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# Inert extension of the Zee model

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**Abstract**

We propose a simple extension of the Zee model in order to solve the dark matter issue. It is achieved by adding one generation of two vectorlike leptons and introducing an exact $Z_2$ symmetry. We establish the parameter space that is consistent with the constraints coming from lepton flavor violation processes, neutrino oscillation and dark matter.

*Keywords:* Majorana neutrino, Zee model, Inert doublet model, Lepton flavor violation.

## 1. Introduction

Neutrino masses and dark matter are two of the evidences for physics beyond the Standard Model (SM). Until now, many mechanisms have been proposed in order to accommodate the neutrino masses. These mechanisms are implemented at tree level, like seesaw type I [1], type II [2] and type III [3] or at higher order in perturbation theory, like the *Zee Model* [4]. On the other hand, E. Ma and G. Deshpande showed that by adding a $SU(2)_L$ scalar doublet to the SM and imposing a $Z_2$ symmetry, it is possible to obtain a viable cold dark matter (DM) candidate: the lightest scalar carrying $Z_2$ charge. That idea was know as the *Inert Doublet Model* (IDM) [5].

In this work we propose a model that combines the ideas mentioned above, generating small masses for neutrinos at one loop and including a viable dark matter candidate. For this, we extend the Zee model introducing an unbroken $Z_2$ symmetry. In addition, we add two vector like leptons, one singlet and one doublet under $SU(2)_L$. Only the new fields are odd under $Z_2$ and the DM particle is the lightest neutral component of the scalar doublet. This realization is called T1-2-A in [6].

## 2. The Model

The particle content and the charges assignment are shown in the Table 1. Under the $SU(2)_L \otimes U(1)_Y \otimes Z_2$ invariance, the most general scalar potential and lepton Yukawa Lagrangian are given by:

| | $L_i$ | $e_R^i$ | $\epsilon$ | $\Psi$ | $H$ | $\Phi$ | $h^-$ |
|---|---|---|---|---|---|---|---|
| $SU(2)_L$ | 2 | 1 | 1 | 2 | 2 | 2 | 1 |
| $U(1)_Y$ | −1 | −2 | −2 | −1 | 1 | 1 | −2 |
| $Z_2$ | + | + | − | − | + | − | − |

Table 1: *Leptons and scalar fields with their transformation properties under* $SU(2)_L \otimes U(1)_Y \otimes Z_2$.

$$-\mathcal{L}_l^Y = \left\{ Y_{ij}\bar{L}^i H e_R^j + \eta_i \bar{L}^i \Phi\epsilon + \rho_i \bar{\Psi}\Phi e_R^i + \Pi\bar{\Psi}H\epsilon + f_i \overline{L^C}^i \Psi h^+ + \text{h.c} \right\} + M_\psi \bar{\Psi}\Psi + M_\epsilon \bar{\epsilon}\epsilon \,, \tag{1}$$

$$\begin{aligned}\mathcal{V} = {} & \mu_1^2 H^\dagger H + \mu_2^2 \Phi^\dagger\Phi + \frac{\lambda_1}{2}\left(H^\dagger H\right)^2 + \frac{\lambda_2}{2}\left(\Phi^\dagger\Phi\right)^2 \\ & + \lambda_3 H^\dagger H \Phi^\dagger \Phi + \lambda_4 \left(H^\dagger\Phi\right)\left(\Phi^\dagger H\right) + \mu_h^2 \left|h^-\right|^2 \\ & + \lambda_h \left|h^-\right|^4 + \lambda_6 \left|h^-\right|^2 H^\dagger H + \lambda_7 \left|h^-\right|^2 \Phi^\dagger\Phi \\ & + \left\{ \frac{\lambda_5}{2}\left(H^\dagger\Phi\right)^2 + \mu\epsilon_{ab}H^a\Phi^b h^- + \text{h.c} \right\} ,\end{aligned} \tag{2}$$

where $\Psi = (N, E)^T$ and $\epsilon$ are the vectorlike doublet and singlet. $\Phi$, $h^-$ are the new scalar fields and $H$ is the SM scalar doublet. The couplings $\eta_i$, $\rho_i$ and $f_i$ are vectors in flavor space. The parameter $\mu$ controls the lepton number violation, which is necessary for generate a neutrino Majorana mass term.

