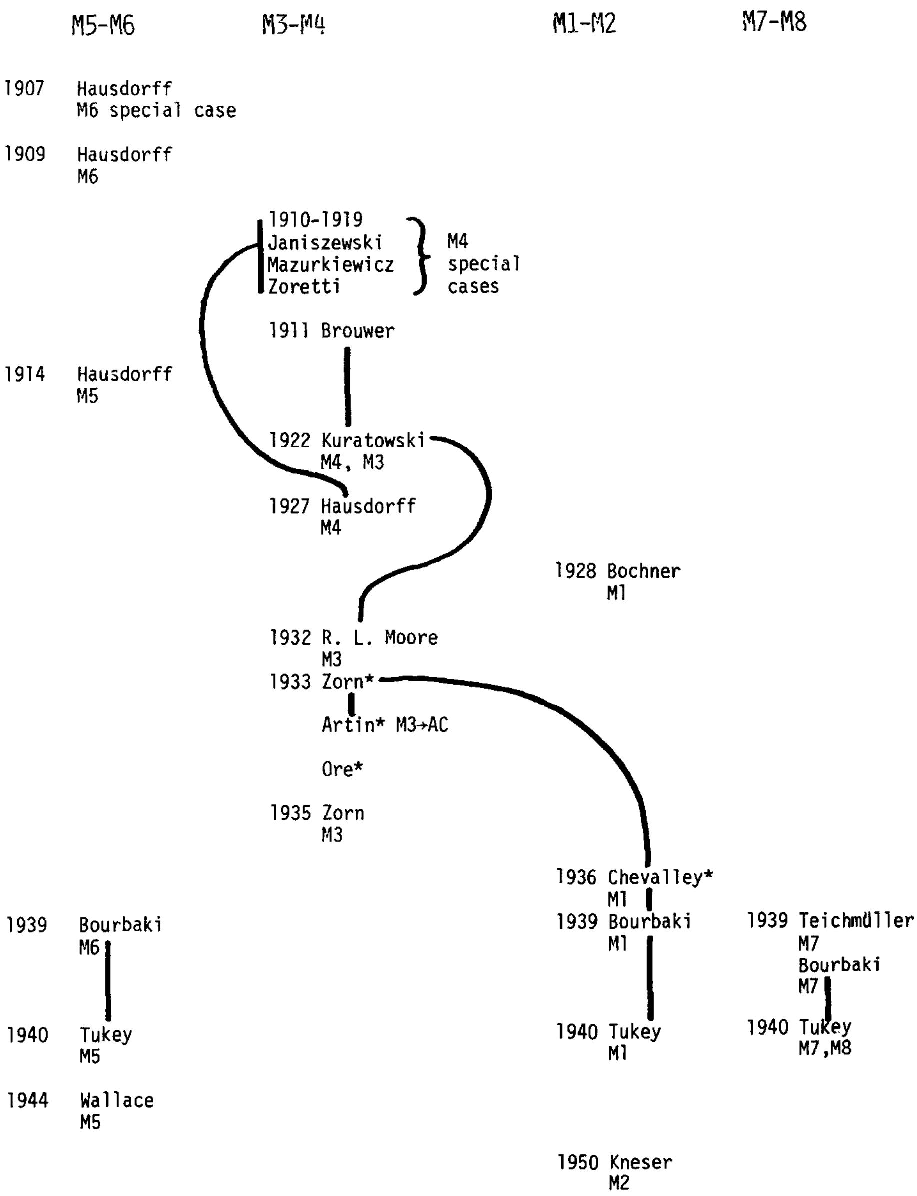

FIGURE 1
Family tree of "Zorn's Lemma." * indicates information supplied by Zorn.
Solid lines indicate known associations.

We quote the original form of "Zorn's Lemma" as given by Zorn in his 1935 paper, retaining the italics as present there:

> DEFINITION 1. A set $\mathcal{B} = \{B\}$ of sets B is called a *chain*, if for every two sets B_1, B_2, either $B_1 \supset B_2$, or $B_2 \supset B_1$.
>
> DEFINITION 2. A set $\mathcal{A}$ of sets A is said to be closed (right-closed) if it contains the union $\sum_{\mathcal{B} \ni B} B$ of every chain $\mathcal{B}$ contained in $\mathcal{A}$.
>
> Then our *maximum principle* is expressible in the following form. (MP). *In a closed set* $\mathcal{A}$ *of sets* A *there exists at least one,* A*, *not contained as a proper subset in any other* $A \in \mathcal{A}$.
>
> [Zorn 1935, 667]

