Physics Letters B 757 (2016) 510–514

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Testing local Lorentz invariance with gravitational waves

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ARTICLE INFO

Article history:
Received 4 April 2016
Accepted 18 April 2016
Available online 20 April 2016
Editor: A. Ringwald

ABSTRACT

The effects of local Lorentz violation on dispersion and birefringence of gravitational waves are investigated. The covariant dispersion relation for gravitational waves involving gauge-invariant Lorentz-violating operators of arbitrary mass dimension is constructed. The chirp signal from the gravitational-wave event GW150914 is used to place numerous first constraints on gravitational Lorentz violation.

The recent discovery of gravitational waves [1], a century after their prediction by Einstein [2], opens the door to a new class of experimental tests of General Relativity (GR). While GR is an impressively successful classical field theory of gravity, incorporating a consistent description of quantum effects is widely expected to involve changes to its underlying principles. An essential foundation of GR is the Einstein equivalence principle, which combines the requirements of local Lorentz invariance with local position invariance and the weak equivalence principle. In this work, we demonstrate that the observation of gravitational waves from coalescing black holes at cosmological distances presents an opportunity for clean tests of local Lorentz invariance in the pure-gravity sector. We use the chirp data from the gravitational-wave event GW150914 to place first constraints on certain types of local Lorentz violation involving the gravitational field.

Experimental studies of local Lorentz invariance, which includes symmetry under local rotations and boosts, have enjoyed a resurgence in popularity in recent decades [3,4], triggered by the demonstration that minuscule Lorentz violation could naturally emerge in quantum-gravity theories such as strings [5]. A general and model-independent approach to describing the effects of Lorentz violation in quantum gravity is provided by effective field theory [6]. Studying specific models offers an alternative approach [7]. We are interested in the model-independent action for pure gravity, which is formed as the sum of the usual Einstein–Hilbert action with cosmological constant together with all possible terms involving operators formed from gravitational fields. This theory is a piece of the general effective field theory for gravity and matter, the gravitational Standard-Model Extension (SME). A Lorentz-violating term in the action is an observer-independent quantity [6] containing a Lorentz-violating operator contracted with a coefficient governing the size of its effects. Using natural units, each operator can be assigned a mass dimension d, and the associated coefficient then has mass dimension $4-d$. Under the plausible assumption that Lorentz violation is suppressed by powers of the Planck mass, which is the natural mass scale associated with the Newton gravitational coupling, operators of higher d can be expected to induce smaller effects.

Observational [8–15] and analytical [16–23] investigations using this pure-gravity effective field theory have largely concentrated on minimal Lorentz-violating operators, which have dimension $d=4$. Theoretical aspects of nonminimal operators with dimensions $d=5,6$ have been studied [24], and constraints on some nonrelativistic combinations of operators with $d=6$ have been obtained via laboratory tests of short-range gravity [25–27]. The tightest constraints to date on local Lorentz violation in the gravity sector have been deduced from the absence of gravitational Čerenkov radiation by cosmic rays [31–37] with a large class of effects for $d=4$, 6, and 8 now being excluded at sharp levels [36]. Reviews can be found, for example, in Refs. [3,4,28–30]. Here, we construct the general quadratic Lagrange density for gravitational waves in the presence of Lorentz-violating operators of arbitrary d, and we extract the covariant dispersion relation involving gauge-invariant effects. We show that observations of gravitational waves provide sensitivities to nonminimal Lorentz violation independent of matter-sector effects, and we use the gravitational-wave event GW150914 to place numerous first limits on gravity-sector operators with $d \geq 5$.

The effective field theory for gravitational Lorentz violation [6] can be linearized in a flat-spacetime background with Minkowski metric, $g_{\mu\nu}=\eta_{\mu\nu}+h_{\mu\nu}$. Our first goal is to construct the general

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http://dx.doi.org/10.1016/j.physletb.2016.04.040
0370-2693/

Table 1
Contributions $\hat{\mathcal{K}}^{(d)\mu\nu\rho\sigma}$ to the gauge-invariant part of the theory (1).