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Perturbativity and vacuum stability imply the following conditions on the scalar parameters [7]:

$$\lambda_i < 8\pi\ ,\ \lambda_1 > 0\ ,\ \lambda_2 > 0\ ,\ \lambda_6 > -\sqrt{\frac{\lambda_1\lambda_h}{2}}\ ,$$
$$\lambda_7 > -\sqrt{\frac{\lambda_2\lambda_h}{2}}\ ,\ \lambda_3 + \lambda_4 - |\lambda_5| > \sqrt{\lambda_1\lambda_2}\ . \quad (3)$$

After electroweak symmetry breaking, the scalar doublets can be parametrized in the form:

$$\Phi = \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}}\left(H^0 + iA^0\right) \end{pmatrix}\ ,\ H = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}\left(h + v + iG^0\right) \end{pmatrix}\ .$$

Note that $\Phi$ doublet does not acquire a vacuum expectation value (vev) and this is in order to held an unbroken $Z_2$. The mass eigenstates $\{\kappa_1^\pm, \kappa_2^\pm\}$ and $\{\chi_1, \chi_2\}$ are defining by the mixing

$$\begin{pmatrix} H^\pm \\ h^\pm \end{pmatrix} = \begin{pmatrix} \cos\delta & \sin\delta \\ -\sin\delta & \cos\delta \end{pmatrix}\begin{pmatrix} \kappa_1^\pm \\ \kappa_2^\pm \end{pmatrix}\ ,$$
$$\begin{pmatrix} E \\ \epsilon \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix}\begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix}\ . \quad (4)$$

In this work we choose $H^0$ as the dark matter particle and we expect the phenomenology to be quite similar to the IDM [8]. However, co-annihilations processes, like $H^0\kappa_2^\pm \to W^\pm h$ through a $W^\pm$ boson exchange, may change the relic density, modifying the allowed region for $m_{H^0}$.

## 3. Neutrino masses

We have derived the Majorana neutrino masses at one loop from the diagram in Figure 1. The mass matrix is given by

$$M^\nu_{ij} = k\left[\eta_i f_j + \eta_j f_i\right]\ , \quad (5)$$

where $k$ is defined as

$$k = \frac{s_{2\alpha}s_{2\delta}}{64\pi^2}\left[I(m^2_{\kappa_1^\pm}, m^2_{\kappa_2^\pm}, m^2_{\chi_2})m_{\chi_2} - I(m^2_{\kappa_1^\pm}, m^2_{\kappa_2^\pm}, m^2_{\chi_1})m_{\chi_1}\right],$$

with $s_{2\alpha} = \sin 2\alpha$ and $s_{2\delta} = \sin 2\delta$. The loop function $I(m_1^2, m_2^2, m_3^2)$ is defined in [9].
The matrix in equation (5) has two non-zero eigenvalues, i.e, that structure only allow two massive neutrinos. $M^\nu$ is diagonalized by introducing the Pontecorvo-Maki-Nakagawa-Sakata matrix $U$:

$$U^T M^\nu U = \mathrm{diag}(0, m_2, m_3)\ , \quad (6)$$

where we have chosen the normal hierarchy spectrum.

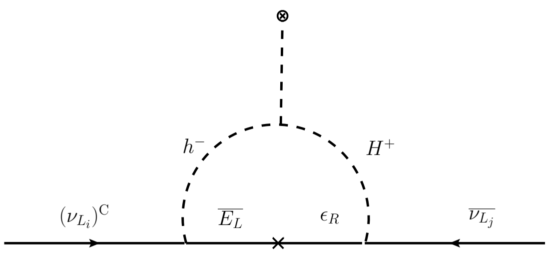


Figure 1: *Loops diagrams for the neutrino mass generation at the one loop level.*