2. HISTORY OF ZORN'S LEMMA AND OTHER CLOSELY RELATED MAXIMAL PRINCIPLES

Rubin and Rubin [1963, 10-13] do a careful job of distinguishing the various maximal principles of interest to us here [1]. The specific principle formulated by Zorn is listed as principle M3. Rubin and Rubin first define a *nest* as a class which is linearly ordered *under inclusion*, and then formulate Zorn's principle as:

> *M3: If every non-empty nest which is a subset of a non-empty set x has its union an element of x, then x has a maximal element.*
>
> [Rubin and Rubin 1963, 12]

They then give:

> *M4: If every well-ordered nest which is a subset of a non-empty set x has its union an element of x, then x has a maximal element.*
>
> [Rubin and Rubin 1963, 12]

Note that M4 simply implies M3, since the hypothesis of the latter is at least as strong as that of the former. By the same token, Zorn's principle M3 is the apparently weaker principle to posit, though in fact both are logically equivalent.

A curious adumbration of M4 occurred as early as 1910, in the form of a theorem in topology that every continuum (or arc) between two points contains a subset which is an irreducible (minimal) continuum between the points. This theorem was first announced by Janiszewski [1910], who sketched a proof based on the Well-Ordering Theorem. But Zoretti [1910] announced a proof that does not use ordinals, and Mazurkiewicz [1910] published such a proof. Zoretti's proof appeared in [1912], but it is clear already from Mazurkiewicz's that the theorem in question can be proved without appeal to any form or equivalent of the

Axiom of Choice. Janiszewski produced the details of his own proof [1911, 85-89; 1912, 163-167] and also reproduced (with one slip) [1911, 30-34; 1912, 108-112] Mazurkiewicz's proof. Subsequently, Mazurkiewicz [1919] gave yet another proof, this one employing the Well-Ordering Theorem.

The theorem was mentioned by Brouwer [1911, 138], who noted that it was a special case of a more general theorem which he proved using the Well-Ordering Theorem. Brouwer described a closed set of points which is replaced by a closed subset as having been *lopped*, and defined an *inductible property of closed sets of points* to be "a property which, when possessed by each term of a lopping series [decreasing sequence of closed sets], holds also for the limiting set of that series." His theorem is:

> *Let* μ *be a closed set of points of* Sp_n [ordinary n-space] *possessing the inductible property* α; *we can reduce it by a denumerable number of loppings of a definite kind* β *to a closed set of points* μ_i *possessing still the property* α *but losing it by any new lopping of kind* β.

He pointed out that the Janiszewski-Mazurkiewicz-Zoretti result can be obtained by taking α to be the property of containing the two points and being continuous, and as β "the most general" kind of lopping.

The one who perceived an even more general pattern was Kuratowski, who in [1922, 88-90] stated and proved M4 and then related the earlier work mentioned above. He also acknowledged [1922, 77] earlier work by Hessenberg [1909] on the properties of chains of sets.

A summary of the historical developments and interconnections of the principles M3 and M4 is displayed in the second column of Figure 1.

Another family of closely-related maximal principles began with Hausdorff [1907, 110], who proved a special case of Rubin and Rubin's M6:

> *M6: For every set x, there exists a maximal subset* [footnote: maximal with respect to inclusion, $\subseteq$] *of x which is a nest.* [Rubin and Rubin 1963, 12]

Later Hausdorff [1909, 300-301] formulated and proved the general statement. His book [1914, 140] improved the result by extending the conclusion; Rubin and Rubin denominate the result as M5:

> *M5: If R is a transitive relation on x, then there exists a maximal subset* [footnote: maximal with respect to inclusion, $\subseteq$] *of x which is linearly ordered by R.* [Rubin and Rubin 1963, 12]

In 1927 Hausdorff published an abbreviated edition of [1914] in which the chapter on ordered sets was substantially reduced. The new version did not include M5 but did clearly state and prove M4 on pp. 173-174, and reference was made on pp. 280-281 to papers by Janiszewski [1910, 1911, and 1912] and Brouwer [1910] (though curiously not to Brouwer [1911], which is the second half of Brouwer [1910]). (The author is indebted to Zorn for pointing out the result on pp. 173-174.)