Operator $\hat{\mathcal{K}}^{(d)\mu\nu\rho\sigma}$	Tableau	CPT	d	Number
$s^{(d)\mu\rho\circ\nu\sigma\circ\circ^{d-4}}$	μ ν $\cdots$ ρ σ $\circ$ $\circ$	even	even, ≥ 4	$(d-3)(d-2)(d+1)$
$q^{(d)\mu\rho\circ\nu\circ\sigma\circ\circ^{d-5}}$	μ ν σ $\cdots$ ρ $\circ$ $\circ$ $\circ$	odd	odd, ≥ 5	$\frac{5}{2}(d-4)(d-1)(d+1)$
$k^{(d)\mu\circ\nu\circ\rho\circ\sigma\circ\circ^{d-6}}$	μ ν ρ σ $\cdots$ $\circ$ $\circ$ $\circ$ $\circ$	even	even, ≥ 6	$\frac{5}{2}(d-5)d(d+1)$

Lagrange density quadratic in the dimensionless metric perturbation $h_{\mu\nu}$, allowing for both Lorentz-invariant and Lorentz-violating terms. A generic term of this type takes the form

$$\mathcal{L}_{\mathcal{K}^{(d)}} = \tfrac{1}{4} h_{\mu\nu} \hat{\mathcal{K}}^{(d)\mu\nu\rho\sigma} h_{\rho\sigma}, \tag{1}$$

where

$$\begin{aligned}\hat{\mathcal{K}}^{(d)\mu\nu\rho\sigma} &= \mathcal{K}^{(d)\mu\nu\rho\sigma\varepsilon_1\varepsilon_2\ldots\varepsilon_{d-2}} \partial_{\varepsilon_1}\partial_{\varepsilon_2}\ldots\partial_{\varepsilon_{d-2}} \\ &\equiv \mathcal{K}^{(d)\mu\nu\rho\sigma\circ^{d-2}}\end{aligned} \tag{2}$$

is an operator of mass dimension $d \geq 2$ and the coefficients $\mathcal{K}^{(d)\mu\nu\rho\sigma\varepsilon_1\varepsilon_2\ldots\varepsilon_{d-2}}$ have mass dimension $4-d$ and are assumed both small and constant over the scales relevant here. In this equation, we have introduced a convenient notation by which indices contracted into a derivative are denoted with a circle index $\circ$, and n-fold contractions are denoted as $\circ^n$. The Lorentz-invariant pieces of the expression (2) consist of complete traces of the coefficients $\mathcal{K}^{(d)\mu\nu\rho\sigma\varepsilon_1\varepsilon_2\ldots\varepsilon_{d-2}}$. Varying the action reveals that only operators satisfying the condition $\hat{\mathcal{K}}^{(d)(\mu\nu)(\rho\sigma)} \pm \hat{\mathcal{K}}^{(d)(\rho\sigma)(\mu\nu)} \neq 0$ can contribute to the equations of motion, where the upper sign is for even d and lower one for odd d.

To construct explicitly the operators $\hat{\mathcal{K}}^{(d)\mu\nu\rho\sigma}$, we perform a decomposition into irreducible pieces and examine the properties of each. This reveals that 14 independent classes of operators can control the behavior of gravitational waves. However, many violate the usual gauge symmetry of GR under the transformation $h_{\mu\nu} \to h_{\mu\nu} + \partial_\mu\xi_\nu + \partial_\nu\xi_\mu$. Performing this transformation on the term (2) shows that the condition for gauge invariance is $\big(\hat{\mathcal{K}}^{(d)(\mu\nu)(\rho\sigma)} \pm \hat{\mathcal{K}}^{(d)(\rho\sigma)(\mu\nu)}\big)\partial_\nu = 0$. Only three of the 14 classes of irreducible operators obey this condition, and they are therefore of particular interest. Their existence can be understood as following from spontaneous breaking of the diffeomorphism and Lorentz invariance [38], which hides symmetry rather than explicitly violating it and is therefore automatically compatible with the Bianchi identities [6]. The gauge invariance maintains the standard counting of degrees of freedom in $h_{\mu\nu}$, insuring that the three classes of operators induce perturbative modifications to the two usual propagating modes in a gravitational wave. Note that in principle the higher derivatives occurring in the term (2) introduce additional modes, but these are nonperturbative in Lorentz violation and occur only at high energies outside the domain of validity of the effective field theory [39].

