Pure mode II fracture characterization of composite bonded joints

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Abstract

A new data reduction scheme is proposed for measuring the critical fracture energy of adhesive joints under pure mode II loading using the End Notched Flexure test. The method is based on the crack equivalent concept and does not require crack length monitoring during propagation, which is very difficult to perform accurately in these tests. The proposed methodology also accounts for the energy dissipated at the Fracture Process Zone which is not negligible when ductile adhesives are used. Experimental tests and numerical analyses using a trapezoidal cohesive mixed-mode damage model demonstrated the good performance of the new method, namely when compared to classical data reduction schemes. An inverse method was used to determine the cohesive properties, fitting the numerical and experimental load–displacement curves. Excellent agreement between the numerical and experimental R-curves was achieved demonstrating the effectiveness of the proposed method.

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1. Introduction

Adhesively bonded joints present several advantages relative to alternative joining methods. They present fewer sources of stress concentrations, behave well under fatigue loads, and allow joining different materials. However, their use on structural applications is still limited. Most of the design approaches are based on strength of materials concepts which are clearly inadequate when singularities are present. In fact, stresses at the singularity points are markedly mesh dependent when a finite element analysis is used. The alternative is the use of fracture mechanics concepts where the energetic based approaches acquire special relevance. At the approaches based on stress intensity factors. Energetic analyses provide a suitable measurement of the critical fracture energy using non-local parameters such as applied load and displacement. The fracture mechanics approaches rely on the definition of an initial flaw or crack length. However, in many structural applications the locus of damage initiation is not obvious. On the other hand, stress-based methods behave well at predicting damage initiation, and fracture mechanics behaves well in modelling damage propagation. In order to overcome the drawbacks of each method and exploit the usefulness of the described advantages, cohesive damage models become suitable options (Blackman et al., 2003; Andersson and Stigh, 2004; Bader et al., 2000; Ducept et al., 2000; Nairn, 2000). However, mode II is still not well addressed owing to some particular aspects inherent to the most popular tests: the End Notched Flexure (ENF), the End Loaded Split (ELS) and the Four-Point End Notched Flexure (4ENF). The ELS test involves a clamp which is a source of variability and increases the complexity of data reduction (Blackman et al., 2005). On the other hand, the 4ENF test requires a complex setup and presents some problems related to large friction effects (Schuecker and Davidson, 2000). As a consequence, the ENF appears to be the most suitable test for mode II characterization of bonded joints (Leffler et al., 2007). However, problems related to unstable crack growth and to crack monitoring during propagation have not been adequately solved yet. In fact, in the mode II fracture characterization tests, the crack tends to close due to the applied load, which hinders a clear visualization of its tip. In addition, the classical data reduction schemes, based on beam theory analysis and compliance calibration, require crack monitoring during propagation. On the other hand, a quite extensive Fracture Process Zone (FPZ) ahead of crack tip exists under mode II loading for ductile adhesives. This non-negligible FPZ affects the measured fracture energy. Consequently, its influence should be taken into account, which does not occur when a real crack length is used in the selected data reduction scheme. To overcome these limitations, a new data reduction scheme based on the...
crack equivalent concept and depending only on the specimen compliance is presented in this work. Similar approaches were proposed by Tamuzs et al. (2003) and Biel and Stigh (2008) for the Double Cantilever Beam (DCB) specimen. The main objective of the proposed methodology is to increase the accuracy of the critical fracture energy measurements resulting from experimental mode II tests. The method was applied to experimental tests and validated by a numerical approach that uses a trapezoidal cohesive mixed-mode damage model to account for the adhesive ductility. The cohesive parameters defining the constitutive trapezoidal law were obtained using an inverse method, fitting the load–displacement curves. Classical data reduction schemes based on specimen compliance calibration and corrected beam theory were also used and were compared with the proposed method.