From equations (5) and (6) we can write the Yukawa couplings in the form

$$\eta = \eta_1\begin{pmatrix} 1 \\ \sqrt{a^2+b^2}e^{i\theta} \\ \sqrt{c^2+d^2}e^{i\theta'} \end{pmatrix}\ ,\ f = \frac{1}{2k\eta_1}\begin{pmatrix} \beta_1 \\ \frac{\beta_2}{\sqrt{a^2+b^2}}e^{-i\theta} \\ \frac{\beta_3}{\sqrt{c^2+d^2}}e^{-i\theta'} \end{pmatrix}\ , \quad (7)$$

where the parameters $a, b, c, d, \theta, \theta', \beta_1, \beta_2, \beta_3$ are functions of the neutrino masses and mixing angles [9].
It is worth to mention that neutrino physics restricts five of six parameters in $\eta_i$ and $f_i$. Only $\eta_1$ remains as a free parameter and can be restricted using low energy observables, like lepton flavor violation processes. We obtained that the restriction coming from $\mu \to e\gamma$ radiative process impose $\eta_1 \lesssim 10^{-2}$. On the other hand, if we want to have a natural hierarchy between $\eta_i$ and $f_i$ we must choose an small mixing in charged scalar sector as well as in fermionic sector.

## 4. S, T and U parameters

The new fields may modify the vacuum polarization of gauge boson and this effects are parametrized by the oblique parameters $S$, $T$ and $U$ [10]. In our model, the new contributions are given by:

$$\begin{aligned} T = {}& \frac{1}{4\pi m_W^2 s_W^2}\Bigg\{\frac{\left(m_{\chi_1} - m_{\chi_2}\right)^2}{2}\left[2c_\alpha^4\log\left(\frac{m^2_{\chi_2}}{m_N m_{\chi_1}}\right)\right. \\ & \left. + c_\alpha^2\log\left(\frac{m_N^2}{m^2_{\chi_2}}\right) + c_\alpha^6\log\left(\frac{m^2_{\chi_1}}{m^2_{\chi_2}}\right)\right] + 2c_\alpha^2 F_T\left(m^2_{\chi_1}, m_N^2\right) \\ & + 2s_\alpha^2 F_T\left(m^2_{\chi_2}, m_N^2\right) - 2s_\alpha^2 c_\alpha^2 F_T\left(m^2_{\chi_1}, m^2_{\chi_2}\right)\Bigg\} \\ & + \frac{1}{16\pi m_W^2 s_W^2}\Bigg\{c_\delta^2 F\left(0; m_{\kappa_1^\pm}, m_{H^0}\right) - F\left(0; m_{A^0}, m_{H^0}\right) \\ & + s_\delta^2 F\left(0; m_{\kappa_2^\pm}, m_{H^0}\right) + s_\delta^2 F\left(0; m_{\kappa_2^\pm}, m_{A^0}\right) \\ & + c_\delta^2 F\left(0; m_{\kappa_1^\pm}, m_{A^0}\right) - 2s_\delta^2 c_\delta^2 F\left(0; m_{\kappa_1^\pm}, m_{\kappa_2^\pm}\right)\Bigg\}\ , \end{aligned} \quad (8)$$

$$
\begin{aligned}
S = \frac{1}{3\pi}\Bigg\{ & 2s_\alpha^2 c_\alpha^2 \left[ 3F_S\left(m_{\chi_1}^2, m_{\chi_2}^2\right) - 1 \right] + \log\left(\frac{m_{\chi_2}^2}{m_N^2}\right) \\
& + c_\alpha^2 \log\left(\frac{m_{\chi_1}^2}{m_{\chi_2}^2}\right) \Bigg\} + \frac{1}{4\pi m_Z^2} \Bigg\{ F\left(m_Z^2; m_{H^0}, m_{A^0}\right) \\
& + c_\delta^2\left(c_\delta^2 - 2\right) F\left(m_Z^2; m_{\kappa_1^\pm}, m_{\kappa_1^\pm}\right) - F\left(0; m_{H^0}, m_{A^0}\right) \\
& + 2s_\delta^2 c_\delta^2 \left[ F\left(m_Z^2; m_{\kappa_1^\pm}, m_{\kappa_2^\pm}\right) - F\left(0; m_{\kappa_1^\pm}, m_{\kappa_2^\pm}\right) \right] \\
& + s_\delta^2 \left(s_\delta^2 - 2\right) F\left(m_Z^2; m_{\kappa_2^\pm}, m_{\kappa_2^\pm}\right) \Bigg\} , \qquad (9)
\end{aligned}
$$

where the loop functions $F$, $F_S$ and $F_T$ are defined in [9]. The $U$ parameter is suppressed by the new physics scale $U \sim (M_W/\Lambda)^2\, T$ and we do not take it into account [10]. The experimental deviations from the SM predictions in the $S$ and $T$ parameters under $m_h = 126$ GeV, $m_t = 173$ GeV and $U = 0$ are given by [11]:

$$S = 0.05 \pm 0.09 \ , \ T = 0.08 \pm 0.07 \ ,$$

where the correlation factor between $S$ and $T$ is 0.91. The oblique parameters are easy satisfied if $\delta = \alpha \lesssim 0.1$ or if $m_{\kappa_2^\pm} - m_{\kappa_1^\pm} \lesssim 60$ GeV and $m_{\chi_2} - m_{\chi_1} \lesssim 300$ GeV.

