Physics Letters B 743 (2015) 315–324

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Determination of $f_+^\pi(0)$ or extraction of $|V_{cd}|$ from semileptonic D decays

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ARTICLE INFO

Article history:
Received 13 October 2014
Received in revised form 26 January 2015
Accepted 20 February 2015
Available online 3 March 2015
Editor: M. Doser

ABSTRACT

By globally analyzing all existing measured branching fractions for $D \to \pi e^+ \nu_e$ decays, partial decay rates in different four-momentum transfer-squared q^2 bins, as well as products of the decay form factor $f_+^\pi(q^2)$ and the Cabibbo–Kobayashi–Maskawa (CKM) quark-mixing matrix element $|V_{cd}|$, we obtain $f_+^\pi(0)|V_{cd}| = 0.1428 \pm 0.0019^{+0.0019}_{-0.0011}$. This product, in conjunction with the $|V_{us}|$ determined from (semi-)leptonic K decays and the relation of $|V_{cd}| = |V_{us}| = \lambda$ from the unitarity of the CKM matrix, implies a value for the $D \to \pi$ semileptonic form factor $f_+^\pi(0) = 0.634^{+0.012}_{-0.010} \pm 0.002$, which is consistent within error with those calculated in theory based on quantum chromodynamics (QCD). Alternately, using this product together with the most accurate form factor calculated in Lattice QCD (LQCD), we find $|V_{cd}|^{D\to\pi e^+\nu_e} = 0.2144^{+0.0040}_{-0.0033} \pm 0.0093$. Combining this $|V_{cd}|^{D\to\pi e^+\nu_e}$ with $|V_{cd}|^{D^+\to\mu^+\nu_\mu} = 0.2160 \pm 0.0049 \pm 0.0014$ extracted from both the BESIII and CLEO-c measurements of $D^+ \to \mu^+\nu_\mu$ decays, we find the most precisely extracted $|V_{cd}|$ to be $|V_{cd}| = 0.2157 \pm 0.0045$ up to date. From these determined quantities we find $[m_{D^+} f_+^\pi(0)/f_{D^+}]^{\rm exp} = 5.81 \pm 0.17$, which is in excellent agreement with $[m_{D^+} f_+^\pi(0)/f_{D^+}]^{\rm LQCD} = 5.85 \pm 0.26$ calculated in LQCD, indicating that the LQCD approach to the charm quark sector is excellent. Using this $|V_{cd}|$ together with the PDG'2014 $|V_{ud}|$ and $|V_{td}|$, we check for the first column unitarity and find $|V_{ud}|^2 + |V_{cd}|^2 + |V_{td}|^2 - 1 = -0.004 \pm 0.002$, which deviates from unitarity by 2σ.

1. Introduction

In the Standard Model (SM) of particle physics, the mixing between the quark flavors in weak interaction is parameterized by the Cabibbo–Kobayashi–Maskawa (CKM) matrix $\hat{V}_{\rm CKM}$, which is a 3×3 unitary matrix. Since the CKM matrix elements are fundamental parameters of the SM, they should be measured as accurately as possible. Precise measurements of these elements are very important in testing the SM and searching for New Physics (NP) beyond the SM. Any improved measurement of these elements would be the important input for precision test of the SM.

Three-generation unitarity can be checked to see whether $\hat{V}_{\rm CKM} * \hat{V}^\dagger_{\rm CKM} = \hat{I}$ is satisfied, which leads to test the first, second and third column/row unitarity. The unitarity also gives rise to unitarity triangle (UT) relation $V_{ud}V^*_{ub} + V_{cd}V^*_{cb} + V_{td}V^*_{tb} = 0$. To check for this column/row unitarity and the UT relation, many experimental measurements and theoretical efforts have been made in flavor physics for many years. If any of these consistency checks significantly deviate from unitarity, it may indicate some evidence for NP effects. In addition, the unitarity of the CKM matrix implies that $|V_{cd}|$ coincides with $|V_{us}|$. Comparing a more precisely extracted value of $|V_{cd}|$ from experimental measurements with the value of $|V_{cd}|$ obtained by imposing unitarity of the CKM matrix would also represent an indication of presence or absence of NP in this decay.