The three classes of gauge-invariant operators are determined by their symmetries, which can be determined via standard methods in group theory [40] and are specified by the Young tableaux shown in Table 1. It is convenient to denote these operators by the three specific symbols shown in the first column of the table, instead of the generic form $\hat{\mathcal{K}}^{(d)\mu\nu\rho\sigma}$. The CPT handedness of the corresponding terms in the quadratic Lagrange density is given in the third column. The operators exist only in the dimensions listed in the fourth column of the table. The number of independent components of each is displayed in the fifth column of the table. In what follows, it is also useful to define quantities that are the sums over d of each of these sets of operators,

$$\hat{s}^{\mu\rho\nu\sigma} = \sum_d s^{(d)\mu\rho\circ\nu\sigma\circ^{d-3}}, \quad \hat{q}^{\mu\rho\nu\sigma} = \sum_d q^{(d)\mu\rho\circ\nu\circ\sigma\circ^{d-4}},$$
$$\hat{k}^{\mu\nu\rho\sigma} = \sum_d k^{(d)\mu\circ\nu\circ\rho\circ\sigma\circ^{d-5}}. \tag{3}$$

The operator $\hat{q}^{\mu\rho\nu\sigma}$ is antisymmetric in the first pair of indices and symmetric in the second, while the operator $\hat{s}^{\mu\rho\nu\sigma}$ is antisymmetric in both the first and second pairs of indices, and $\hat{k}^{\mu\nu\rho\sigma}$ is totally symmetric. Any contraction of these operators with a derivative vanishes.

The complete gauge-invariant quadratic Lagrange density, including all Lorentz-violating and Lorentz-invariant terms of arbitrary mass dimension d, then takes the form

$$\begin{aligned}\mathcal{L} &= \mathcal{L}_0 + \tfrac{1}{4} h_{\mu\nu}(\hat{s}^{\mu\rho\nu\sigma} + \hat{q}^{\mu\rho\nu\sigma} + \hat{k}^{\mu\nu\rho\sigma}) h_{\rho\sigma}, \\ \mathcal{L}_0 &= \tfrac{1}{4}\epsilon^{\mu\rho\alpha\kappa}\epsilon^{\nu\sigma\beta\lambda}\eta_{\kappa\lambda} h_{\mu\nu}\partial_\alpha\partial_\beta h_{\rho\sigma},\end{aligned} \tag{4}$$

where $\mathcal{L}_0$ is the quadratic approximation to the Einstein–Hilbert action. The symmetries of $\mathcal{L}_0$ involving the double Levi-Civita tensor show that this term is a subset of the $d=4$ component of $\hat{s}^{\mu\rho\nu\sigma}$. We remark in passing that the introduction of a dual operator via $\hat{s}^{\mu\rho\nu\sigma} = -\epsilon^{\mu\rho\alpha\kappa}\epsilon^{\alpha\nu\sigma\beta\lambda}\hat{\hat{s}}_{\kappa\lambda}\partial_\alpha\partial_\beta$ reveals that $\hat{s}^{\mu\rho\nu\sigma}$ contributes as a momentum-dependent metric perturbation, $\eta_{\kappa\lambda} \to \eta_{\kappa\lambda} - \hat{\hat{s}}_{\kappa\lambda}$. The effects of $\hat{\hat{s}}_{\kappa\lambda}$ on gravitational Čerenkov radiation are the subject of Ref. [36].

Following methods developed for the study of Lorentz invariance in the photon sector of the SME [39], the covariant dispersion relation for propagation of a gravitational wave of 4-momentum $p^\mu = (\omega, \boldsymbol{p})$ can be derived from the Lagrange density (4). Some algebra reveals that the leading-order dispersion relation for the relevant two perturbative modes takes the form

$$\omega = \Big(1 - \varsigma^0 \pm \sqrt{(\varsigma^1)^2 + (\varsigma^2)^2 + (\varsigma^3)^2}\Big)|\boldsymbol{p}|, \tag{5}$$