2. Experimental work

Fig. 1 shows the geometry and dimensions of the ENF specimens. In order to provide crack growth stability (Carlsson et al., 1986), the initial crack length was considered to be equal to 70% of the half-length of the specimen. Unidirectional 0° lay-ups of carbon/epoxy prepreg (TEXIPREG HS 160 RM) with 0.15 mm ply thickness were used as adherends, whose mechanical properties are presented in Table 1 (Campilho et al., 2005). The laminates were manufactured by the hand lay-up technique and cured in a hot-plates press during 1 h at 130°C and 4 bar pressure. The ductile epoxy adhesive Araldite® 2015, whose elastic properties were measured experimentally in bulk tests ($E = 1850$ MPa, $\nu = 0.3$), was used. The bonding surfaces were roughened with sandpaper and cleaned with acetone prior to bonding, in order to avoid unwanted adhesive failures. Assembly was achieved by holding with contact pressure and curing at room temperature. Five specimens were tested using an INSTRON testing machine at room temperature under displacement control (2 mm/min). Fig. 2a shows the experimental setup. The load–displacement ($P$–$\delta$) curve was registered during the test. In mode II fracture characterization tests, crack length monitoring during propagation is very difficult to perform, as the crack grows without a clear opening (Blackman et al., 2005; de Moura et al., 2006). Nevertheless, the crack length was monitored by bonding to the specimen’s edge a strip of paper with the graduations printed on it and by taking photos during the tests with 5 s intervals using a 10 mega pixel digital camera (Fig. 2b). For a better crack tip visualization a white correction fluid layer was used. These procedures help minimize the reading errors made when measuring crack length by visual inspection during the course of the test. The experimental values of $P$–$\delta$–$a$ as a function of time were obtained. The time of each $P$–$\delta$ data point was calculated from the applied displacement and the chosen loading rate. The time for each value of $a$ is the one at which the corresponding photo was taken.

3. Data reduction schemes

3.1. Classical methods

The classical data reduction schemes to obtain the critical fracture energy in pure mode II ($J_{IIc}$) are usually based on compliance calibration or beam theory. The Compliance Calibration Method (CCM) is based on the Irwin–Kies equation (Kanninen and Popelar, 1985)

$$J_{IIc} = \frac{P^2}{2B} \left( \frac{dC}{da} \right)$$

where $B$ is the width and $C = \delta/P$ the compliance of the specimen. Cubic polynomials ($C = C_1 a^3 + C_0$) were used to fit the $C = f(a)$ curves, leading to

$$J_{IIc} = \frac{3P^2 C_1 a^2}{2B}$$

In the case of the ENF test, the Corrected Beam Theory (CBT) proposed by Wang and Williams (1992) leads to

$$J_{IIc} = \frac{9(a + 0.42 \Delta a)^2 P^2}{16B^2 h E_1}$$

where $E_1$ is the axial modulus, $h$ is the half height of the specimen and $\Delta a$ a crack length correction to account for shear deformation

$$\Delta a = h \left[ \frac{E_1}{11G_{12}} \left( 3 - \frac{2}{1 + \frac{G_{12}}{E_1}} \right) \right]$$

Table 1
Carbon–epoxy elastic properties.

<table>
<thead>
<tr>
<th></th>
<th>$E_1$ (GPa)</th>
<th>$E_2$ (GPa)</th>
<th>$G_{13}$ (GPa)</th>
<th>$G_{12}$ (GPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\nu_{12}$</td>
<td>0.342</td>
<td>0.342</td>
<td>3415</td>
<td>4315</td>
</tr>
<tr>
<td>$\nu_{23}$</td>
<td>0.380</td>
<td>0.380</td>
<td>3200</td>
<td>3200</td>
</tr>
<tr>
<td>$\nu_{13}$</td>
<td>0.342</td>
<td>0.342</td>
<td>4215</td>
<td>4315</td>
</tr>
</tbody>
</table>

Fig. 1. Schematic representation of the ENF test (dimensions in mm).

Fig. 2. Experimental setup (a) and crack length measurement during propagation (b).
with

\[ I^\prime = 1.18 \sqrt{\frac{E_3 E_1}{G_{13}}} \]  

(5)

where \( E_3 \) and \( G_{13} \) are the transverse and shear moduli, respectively.

### 3.2. Compliance Based Beam Method

The methods described in the previous section require accurate crack length measurements during propagation, which are not easy to obtain. As a result, important errors can occur during fracture characterization of bonded joints under pure mode II loading. On the other hand, modern adhesives usually present a significant ductile behaviour. In these cases, a large FPZ develops ahead of the crack tip, which is responsible for a non-negligible amount of energy dissipation. Consequently, the used data reduction schemes should take this issue into account. The CBT depends on the real crack length measured experimentally, thus not including the energy dissipated in the FPZ. The CCM is based on specimen compliance calibration, which is affected by the energy being dissipated in the FPZ. However, in view of the used cubic polynomial approach, the fracture energy equation also depends on the real crack length (Eq. (2)).