Each matrix element can be extracted from measurements of different processes supplemented by theoretical calculations for corresponding hadronic matrix elements. Since the effects of strong interactions and weak interaction can be well separated in semileptonic D decays, these decays are excellent processes from which one can determine the magnitude of the CKM matrix element $V_{cd(s)}$. In the SM, neglecting the lepton mass, the differential decay rate for $D \to \pi e^+\nu_e$ process is given by

$$\frac{d\Gamma}{dq^2} = X \frac{G_F^2}{24\pi^3} |V_{cd}|^2 \boldsymbol{p}^3 |f_+^\pi(q^2)|^2, \tag{1.1}$$

where G_F is the Fermi constant, $\boldsymbol{p}$ is the three-momentum of the π meson in the rest frame of the D meson, q^2 is the four-momentum transfer-squared, i.e. the invariant mass of the lepton and neutrino system, and $f_+^\pi(q^2)$ is the form factor which parameterizes the effect of strong interaction in the decay. In Eq. (1.1),

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http://dx.doi.org/10.1016/j.physletb.2015.02.049

X is a multiplicative factor due to isospin, which equals to 1 for mode $D^0 \to \pi^- e^+ \nu_e$ and 1/2 for mode $D^+ \to \pi^0 e^+ \nu_e$.

In addition to extraction of $|V_{cd}|$, precise measurements of the $D \to \pi$ semileptonic form factor, which is given by z-expansion coefficients a_i (see Section 3.1), is also very important to fit $B \to \pi \ell^+ \nu_\ell$ decays which would help in reducing the uncertainty of the measured $|V_{ub}|$ from the semileptonic B decays [1]. The improved measurement of $|V_{ub}|$ from the semileptonic B decay will improve the determination of the B_d UT, from which one can more precisely test the SM and search for NP beyond the SM.

Furthermore, another way to test the consistency of the SM is to measure the ratio of D semileptonic form factor and D^+ decay constant $f_+^\pi(0)/f_{D^+}$, and compare this ratio with that calculated in Lattice QCD (LQCD). This comparison can also be used to check how well the LQCD approach to the charm sector is.

In the past decades, copious measurements of decay branching fractions and/or decay rates for $D \to \pi e^+ \nu_e$ were performed at different experiments. To obtain the knowledge about $f_+^\pi(0)$ and $|V_{cd}|$ as good as possible, we analyze all of these existing measurements to get the world averages for these two quantities. By a comprehensive analysis of these existing measurements together with the most precise determination of $|V_{us}| = 0.2253 \pm 0.0008$ [2] directly extracted from semileptonic K decays and leptonic K^+ decays and assuming $|V_{cd}| = |V_{us}| = \lambda$ [2] from the unitarity of the CKM matrix or together with the form factor $f_+^\pi(0)$ calculated in LQCD, we precisely determine $f_+^\pi(0)$ or extract $|V_{cd}|$. With these quantities together with D^+ decay constant f_{D^+} and $|V_{cd}|$ extracted from $D^+ \to \mu^+ \nu_\mu$ decays we check the consistency of the SM.

In the following sections, we first review the experimental measurements of decay branching fractions and decay rates for $D \to \pi e^+ \nu_e$ and pre-deal with these measurements to get decay rates to be used in the comprehensive analysis of all these existing measurements in Section 2. We then describe our comprehensive analysis procedure for dealing with these measurements to obtain the product of $f_+^\pi(0)$ and $|V_{cd}|$ in Section 3. In Section 4, we present the final results of our comprehensive analysis of these measurements and check the consistency of the SM with these determined quantities. We finally give a summary for the determination of $f_+^\pi(0)$, the extraction of $|V_{cd}|$ and check the consistency of the SM in Section 5.

2. Experiments

There are different kinds of measurements of $D \to \pi e^+ \nu_e$ decays performed at many experiments during last 25 years, some of which cannot directly be used to determine $f_+^\pi(0)$ or extract $|V_{cd}|$. To determine these quantities from all of these existing measurements, some of these measurements are needed to be preprocessed.

2.1. Relative measurements

In 1995, by analyzing 3.0 fb^{-1} data collected with the CLEO-II detector at the Cornell Electron Storage Ring (CESR), the CLEO Collaboration made a measurement of the branching ratio of the Cabibbo suppressed semileptonic D^0 decays. The CLEO Collaboration observed 87 ± 33 signal events for $D^0 \to \pi^- e^+ \nu_e$ decays and obtained the ratio of branching fractions $R_0 \equiv B(D^0 \to \pi^- e^+ \nu_e)/B(D^0 \to K^- e^+ \nu_e) = 0.103 \pm 0.039 \pm 0.013$ [3].