with

$$\begin{aligned}\varsigma^0 &= \tfrac{1}{4\boldsymbol{p}^2}\Big(-\hat{s}^{\mu\nu}{}_{\mu\nu} + \tfrac{1}{2}\hat{k}^{\mu\nu}{}_{\mu\nu}\Big), \\ (\varsigma^1)^2 + (\varsigma^2)^2 &= \tfrac{1}{8\boldsymbol{p}^4}\Big(\hat{k}^{\mu\nu\rho\sigma}\hat{k}_{\mu\nu\rho\sigma} - \hat{k}^{\mu\rho}{}_{\nu\rho}\hat{k}_{\mu\sigma}{}^{\nu\sigma} \\ &\qquad + \tfrac{1}{8}\hat{k}^{\mu\nu}{}_{\mu\nu}\hat{k}^{\rho\sigma}{}_{\rho\sigma}\Big), \\ (\varsigma^3)^2 &= \tfrac{1}{16\boldsymbol{p}^4}\Big(-\tfrac{1}{2}\hat{q}^{\mu\rho\nu\sigma}\hat{q}_{\mu\rho\nu\sigma} - \hat{q}^{\mu\nu\rho\sigma}\hat{q}_{\mu\rho\nu\sigma} \\ &\qquad + (\hat{q}^{\mu\rho\nu}{}_\rho + \hat{q}^{\nu\rho\mu}{}_\rho)\hat{q}_{\mu\sigma\nu}{}^\sigma\Big),\end{aligned} \tag{6}$$

where now the derivative factors ∂_μ in the coefficients (3) are understood to be replaced by their 4-momentum equivalent, $\partial_\mu \to ip_\mu$. The observer covariance of the dispersion relation is manifest in the complete index contractions in this equation.

The structure of the dispersion relation (5) indicates that Lorentz-violating modifications to the propagation of gravitational waves can be classified in terms of anisotropy, dispersion, and birefringence. Anisotropy is a consequence of the breaking of rotation symmetry, and in a specified observer frame it is controlled by coefficients for Lorentz violation with spatial indices. All three classes of gauge-invariant operators can produce anisotropic effects. Dispersion arises when the speed of the gravitational wave depends on its frequency. Since every coefficient for Lorentz violation with $d > 4$ is associated with powers of momenta in the dispersion relation, only coefficients with $d = 4$ produce dispersion-free propagation. These are all contained in $\hat{s}^{\mu\rho\nu\sigma}$. Finally, the separation of polarization modes evident in the dispersion relation through the two branches of the square root implies that birefringence of gravitational waves can be caused only by the operators $\hat{q}^{\mu\rho\nu\sigma}$ and $\hat{k}^{\mu\rho\nu\sigma}$ and hence only for $d > 4$.

A gravitational wave traveling along $\hat{\boldsymbol{p}}$ and detected by a laboratory in the vicinity of the Earth appears to emanate from a source located in the direction of the unit radial vector $\hat{\boldsymbol{n}} = -\hat{\boldsymbol{p}}$ in spherical polar coordinates centered on the Earth. For example, the most likely location of the source of the gravitational-wave event GW150914 is a region of about ~ 600 square degrees in the southern hemisphere around declination $-70°$ and right ascension 8 hr [41], so that $\hat{\boldsymbol{n}}$ has spherical polar angles $\theta \simeq 160°$, $\phi \simeq 120°$ in the Sun-centered celestial–equatorial frame canonically used to report results of searches for Lorentz violation [42]. For practical applications, it is therefore useful to perform a decomposition of the dispersion relation (5) in spherical harmonics. Since the metric perturbation $h_{\mu\nu}$ has helicity-2 components, and since the Lagrange density (4) is quadratic in $h_{\mu\nu}$, the decomposition involves spin-weighted spherical harmonics [43] of spin weight $|s| \le 4$, which we denote as ${}_sY_{jm}(\hat{\boldsymbol{n}})$. A summary of the properties of these harmonics can be found in Appendix A of Ref. [39]. Note in particular that ${}_0Y_{jm}(\hat{\boldsymbol{n}}) \equiv Y_{jm}(\hat{\boldsymbol{n}})$, the usual scalar spherical harmonics.

Investigation shows that ς^0, ς^3 are rotation scalars while ς^1, ς^2 are helicity-4 tensors. The decomposition then can be written as

$$\begin{aligned}
\varsigma^0 &= \sum_{djm} \omega^{d-4}\, Y_{jm}(\hat{\boldsymbol{n}})\, k^{(d)}_{(I)jm}, \\
\varsigma^1 \mp i\varsigma^2 &= \sum_{djm} \omega^{d-4}\, {}_{\pm4}Y_{jm}(\hat{\boldsymbol{n}}) \left(k^{(d)}_{(E)jm} \pm i k^{(d)}_{(B)jm}\right), \\
\varsigma^3 &= \sum_{djm} \omega^{d-4}\, Y_{jm}(\hat{\boldsymbol{n}})\, k^{(d)}_{(V)jm},
\end{aligned} \tag{7}$$