A paper on abstract Riemann surfaces by S. Bochner in 1928 made use of a "mengentheoretischer Hilfssatz" [Bochner 1928, 408-409] which is almost what Rubin and Rubin call M1:

> *M1: If R is a transitive relation on a non-empty set x and if every subset of x which is linearly ordered by R has an R-upper bound then there is an R-maximal element in x.*

The difference is that Bochner included the condition that the relation be asymmetric (not both aRb and bRa). The form M1 is the statement probably most often referred to as "Zorn's Lemma" (cf., for instance, Kelley [1955, 33]). Zorn himself has acquiesced to the prevailing terminology: he consistently refers to M3 as the "the Maximal Principle" and to M1 as "Zorn's Lemma." [Zorn 1976-1977] Bochner's proof was by transfinite induction on a well-ordered set, for which he referred the reader to [Hausdorff 1914]. Note that M3 is a special case of M1. (The author is indebted to [Veech 1976] for the reference to Bochner's paper.) Finally, in 1932 R. L. Moore published a proof of M3 [Moore 1932, 84], but Moore's bibliography lists [Kuratowski 1922].

The historical interrelations of all of the above-mentioned principles are indicated in Figure 1. In addition, the chart includes data on several principles (M2, M7, M8) that lie outside the main line of our investigation but whose definitions are included in Note 2.

All of the mathematicians prior to Zorn proved results of the same nature; in each case a maximal principle was shown to follow from the Well-Ordering Theorem or from the Axiom of Choice. Rubin and Rubin [1963, 11] note that Zorn "was the first to state that a maximal principle implies the Axiom of Choice" (but, cf. §4 below). He asserted the equivalence of the Axiom of Choice, the Well-Ordering Theorem, and his Maximal Principle but did not then publish his proof, though he expressed the intention of doing so [1935, 669]. Fraenkel and Bar-Hillel [1958, 69] give references to the papers which prove the equivalence of Zorn's Maximal Principle with other maximal principles, the Well-Ordering Theorem, and the Axiom of Choice.

Moreover, the applications of the maximal principles before Zorn were all in the realm of topology (although there were instances of direct use of the Axiom of Choice and Well-Ordering Theorem in algebra [Fraenkel and Bar-Hillel 1958, 68]). Rubin and Rubin [1963, 11] note that Zorn was the first to apply a maximal principle in algebra, and that he was apparently unaware of the earlier work of Hausdorff and Kuratowski.

3. WHENCE THE TERM "ZORN'S LEMMA?"

How the term "Zorn's Lemma" came to be is still something of a mystery, but there are at least some grounds for speculation. Zorn himself intended to offer his maximal principle as an alternative set-theoretical axiom to the Axiom of Choice. In no sense was his maximal principle for him a lemma to anything. Bourbaki [1939, 37] talked about "le théorème de Zorn," while Tukey [1940, 7] seems to have been the first in print to use the words "Zorn's Lemma." Actually, neither Bourbaki nor Tukey was referring to the specific formulation by Zorn (specifically, M3). Instead, both were referring to variants of the more general M1, even though they attached Zorn's name to it. (For example, Bourbaki's theorem uses R-least-upper-bound in place of R-upper-bound.) Neither appears to have known of Bochner's "Hilfssatz," which would otherwise seem a likely source for the "lemma" in "Zorn's Lemma."

Bourbaki did state a "lemme fondamental" in the context of discussing "le théorème de Zorn," to wit:

> *Soit E un ensemble ordonné inductif, et f une application de E dans E, telle que, pour tout $x \in E$, on ait $f(x) \geq x$; il existe au moins un élément $x \in E$ tel que $f(x) = x$.* [Bourbaki 1939, 37]

The lemma does not require the Axiom of Choice. Kuratowski had proved and used the set-inclusion version of the result in his proof of M4.

One could advance the conjecture that the term "Zorn's Lemma" found its origin in a melting together of "le théorème de Zorn" and "le lemme fondamental" found on the same page of Bourbaki. This transfer seems far-fetched.