The Cabibbo suppressed semileptonic $D^0 \to \pi^- \ell^+ \nu_\ell$ ($\ell = e, \mu$) decays were studied at the E687 experiment in 1996. The E687 Collaboration observed 45.4 ± 13.3 and 45.6 ± 11.8 signal events for $D^0 \to \pi^- e^+ \nu_e$ and $D^0 \to \pi^- \mu^+ \nu_\mu$ decays, respectively. After making a small correction to the muon events, the E687 Collaboration combined the branching ratio measurements for the electron and muon modes together and determined the ratio of decay branching fractions to be $R_0 \equiv B(D^0 \to \pi^- e^+ \nu_e)/B(D^0 \to K^- e^+ \nu_e) = 0.101 \pm 0.020 \pm 0.003$ [4].

Table 1
The partial rates $\Delta\Gamma$ of the $D^0 \to \pi^- e^+ \nu_e$ decays in q^2 ranges obtained from different experiments. q^2_{max} is the maximum value of q^2.

Experiment	Index	q^2 (GeV/c^2)	$\Delta\Gamma$ (ns^{-1})
CLEO-II [3]	1	$(0.0, q^2_{max})$	8.79 ± 3.51
E687 [4]	2	$(0.0, q^2_{max})$	8.62 ± 1.73
CLEO-III [6]	3	$(0.0, q^2_{max})$	7.00 ± 0.67
BaBar [7]	4	$(0.0, 0.3)$	1.23 ± 0.07
	5	$(0.3, 0.6)$	1.14 ± 0.09
	6	$(0.6, 0.9)$	1.11 ± 0.08
	7	$(0.9, 1.2)$	0.93 ± 0.07
	8	$(1.2, 1.5)$	0.74 ± 0.07
	9	$(1.5, 1.8)$	0.65 ± 0.07
	10	$(1.8, 2.1)$	0.51 ± 0.07
	11	$(2.1, 2.4)$	0.30 ± 0.06
	12	$(2.4, 2.7)$	0.12 ± 0.05
	13	$(2.7, q^2_{max})$	0.02 ± 0.02
Mark-III [8]	14	$(0.0, q^2_{max})$	9.51 ± 4.26
BES-II [9]	15	$(0.0, q^2_{max})$	8.05 ± 3.25
CLEO-c [10]	16	$(0.0, 0.3)$	1.39 ± 0.10
	17	$(0.3, 0.6)$	1.22 ± 0.09
	18	$(0.6, 0.9)$	1.02 ± 0.08
	19	$(0.9, 1.2)$	0.98 ± 0.08
	20	$(1.2, 1.5)$	0.79 ± 0.07
	21	$(1.5, 2.0)$	0.84 ± 0.07
	22	$(2.0, q^2_{max})$	0.80 ± 0.07

By analyzing 4.8 fb^{-1} data taken with the CLEO-II detector, the CLEO Collaboration performed a measurement of the branching fraction for $D^+ \to \pi^0 e^+ \nu_e$ decay. The CLEO Collaboration found $65 \pm 15 \pm 20$ signal events for $D^+ \to \pi^0 e^+ \nu_e$ decay and obtained the ratio of the branching fractions to be $R_+ \equiv B(D^+ \to \pi^0 e^+ \nu_e)/B(D^+ \to \bar{K}^0 e^+ \nu_e) = (4.5 \pm 1.6 \pm 1.9)\%$ [5] in 1997.

In 2005, the CLEO Collaboration measured the branching ratios of the semileptonic $D^0 \to \pi^- \ell^+ \nu_\ell$ decays by analyzing about 7 fb^{-1} of data collected around the $\Upsilon(4S)$ resonance with the CLEO-III detector. Combining their measurements for electron mode and muon mode with considering the differences in phase spaces of these two decay modes, the CLEO Collaboration obtained the ratio of branching fractions to be $R_0 \equiv B(D^0 \to \pi^- e^+ \nu_e)/B(D^0 \to K^- e^+ \nu_e) = 0.082 \pm 0.006 \pm 0.005$ [6].