In order to overcome these limitations, a data reduction scheme based on the crack equivalent concept is proposed. It is named Compliance Based Beam Method (CBBM) and depends only on the specimen’s compliance during the test, which, using the beam theory, can be written as

\[ C = \frac{3a^3 + 2L^3}{8E_1Bh} + \frac{3L}{10G_{13}Bh} \]  

(6)

Since the flexural modulus of the specimen plays a fundamental role on the \( P-\dot{a} \) relationship, it can be calculated from Eq. (6) using the initial compliance \( C_0 \) and the initial crack length \( a_0 \).

\[ E_1 = \frac{3a_0^3 + 2L^3}{8Bh C_{0corr}} \]  

(7)

where \( C_{0corr} \) is given by

\[ C_{0corr} = C_0 - \frac{3L}{10G_{13}Bh} \]  

(8)

This procedure takes into account the variability of the material properties between different specimens and several effects that are not included in beam theory, e.g., stress concentrations near the crack tip, contact between the specimen arms at the pre-crack region and root displacement and rotation effects. In fact, these phenomena affect the specimen behaviour and consequently the \( P-\dot{a} \) curve, even in the elastic regime. Using this methodology, their influence is accounted for through the calculated flexural modulus.

The effect of the FPZ can be included considering the compliance and the equivalent crack concept during propagation. Consequently, during crack propagation, a correction of the real crack length is considered in the equation of compliance (6) to account for the FPZ influence. Substituting \( E_1 \) (Eq. (7)) and \( a_0 \) in the place of \( E_1 \) and \( a_0 \), respectively, in Eq. (6) it can be written

\[ a_\varepsilon = a + \Delta a_{FPZ} = \left[ \frac{C_{corr}}{C_{0corr}} a_0^3 + \frac{2}{3} \left( \frac{C_{corr}}{C_{0corr}} - 1 \right) L^3 \right]^{1/3} \]  

(9)

where \( C_{corr} \) is given by Eq. (8) using \( C \) instead of \( C_0 \). \( J_{IC} \) can now be obtained using the Irwin–Kies relation (Eq. (11))

\[ J_{IC} = \frac{9P^2a_\varepsilon^2}{16B^2E_1h} \]  

(10)

Using the methodology presented above, the critical fracture energy \( J_{IC} \) is obtained just from the \( P-\dot{a} \) curve. Note that the modulus of the specimen is not an inputted property but a computed one (Eq. (7)) and is a function of the initial compliance and \( G_{13} \), which is the only material property needed in this approach. Previous studies (de Moura et al., 2006) showed that \( G_{13} \) has much less influence than the longitudinal modulus, which means that a typical value can be used. Note also that in the above approach it is not necessary to measure the crack length during propagation because the calculated equivalent crack length is used instead of the real one. Another advantage is related to the fact that \( a_\varepsilon \) includes the effect of the FPZ, which is not taken into account when the real crack length is considered. Moreover, a complete \( R \)-curve can be obtained using the proposed methodology. Leffler et al. (2007) also proposed a methodology that does not require crack length monitoring. However, shear deformation measurements at the crack tip are required which are not easy to perform when thin adherends are used.

### 4. Experimental results

The experimental \( P-\dot{a} \) parameters were used to obtain the critical fracture energy in pure mode II as a function of the crack length using the CCM and the CBT. The crack length correction \( \Delta a \) used in the CBT was calculated individually for all specimens, taking into account the Young’s modulus variation between specimens. An average value of 4.706 mm was obtained. The complete \( R \)-curve was determined using the CBBM from the respective experimental \( P-\dot{a} \) curves. The flexural modulus of all tested specimens, necessary for the CBBM, was calculated using Eq. (7) and an average value of 107.5 GPa was obtained. This value is not very different than the nominal Young’s modulus presented in Table 1.

A comparison between the three methods is presented in Fig. 3 for one of the specimens tested. In order to plot the CBBM \( R \)-curve as a function of \( a \), it was necessary to establish a correspondence between \( a_\varepsilon \) and \( a \) using the applied displacement. The global results of all specimens are summarized in Table 2. Fig. 3 shows, for the three data reduction schemes, a short plateau region followed by an increasing trend of \( J_H = f(a) \), especially when the crack approaches the loading cylinder. This effect is explained by the large FPZ of this particular adhesive, due to its high fracture toughness in pure mode II. In fact, the quite large FPZ quickly attains the central loaded region where the local effects of compressive stresses hinder self-similar crack propagation for higher values of crack length. The CCM and the CBBM present a similar plateau. The CBT underestimates the critical fracture energy (de Moura, 2006). In analysing the average results of all specimens (Table 2), \( J_{IC} \) is slightly higher for the CBBM and smaller for the CBT.
5. Numerical analysis

With the objective of numerically simulating the behaviour of the adhesive layer in pure mode II, a trapezoidal cohesive damage model was developed and implemented within interface finite elements in the ABAQUS® software. The elements have zero thickness and intend to replace the solid finite elements traditionally used to represent the adhesive layer. The complete cohesive law in pure mode II was determined using an inverse method, fitting the numerical and experimental \(P-\delta\) curves. This law was then used to assess the adequacy of the three data reduction schemes utilized in the work of Campilho et al. (2008).