where $|s| \le j \le d-2$. The CPT-odd operators in the Lagrange density are controlled by the coefficients $k^{(d)}_{(V)jm}$. The dimension $d \ge 4$ is even for the spherical coefficients $k^{(d)}_{(I)jm}$ for Lorentz violation, while $d \ge 5$ is odd for $k^{(d)}_{(V)jm}$ and $d \ge 6$ is even for $k^{(d)}_{(E)jm}$ and $k^{(d)}_{(B)jm}$. The number of independent components for each of $k^{(d)}_{(I)jm}$ and $k^{(d)}_{(V)jm}$ is $(d-1)^2$, and the number for each of $k^{(d)}_{(E)jm}$ and $k^{(d)}_{(B)jm}$ is $(d-1)^2 - 16$. In the language of spherical coefficients, anisotropic effects are governed by all coefficients with $j \neq 0$, dispersion manifests for all coefficients except $k^{(4)}_{(I)jm}$, while birefringence occurs for all coefficients except $k^{(d)}_{(I)jm}$. We remark in passing that this implies birefringence for even d can occur only for nonminimal operators, $d \ge 6$, unlike the case of Lorentz violation in the pure-photon sector for which minimal $d = 4$ birefringent operators exist [44].

Armed with the above tools, we can use the gravitational-wave event GW150914 to test local Lorentz invariance in gravity. Since operators with larger d are expected to be more suppressed, it might seem natural to study first effects involving the coefficients $k^{(4)}_{(I)jm}$. However, the corresponding operators are nondispersive and nonbirefringent, so detecting their effects is more challenging and typically would require a comparison to light or neutrinos propagating from the same source [36,45]. Instead, we consider in turn the $d = 5$ coefficients $k^{(5)}_{(V)jm}$ and the $d = 6$ coefficients $k^{(6)}_{(E)jm}$, $k^{(6)}_{(B)jm}$. Since polarimetric information for GW150914 and its source is unavailable, we focus here on dispersive effects.

Consider first the generic situation involving a source producing gravitational waves, such as the merger and ringdown of a black-hole binary. We can reasonably assume the observed wave is generated in a superposition of the two propagating modes. For example, the eigenmodes in the presence of $d = 5$ Lorentz violation are circularly polarized, so special physical circumstances would be required for a source to produce only one eigenmode. Note that this assumption is independent of possible Lorentz-violating modifications to the black-hole structure or merger itself. The dispersion between the two modes evinced in the dispersion relation (5) can be used to constrain Lorentz violation by comparing their arrival times. The difference in their velocities generically depends on both the frequency ω of the wave and the location $\hat{\boldsymbol{n}}$ of its source. Both the source and the detector can be taken as comoving objects. The coordinate interval between them is therefore $dl_c = (1+z)dl_p = -v_z dz/H_z$, where v_z is the velocity of the source at redshift z, and $H_z = H_0(\Omega_r\zeta^4 + \Omega_m\zeta^3 + \Omega_k\zeta^2 + \Omega_\Lambda)^{1/2}$, $\zeta \equiv 1+z$, is the Hubble expansion rate at z expressed in terms of the Hubble constant $H_0 \simeq 67.3$ km/s/Mpc, radiation density $\Omega_r \simeq 0$, matter density $\Omega_m \simeq 0.315$, vacuum density $\Omega_\Lambda \simeq 0.685$, and curvature density $\Omega_k = 1 - \Omega_r - \Omega_m - \Omega_\Lambda$. Although the coordinate distance is identical for both modes, their velocities and hence their travel times differ in the presence of Lorentz violation. For example, the arrival-time difference between the two modes for coefficients $k^{(d)}_{(V)jm}$ with fixed d is given by

$$\Delta t \approx 2\omega^{d-4} \int_0^z \frac{(1+z)^{d-4}}{H_z} dz \sum_{jm} Y_{jm}(\hat{\boldsymbol{n}}) k^{(d)}_{(V)jm}. \tag{8}$$

For GW150914, the linear combinations of $k^{(5)}_{(V)jm}$ or $k^{(6)}_{(E)jm}$, $k^{(6)}_{(B)jm}$ appearing in the corresponding expression for Δt are determined by spin-weighted spherical harmonics with approximate angular arguments $\theta \simeq 160°$, $\phi \simeq 120°$ in the Sun-centered frame. The source is located at redshift $z = 0.09^{+0.03}_{-0.04}$ [1]. At the maximum amplitude of the observed chirp signal, the width of the peak is approximately 0.003 s and no indication of splitting is evident. We can therefore reasonably take $\Delta t \lesssim 0.003$ s. The frequency $f = \omega/2\pi$ of the chirp spans the range 35–250 Hz [1], and we can adopt a conservative value of $f \simeq 100$ Hz.