All above mentioned measurements are relative measurements which could not be used directly to determine $f_+^\pi(0)$ or $|V_{cd}|$. To use these measurements to determine $f_+^\pi(0)$ or $|V_{cd}|$, we should first transfer these measurements into absolute decay rates in certain q^2 range. The absolute decay rate $\Delta\Gamma$ can be obtained from the measured relative decay branching ratio R by

$$\Delta\Gamma = R \times B(D \to K e^+ \nu_e) \times \frac{1}{\tau_D}, \tag{2.1}$$

where $B(D \to K e^+ \nu_e)$ is the branching fraction for $D^0 \to K^- e^+ \nu_e$ or $D^+ \to \bar{K}^0 e^+ \nu_e$ decays, and τ_D is the lifetime of D meson. Using the lifetime of D meson, $\tau_{D^0} = (410.1 \pm 1.5) \times 10^{-15}$ s, and $\tau_{D^+} = (1040 \pm 7) \times 10^{-15}$ s, the branching fractions of $B(D^0 \to K^- e^+ \nu_e) = (3.50 \pm 0.05)\%$ and $B(D^+ \to \bar{K}^0 e^+ \nu_e) = (8.83 \pm 0.22)\%$ quoted from PDG'2014 [2], we translate these measurements of relative branching fractions into absolute partial decay rates as shown in Tables 1 and 2.

In 2014, the BaBar Collaboration studied the $D^0 \to \pi^- e^+ \nu_e$ decay by analyzing 347.2 fb^{-1} data collected at 10.6 GeV [7]. They selected $D^0 \to \pi^- e^+ \nu_e$ decays from $e^+ e^- \to c\bar{c}$ events and

Table 2
The partial rates of the $D^+ \to \pi^0 e^+ \nu_e$ decays in q^2 ranges obtained from different experiments. q^2_{max} is the maximum value of q^2.

Experiment	Index	q^2 (GeV/c^2)	$\Delta\Gamma$ (ns^{-1})
CLEO-II [5]	23	$(0.0, q^2_{max})$	3.82 ± 2.11
CLEO-c [10]	24	$(0.0, 0.3)$	0.71 ± 0.07
	25	$(0.3, 0.6)$	0.66 ± 0.07
	26	$(0.6, 0.9)$	0.56 ± 0.07
	27	$(0.9, 1.2)$	0.57 ± 0.07
	28	$(1.2, 1.5)$	0.48 ± 0.07
	29	$(1.5, 2.0)$	0.54 ± 0.07
	30	$(2.0, q^2_{max})$	0.37 ± 0.07

divided the candidate events into ten q^2 bins. In each q^2 bin, the branching fraction is measured relative to the normalization mode, $D^0 \to K^-\pi^+$. The partial decay rate in ith q^2 bin is given by

$$\Delta\Gamma_i = \Delta B_i \times \frac{1}{\tau_{D^0}}, \tag{2.2}$$

where ΔB_i is the branching fraction measured in ith q^2 bin. Inserting the lifetime of D^0 meson, $\tau_{D^0} = (410.1 \pm 1.5) \times 10^{-15}$ s and the branching fraction values presented in Ref. [7] into Eq. (2.2), we translate these measurements of branching fractions in ten q^2 bins into absolute partial decay rates, which are shown in Table 1.

2.2. Absolute measurements

In 1989, the Mark III Collaboration performed a measurement of absolute branching fraction for semileptonic $D^0 \to \pi^- e^+ \nu_e$ decay by analyzing data taken at the peak of $\psi(3770)$ resonance with the Mark-III detector. They tagged $3636 \pm 54 \pm 195$ $\bar{D}^0$ mesons and found 7 $D^0 \to \pi^- e^+ \nu_e$ signal events in the system recoiling against the $\bar{D}^0$ tags. With these events, they measured the absolute decay branching fraction $B(D^0 \to \pi^- e^+ \nu_e) = (0.39^{+0.23}_{-0.11} \pm 0.04)\%$ [8].

Using the similar method as the one used in Mark-III, the BES-II Collaboration measured the branching fractions of $D^0 \to \pi^- e^+ \nu_e$ decays by analyzing about 33 pb^{-1} data taken around 3.773 GeV with the BES-II detector at the BEPC collider. In the system recoiling against the $\bar{D}^0$ tags, 9.0 ± 3.6 events from $D^0 \to \pi^- e^+ \nu_e$ decays were observed. With these events, the branching fraction is measured to be $B(D^0 \to \pi^- e^+ \nu_e) = (0.33 \pm 0.13 \pm 0.03)\%$ [9].

The partial decay rate relates to the decay branching fraction by

$$\Delta\Gamma = B(D^0 \to \pi^- e^+ \nu_e) \times \frac{1}{\tau_{D^0}}. \tag{2.3}$$

Using the lifetime of D^0 meson quoted from PDG'2014 [2], $\tau_{D^0} = (410.1 \pm 1.5) \times 10^{-15}$ s, we translate these absolute measurements of branching fractions for $D^0 \to \pi^- e^+ \nu_e$ decays into the partial decay rates, which are shown in Table 1.