Mycielski [1970] attributed to Semadeni [1968] the following "convincing explanation of the fact that Zorn's name became linked with this proposition" (though a careful reading of Semadeni by this author does not bear out the attribution):

> *Namely, in science the consumer decides upon the name of the tools which he uses and the consumer is not always the best informed person. The name of Cardano's formula is a classical example of this state of affairs. (In this case even Cardano's quotation of Tartaglia (in his* Ars magna*) has not influenced this ever-used name.)* [Mycielski 1970, 244]

Semadeni offered a plausible explanation of how Kuratowski's name disappeared from the picture:

> *Unfortunately, the Polish school of mathematics did not come to an awareness of the importance of*

the maximal principle as enunciated by Kuratowski; so Polish mathematicians learned the use of Zorn's lemma a quarter of a century later from Americans.
[Semadeni 1968, 146]

Thus, there seems no doubt that Zorn provided a great service in directing attention to the largely unrealized potential in maximal principles.

4. WHAT DOES ZORN SAY?

The above information is comprised of what one can learn from searching the literature. Since the mathematics itself is modern, however, there is available an additional potentially rich source of historical information: individuals currently living who participated in the making of the mathematics. We are fortunate indeed that Max Zorn himself, Professor Emeritus at Indiana University, is available to offer his description of development of "Zorn's Lemma." We are particularly indebted to him, because his account reveals many facets of the situation that add substantially to the written record.

I did my work on the Maximal Principle in Hamburg around 1933. I proposed it as a "working principle" --a special case in algebra. It was well-received and became a "folk-affair." Artin would use it, and Chevalley would take it up.

At some point I had Moore's book [1932] in my hands. I would say that I perhaps looked through the first few pages in Germany, but I did not see the theorem. I saw the theorem only after it was pointed out to me long after, in America. From then on I remember believing that Moore used well-ordered chains. Such is not the case. I tell you this to show you how reliable my memory is!

I knew about Kuratowski [1922], but I formed an erroneous impression about the thrust of the article.

Only in 1976 did I realize the contents of Hausdorff [1927]. Of the people I have had contact with, I may have been the first person to have caught that.

I arrived in America the first week of August, 1934, where I was a Sterling Fellow at Yale. I talked with Øystein Ore, who was the chairman there, about a week after I arrived. He told me, "I have used that principle myself"--I took it to mean that that he had privately found the Maximal Principle himself. I probably heard the words "Zorn's Lemma" first from him. It had that name before October, 1934, when I presented the paper [Zorn 1935] to the American Mathematical Society in New York. There

Lefschetz told me I should publish the results.

I used sets and set inclusion. I got to UCLA in 1936, and in a seminar there I conjectured that a certain result was due to Bourbaki. In the fall of 1936 I didn't know who Bourbaki was; I thought Bourbaki was a young man who published occasionally in the Comptes Rendus. *It must have been after 1936 (but I don't really know when) when I heard the so-called "théorème de Zorn" (of Bourbaki [1939])--that is, the theorem for an arbitrary partial order, or "Zorn's Lemma."*

I believed that it was Chevalley's work. Chevalley simply refused to acknowledge authorship, because he wanted it to be a Bourbaki affair. Dieudonné told at least one, if not a group, of my colleagues that it was indeed Chevalley who was responsible for the result in Bourbaki; Ziemer passed on to me what he had heard from Dieudonnné. It wasn't news to me, but at that moment it became "official": I had a source.

A point of personal importance to me is the terminology "Zorn's Lemma." I refer to the principle enunciated in my paper as the "Maximal Principle." For the result in Bourbaki, I used to say "the theorem of Bourbaki." After the news from Dieudonné via Ziemer, I said "the theorem of Chevalley." Now perhaps I should say "the theorem of Kuratowski." I occasionally accept the term "Zorn's Lemma," or use it; but I always smile, to supply the quote marks.

At least the "Lemma" part is apt. If I were presented with a theorem proved and applied, I wouldn't mind calling it a lemma. When it is not an axiom, it looks like a lemma to me. In any case, it is *a lemma, in the Bochner paper. I was greatly surprised when that paper was brought to my attention recently. You see, Bochner and I conversed in 1934-1935, or perhaps later; and yet somehow I must not have mentioned the Maximal Principle to him, and he did not speak of his result to me!*

It was Artin, however, who realized that the Maximal Principle implies the Axiom of Choice. I proved the existence of a system of representatives modulo a subgroup of a group by means of the Maximal Principle. Artin knew me quite well, and he stated to me: "If that is so, then it will work for the Axiom of Choice."

But--you can easily deform memories.