With the above values, we obtain the constraints

$$\left|\sum_{jm} Y_{jm}(\theta,\phi) k^{(5)}_{(V)jm}\right| \lesssim 2 \times 10^{-14} \text{ m}, \tag{9}$$

$$\left|\sum_{jm} {}_{\pm4}Y_{jm}(\theta,\phi)\left(k^{(6)}_{(E)jm} \pm i k^{(6)}_{(B)jm}\right)\right| \lesssim 8 \times 10^{-9} \text{ m}^2. \tag{10}$$

The result (9) represents the first constraint on pure-gravity Lorentz-violating operators with $d = 5$. It also represents the first limit on CPT violation in gravitational waves, which here corresponds to a difference in propagation speed between the two circularly polarized CPT-conjugate eigenmodes. Note that sensitivity to $d = 5$ effects is empirically unavailable in the nonrelativistic limit and hence to typical laboratory experiments on Newton gravity, because the presence of $d = 5$ Lorentz-violating operators in

Table 2
Constraints on coefficients for Lorentz violation.

d	j	Coefficient	Constraint
5	0	$\lvert k^{(5)}_{(V)00}\rvert$	$< 6\times 10^{-14}$ m
5	1	$\lvert k^{(5)}_{(V)10}\rvert$	$< 4\times 10^{-14}$ m
		$\lvert k^{(5)}_{(V)11}\rvert$	$< 1\times 10^{-13}$ m
5	2	$\lvert k^{(5)}_{(V)20}\rvert$	$< 3\times 10^{-14}$ m
		$\lvert k^{(5)}_{(V)21}\rvert$	$< 7\times 10^{-14}$ m
		$\lvert k^{(5)}_{(V)22}\rvert$	$< 4\times 10^{-13}$ m
5	3	$\lvert k^{(5)}_{(V)30}\rvert$	$< 3\times 10^{-14}$ m
		$\lvert k^{(5)}_{(V)31}\rvert$	$< 4\times 10^{-14}$ m
		$\lvert k^{(5)}_{(V)32}\rvert$	$< 2\times 10^{-13}$ m
		$\lvert k^{(5)}_{(V)33}\rvert$	$< 1\times 10^{-12}$ m
6	4	$\lvert k^{(6)}_{(E)40}\rvert$, $\lvert k^{(6)}_{(B)40}\rvert$	$< 1\times 10^{-6}$ m^2
		$\lvert k^{(6)}_{(E)41}\rvert$, $\lvert k^{(6)}_{(B)41}\rvert$	$< 3\times 10^{-7}$ m^2
		$\lvert k^{(6)}_{(E)42}\rvert$, $\lvert k^{(6)}_{(B)42}\rvert$	$< 6\times 10^{-8}$ m^2
		$\lvert k^{(6)}_{(E)43}\rvert$, $\lvert k^{(6)}_{(B)43}\rvert$	$< 2\times 10^{-8}$ m^2
		$\lvert k^{(6)}_{(E)44}\rvert$, $\lvert k^{(6)}_{(B)44}\rvert$	$< 1\times 10^{-8}$ m^2

the action leaves unaffected Newton's law [24]. This fact underscores the added value of the discovery of gravitational waves in the context of studies of the foundations of relativistic gravity. The result (10) represents the first bound on all birefringent coefficients at $d = 6$ and is competitive with existing laboratory bounds [25–27].