## 5. Dark matter

The neutral component of $\Phi$ is the unique DM candidate in the model. We identify it as $H^0$, which must satisfy the observed thermal relic density [8]. For our analysis we have chosen the following set of free parameters, $m_{H^0}, m_{A^0}, m_{\kappa_1^\pm}, m_{\kappa_2^\pm}, m_{\chi_1}, m_{\chi_2}, \lambda_L, \lambda_2, \lambda_6, \lambda_7, \lambda_h, \delta, \alpha, \eta_1, \rho_i$. In order to satisfy $S$ and $T$ parameters we have taken a small mixing: $\sin\delta = \sin\alpha = 10^{-2}$.

The dark matter mass is limited by two regions, low mass regime, $60\,\text{GeV} \lesssim m_{H^0} \lesssim 80\,\text{GeV}$, and high mass regime, $500\,\text{GeV} \lesssim m_{H^0} \lesssim 1\,\text{TeV}$. We will focus our discussion on high mass regime. The results are showed in Figure 2. The horizontal band show the Planck limit for the relic density [12]. We can see (taking the black line) that efficient annihilations in $100\,\text{GeV} \lesssim m_{H^0} \lesssim 500\,\text{GeV}$ give a small relic density. However, as DM mass increases, $m_{H^0} \gtrsim 500\,\text{GeV}$, the $s$-channel propagator give rise to suppression in the cross section that can enhanced the relic density. Note that with co-annihilations (blue and red lines) it is possible to increase the relic density a bit more and reduce the value of $m_{H^0}$ that is compatible with Planck limit. As the mass splitting is reduced, the co-annihilation effects become more important and the increase in the relic density is larger. The viable value for $m_{H^0}$ is reduce from about 550 GeV for $m_{\kappa_2^\pm}, m_{\chi_1} \gg m_{H^0}$ (without co-annihilations, i.e, black line) to about 350 GeV for $m_{\kappa_2^\pm} = m_{\chi_1} = (m_{H^0} + 1)$ GeV (with co-annihilations).

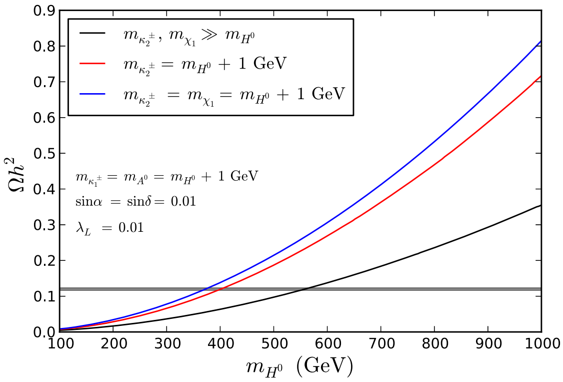


Figure 2: *The relic density as a function of $m_{H^0}$.*

## 6. Conclusions

We have proposed an extension of the Zee model introducing an exact $Z_2$ symmetry that naturally provide a viable dark matter candidate. In order to enclose the one loop neutrino diagram, we need to add two $Z_2$ odd vectorlike leptons. The neutrino oscillation data restricts five couplings on $\eta_i$ and $f_i$. The radiative lepton flavor violation process, $\mu \to e\gamma$, impose a restriction for the free Yukawa parameter: $\eta_1 \lesssim 10^{-2}$. The new contributions to $S$ and $T$ parameters impose that the charged scalar mixing angle must be $\sin\delta \lesssim 10^{-2}$ and the lepton mixing angle must be $\sin\alpha \lesssim 10^{-2}$. On the other hand, we showed that our model reproduces the well known high mass regime that have been studied in the IDM, with the issue that co-annihilations can reduce the value of $m_{H^0}$ compatible with Planck.

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