### 5.1. Trapezoidal cohesive damage model

A mixed-mode (I + II) cohesive damage model implemented within interface finite elements was developed to simulate damage onset and growth. The adhesive layer is simulated by these elements, which have zero thickness. To simulate the behaviour of ductile adhesives, a trapezoidal softening law relating stresses \(\sigma\) and relative displacements \(\delta_i\) between homologous points of the interface elements was employed (Fig. 4). These types of laws accurately reproduced the behaviour of thin ductile adhesive layers in mode I (Andersson and Stigh, 2004) and mode II (Leffler et al., 2007). The constitutive relationship before damage onset is

\[
\sigma = E \delta
\]

where \(E\) is a stiffness diagonal matrix containing the stiffness parameters \(\varepsilon_i\) (\(i = I, II\)), defined as the ratio between the elastic modulus of the material in tension or shear (\(E\) or \(G\), respectively) and the adhesive thickness \(t\). Considering the pure-mode model, after \(\delta_{1,I}\) (the first inflexion point, which leads to the plateau region of the trapezoidal law) the material softens progressively. The softening relationship can be written as

\[
\sigma = (I - D)E\delta_i
\]

where \(I\) is the identity matrix and \(D\) is a diagonal matrix containing, on the position corresponding to mode \(i (i = I, II)\), the damage parameter. In general, bonded joints are under mixed-mode loading. A formulation for interface finite elements should therefore include a mixed-mode damage model (Fig. 4), which can also be applied under pure mode loading. In fact, the pure mode II loading characteristic of the ENF test is a particular case of a general mixed-mode loading.

Damage onset is predicted using a quadratic stress criterion

\[
\left(\frac{\sigma_{1m}}{\varepsilon_{1m}}\right)^2 + \left(\frac{\sigma_{2m}}{\varepsilon_{2m}}\right)^2 = 1 \quad \text{if} \quad \sigma_i > 0
\]

\[
\sigma_{ii} = \sigma_{u,ii} \quad \text{if} \quad \sigma_i \leq 0
\]

where \(\sigma_i (i = I, II)\) represent the stresses in each mode. It is assumed that normal compressive stresses do not induce damage. Considering Eq. (11), the first part of Eq. (13) can be rewritten as a function of the relative displacements

\[
\frac{\delta_{1m}^2}{\varepsilon_{1m}^2} + \frac{\delta_{2m}^2}{\varepsilon_{2m}^2} = 1
\]

where \(\delta_{1m}, \delta_{2m}(i = I, II)\) are the relative displacements in each mode corresponding to damage initiation. Stress softening onset \(\delta_{2j}\) was predicted using a quadratic relative displacements criterion similar to (14), leading to

\[
\frac{\delta_{1m}^2}{\varepsilon_{1m}^2} + \frac{\delta_{2m}^2}{\varepsilon_{2m}^2} = 1
\]

where \(\delta_{2m}, \delta_{2m}(i = I, II)\) are the relative displacements in each mode corresponding to stress softening onset. Crack growth was simulated by the linear fracture energetic criterion

\[
\frac{J_i + J_{uc}}{J_{uc}} = 1
\]

When Eq. (16) is satisfied damage growth occurs and stresses are released, with the exception of normal compressive ones. Using the proposed criteria (Eqs. (14)–(16)), it is possible to define the equivalent mixed-mode displacements \(\delta_{1m}, \delta_{2m}\) and \(\delta_{um}\) and to establish the damage parameter in the plateau region

\[
d_m = 1 - \frac{\delta_{1m}}{\delta_m}
\]

and in the stress softening part of the cohesive law

\[
d_m = 1 - \frac{\delta_{1m} - \delta_{um}}{\delta_{um} - \delta_{2m}}
\]

The damage parameter is introduced in Eq. (12), thus simulating damage propagation. A detailed description of the proposed model is presented in the work of Campilho et al. (2008).