Some insight into the implications of these bounds can be gained by deriving from them the constraint on each individual coefficient in turn, under the assumption that the other components vanish. The resulting estimated bounds on the modulus of each component of $k^{(5)}_{(V)jm}$, $k^{(6)}_{(E)jm}$, and $k^{(6)}_{(B)jm}$ obtained for $\theta \simeq 160°$, $\phi \simeq 120°$ are displayed in Table 2. Note that each entry thereby also represents constraints on the moduli of the real and imaginary parts of each component. This standard practice [3] is useful in comparing limits across different experiments and in constraining specific models. For example, models with rotation-invariant gravitational Lorentz violation [46,47] can involve at most the spherical coefficients $k^{(d)}_{(I)jm}$ and $k^{(d)}_{(V)jm}$ with $jm = 00$, for which it is convenient to define $\mathring{k}^{(d)}_{(I)} \equiv k^{(d)}_{(I)00}/\sqrt{4\pi}$ and $\mathring{k}^{(d)}_{(V)} \equiv k^{(d)}_{(V)00}/\sqrt{4\pi}$. The rotation-invariant limit of the dispersion relation (5) then takes the form

$$\omega = \left(1 - \mathring{k}^{(4)}_{(I)}\right)|\boldsymbol{p}| \pm \mathring{k}^{(5)}_{(V)}\omega^2 - \mathring{k}^{(6)}_{(I)}\omega^3 \pm \mathring{k}^{(7)}_{(V)}\omega^4 - \mathring{k}^{(8)}_{(I)}\omega^5 \pm \ldots, \qquad (11)$$

and the first row of Table 2 constrains $\mathring{k}^{(5)}_{(V)}$. Note that the $\pm$ signs reflect the presence of birefringence and CPT violation for even powers of ω, and they are required to describe physics associated with an effective field theory [48]. Also, the isotropic frequency-independent change in the speed of gravitational waves is governed by $\mathring{k}^{(4)}_{(I)}$, which is related by $\bar{s}^{(4)}_{00} \equiv -2k^{(4)}_{(I)00} = -\sqrt{16\pi}\ \mathring{k}^{(4)}_{(I)}$ to the coefficient $\bar{s}^{(4)}_{00}$ constrained in Ref. [36].

In principle, methods related to the one adopted here could be used to obtain estimated constraints on other coefficients with $d > 5$, which are all associated with dispersive operators. The approach used above can be applied directly to $k^{(d)}_{(E)jm}$, $k^{(d)}_{(B)jm}$, and $k^{(d)}_{(V)jm}$, as these always control birefringent operators. For example, it yields the approximate bounds $|k^{(7)}_{(V)j}m| \lesssim 1\times 10^{-2}$ m^3. In contrast, no birefringence occurs for $k^{(d)}_{(I)jm}$, so a dispersive analysis for this type of Lorentz violation requires a somewhat different approach. One option might be to reverse-propagate the observed signal to the source while allowing for the presence of frequency-dependent Lorentz violation, comparing the result to waveform templates for black-hole coalescence to extract constraints. The resulting limits on the coefficients $k^{(d)}_{(I)jm}$ for $d = 6, 8$ would be significantly weaker than ones already deduced from the absence of gravitational Čerenkov radiation in cosmic rays [36]. Related techniques are applicable to searches for a graviton mass and other nonbirefringent physics [49]. Note that dispersion limits of the type discussed here are particularly clean because they involve comparing the properties of two gravitational modes and hence lie entirely within the pure-gravity sector, whereas bounds from gravitational Čerenkov radiation involve comparative tests between the gravity and matter sectors. Indeed, gravitational Čerenkov radiation may even be forbidden for certain relative sizes of the coefficients for Lorentz violation for gravity and matter, which would obviate any bounds obtained via this technique. We also note in passing that the results in Ref. [36] are presented as limits on components of $\hat{s}^{\mu\nu}{}_{\mu\nu}$, but the analysis in fact bounds ς^0 and hence $k^{(d)}_{(I)jm}$, which contains pieces of both $\hat{s}^{\mu\nu}{}_{\mu\nu}$ and $\hat{k}^{\mu\nu}{}_{\mu\nu}$.

In summary, this work uses the recent event GW150914 to constrain dispersive and birefringent effects associated with Lorentz violation in gravitational waves. The future detection of additional gravitational-wave events will yield direct improvements on the limits obtained in this work. Moreover, the use of dispersion information from multiple astrophysical sources at different sky locations permits extraction of independent constraints on different coefficients, as has already been demonstrated for nonminimal coefficients in the photon and neutrino sectors of the SME [50,51]. Improved sensitivities can also be expected for gravitational waves of higher frequency, as might be emitted in a supernova core collapse. The prospects are evidently bright for future studies of foundational physical principles via measurements of gravitational-wave properties.

Acknowledgements

This work was supported in part by the United States Department of Energy under grant number DE-SC0010120, by the United States National Science Foundation under grant number PHY-1520570, and by the Indiana University Center for Spacetime Symmetries.

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