[Zorn 1976-1977]

Inquiries were sent by this author to Chevalley, Tukey, and Kuratowski along with an earlier version of this paper. At the time of writing Chevalley had not replied, but Tukey had the following to contribute:

> *In view of Zorn's quote from Lefschetz (< 1935) using the very words "Zorn's Lemma" it would seem to me almost certain that the name came to Princeton with Lefschetz. (Chevalley came later, still before my writing.)*
>
> *It would be my impression, though weak, that the term widely used in Princeton at that time (1938-39). Bourbaki was much more a name than an object of study with most of us then.*
>
> *Even more importantly, Lefschetz was revising his Colloquium Lectures on Topology for a second edition. There was a draft first chapter, which Lefschetz later replaced as too abstract, with which Eilenberg and I had a little to do. I suspect that "Zorn's Lemma" might have appeared there.*
>
> [Tukey, 1976]

Kuratowski's reply [1977a] was along the same lines as his subsequently published contribution [1977b], where he notes his formulation of a maximal principle prior to Zorn's work.

5. CONCLUSION

The history of mathematics is rife with a variety of misattributions, whose continued propagation by oral and written tradition is variously due to widespread ignorance of the historical facts, accepted convention, or just plain complication of the situation. All of these play their part in "Zorn's Lemma"; even the term is used by different people to denote different propositions, logically--but not historically--equivalent.

This paper does not offer an authoritative version of the history of the interrelations of the tribe of maximal principles that pass for "Zorn's Lemma." The families of principles whose genealogies are discussed here have multiplied further, with contributions from Teichmüller [1939], Wallace [1944], Kneser [1950], Szele [1950], Birkhoff [1948, 1967], Felgner [1967], Bernays [1974], and still others. Principles can be distinguished according to the conditions on the relation, the structure on the sets of subsets in the inductive clause, what serves to delimit those sets of subsets, and the type of object whose existence is asserted--not to mention the nature of the basic objects under discussion (e.g., in the case of Janiszewski [1910], continua).

Even if no further facts are uncovered which would alter its general outline, the history given here is sufficiently tangled so as to preclude a simple and completely fair solution to the question of attribution. Semadeni suggested to his colleagues in Poland that "without taking to the barricades over priority, maybe it is not out of place to use the term 'Kuratowski-Zorn principle.'" [Semadeni 1968, 146] It is probably better to let the question go begging. A more global solution to the general problem would be that we mathematicians try to educate ourselves and our students more closely in the history of our subject, despite all the tangles that defy easy simplification.

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NOTES

1. An excellent classification of maximal principles is provided by Harper and Rubin [1976]. They denote variations of Zorn's lemma by Z(Q,U) (Every non-empty Q-ordered set, in which each U-ordered subset has an upper bound, has a maximal element) and variations of the Hausdorff maximal principle by H(Q,U) (Every Q-ordered set contains a $\subseteq$-maximal, U-ordered subset). Among the possibilities for Q and U are the following: TR (transitive), AS (antisymmetric), P (partially ordered), W (well-ordered), L (linearly ordered). Thus the following equivalences of notation hold for some of the maximal principles discussed in the present paper: M1 = Z(TR,L), M2 = Z(TR,W), M5 = H(TR,L), M7 = FC (for "finite character"), Bochner's "Hilfssatz" = Z(TR & AS, L). However, Harper and Rubin do not distinguish the historically important forms M3, M4 and M6--which specifically involve set inclusion and unions--from the statements Z(P,L) and H(P,L), whose hypotheses involve an arbitrary partial order and the existence of upper bounds relative to it.

2. We give here definitions of several additional principles which appear in Figure 1.

M2: If R is a transitive relation on a non-empty set x and if every subset of x which is well-ordered by R has an R-upper bound, then there is an R-maximal element in x. [Rubin and Rubin 1963, 12]

Definition: A non-empty property P is of finite character *if a class X has the property P if and only if every finite subset of X has the property P.* [Rubin and Rubin 1963, 12]

> *M7: For every set x and every property of finite character, there exists a maximal subset* [footnote: with respect to inclusion, ⊆] *of x which has the property P.* [Rubin and Rubin 1963, 13]

> *Definition: A condition on a function is a* finite restriction *if, as a property of the graph of the function, it is of finite character. This means that it is the logical sum of conditions each of which depends on the functional values at a finite number of points.* [Tukey 1940, 7-8]

We continue Rubin and Rubin's numbering system with:

> *M8: The class of those functions defined on subsets of a given set and satisfying a given family of finite restrictions, contains a function no one of whose extentions* [sic] *belongs to the class.* [Tukey 1940, 8]

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