### 5.2. Evaluation of the cohesive parameters

The value of \(G\) was obtained experimentally from \(E\) and \(\nu\) which were determined from adhesive bulk tests and was used to define the stiffness parameters \(\varepsilon_i\). The values of \(J_{uc}\) for the five specimens tested were obtained from the respective experimental load–displacement curves using the CBBM. The fracture energy, which corresponds to the plateau value of the \(R\)-curves (Table 2), was an inputted parameter in the numerical approach. The remaining cohesive parameters \(\sigma_{u,II}\) and \(\delta_{2j}\) were determined by an inverse method, fitting the numerical and experimental \(P-\delta\) curves of each specimen. Fig. 5 shows the numerical comparison.
and experimental $P-\delta$ curves for one tested specimen after the fitting procedure.

Fig. 6 shows the deformed shape of the ENF specimen during crack propagation, and the respective boundary and loading conditions. The specimen arms were modelled with plane strain eight-node quadrilateral solid finite elements (CPE8 from the ABAQUS library). The adhesive layer was simulated with six-node interface finite elements compatible with the ABAQUS elements, including the trapezoidal mixed-mode cohesive damage model. Each specimen arm was modelled by eight solid finite elements through-thickness. A more refined mesh was considered at the propagation region and near the cylinders. Boundary conditions included fixing the supporting cylinders in the directions $x$ and $y$ and restraining the loading cylinder in the direction $x$. The lowest node at the specimen mid-section was restrained in the direction $x$.

Fig. 7 shows the average values of $J_{IIc}$, $\sigma_{u,II}$ and $\delta_{2,II}$ and the trapezoidal cohesive laws range obtained by fitting the five experimental $P-\delta$ curves. All of these parameters influence the numerical $P-\delta$ curves profile. $J_{IIc}$, which is the inputted value in the numerical simulations, mainly influences the peak load value. Higher local strengths ($\sigma_{u,II}$) increase the peak load, and the specimen stiffness up to the peak load, leading to a more abrupt post-peak load reduction. Finally, $\delta_{2,II}$ plays an important role on the roundness form at the peak value of the $P-\delta$ curve.

6. Comparison between the numerical and experimental results

A study was performed numerically to assess the adequacy of the three data reduction schemes evaluated to measure $J_{IIc}$ accurately. The objective is to verify how the used methods reproduce the inputted $J_{IIc}$. To accomplish this study, the numerical $P-\delta-a$ parameters were collected during crack propagation and were used to obtain the critical fracture energies. Fig. 8 shows the results for one case. Accurate results were obtained with the CCM and especially the CBBM. The CBT underestimated the inputted $J_{IIc}$, which is explained by the non-negligible amount of energy being dissipated in the FPZ that is not accounted for in this method. Moreover, it should be emphasized that the CCM also requires crack length measurements during the propagation stage, which, even using an optical method, is prone to introduce additional errors. On the other hand, using the CBBM, a complete $R$-curve is obtained only from $P-\delta$ data. Fig. 9 compares the numerical and experimental $R$-curves using the CBBM for the same specimen used in Fig. 5, after the fitting procedure. In this case $J_{II}$ is plotted as a function of equivalent crack instead of real crack length, since the objective of this method is to avoid crack length measurements. As a consequence the $R$-curve presents a different profile. The initial part of the curve (before the plateau) does not correspond to crack propagation, but to FPZ development. In fact, as $a_c$
includes the FPZ effects, its variation up to the plateau region is only due to FPZ development. The crack growth only occurs at the beginning of the plateau region. A summary of the \( J_{\text{IIIc}} \) values predicted by the several data reduction schemes for the five specimens is presented in Table 3. Accurate predictions were obtained with the CBBM (average error of 0.6%) and the CCM (average error of 2.9%). However, a large discrepancy was obtained with the CBT, which underestimated the \( J_{\text{IIIc}} \) by 24.9%. This behaviour is also very different from the one obtained experimentally where the values provided by the three methods were closer.

The explanation for this difference is given in Fig. 10. As it can be seen, for the same specimen compliance there is an almost constant discrepancy between the experimental and numerical crack length. This statement leads to the conclusion that some problems occurred during the experimental identification of the crack tip. Probably, the damage ahead of the crack tip induces material deformation originating some local slip, which fractures the white correction fluid layer used to better follow the crack length. This phenomenon yielded a measured crack longer than the real one by a constant amount, during the process of self-similar propagation with a pronounced FPZ at the crack tip.

7. Conclusions

A methodology for fracture characterization of ductile adhesive layers under pure mode II used in bonded joints was proposed. The respective critical fracture energy \( (J_{\text{IIIc}}) \) was measured by ENF tests, using a new data reduction scheme based on the trapezoidal law, fitting the numerical and experimental \( P-\delta \) curves.

It was verified that the CBBM renders the most accurate results and is a suitable method. Consequently, and due to its advantages, it was considered by the authors to be the best choice to be used on the fracture characterization of bonded joints.

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