In 2009, the CLEO Collaboration studied the semileptonic decays of $D^0 \to \pi^- e^+ \nu_e$ and $D^+ \to \pi^0 e^+ \nu_e$ by analyzing 818 pb^{-1} data collected at 3.773 GeV with the CLEO-c detector. Using double tag method, they measured the decay rates for semileptonic $D^0 \to \pi^- e^+ \nu_e$ and $D^+ \to \pi^0 e^+ \nu_e$ decays in seven q^2 bins [10]. These measurements of decay rates are summarized in Tables 1 and 2.

In 2006, the Belle Collaboration published results on the $D^0 \to \pi^- \ell^+ \nu_\ell$ decays. They accumulated $56\,461 \pm 309 \pm 830$ inclusive D^0 mesons and found $126 \pm 12 \pm 3$ signal events for $D^0 \to \pi^- e^+ \nu_e$ decays and $106 \pm 12 \pm 6$ signal events for $D^0 \to \pi^- \mu^+ \nu_\mu$ decays from 282 fb^{-1} data collected around 10.58 GeV with the Belle detector [11]. Using these selected events from semileptonic D^0 decays, the Belle Collaboration obtained the form factors $f_+^\pi(q^2)$ in ten q^2 bins with the bin size of 0.3 GeV$^2/c^4$. To obtain the product $f_+^\pi(q_i^2)|V_{cd}|$ which will be used in our comprehensive analysis in Section 3, we extrapolate these measurements of form factors at the Belle experiment to the product $f_+^\pi(q_i^2)|V_{cd}|$ using the PDG'2006 value of $|V_{cd}| = 0.2271 \pm 0.0010$ [12] which was originally used in the Belle's paper published. Table 3 lists the form factors $f_+^\pi(q_i^2)$ measured at the Belle experiment and our translated products $f_+^\pi(q_i^2)|V_{cd}|$. These products will be used in our further analysis described in Section 3.

Table 3
Measurements of form factors $f_+^\pi(q_i^2)$ at the Belle experiment and the products $f_+^\pi(q_i^2)|V_{cd}|$ obtained from the Belle and BESIII experiments.

Experiment	q_i^2 (GeV/c^2)	$f_+^\pi(q_i^2)$	$f_+^\pi(q_i^2)\|V_{cd}\|$
Belle [11]	0.15	0.637 ± 0.053	0.145 ± 0.012
	0.45	0.797 ± 0.067	0.181 ± 0.015
	0.75	0.853 ± 0.077	0.194 ± 0.017
	1.05	0.830 ± 0.090	0.188 ± 0.020
	1.35	0.963 ± 0.107	0.219 ± 0.024
	1.65	0.940 ± 0.143	0.213 ± 0.033
	1.95	1.430 ± 0.190	0.325 ± 0.043
	2.25	1.760 ± 0.273	0.400 ± 0.062
	2.55	1.820 ± 0.447	0.413 ± 0.101
	2.85	2.157 ± 1.243	0.490 ± 0.282
BESIII [13]	0.0		0.1420 ± 0.0026

Recently, the BESIII Collaboration reported preliminary results of $D^0 \to \pi^- e^+ \nu_e$ decays obtained by analyzing 2.92 fb^{-1} data taken at 3.773 GeV. The BESIII Collaboration accumulated $(279.3 \pm 0.4) \times 10^4$ $\bar{D}^0$ tags from five hadronic decay modes. In this sample of $\bar{D}^0$ tags, they observed 6297 ± 87 signal events for $D^0 \to \pi^- e^+ \nu_e$ decays [13,14], and measured differential rates of $D^0 \to \pi^- e^+ \nu_e$ decays at fourteen q^2 bins from 0.0 to 3.0 GeV$^2/c^4$. By analyzing these differential decay rates the BESIII Collaboration measured a value of the product [13,14]

$$f_+^\pi(0)|V_{cd}| = 0.1420 \pm 0.0024 \pm 0.0010,$$

which is obtained from a fit to the data in the case of that the form factor is parameterized with three-parameters z-series expansion (see Section 3). The last row of Table 3 lists this $f_+^\pi(0)|V_{cd}|$, where the error is the combined statistical and systematic errors.

3. Analysis

To obtain the product of the semileptonic form factor at four-momentum transfer $q = 0$, $f_+^\pi(0)$, and the magnitude of CKM matrix element $|V_{cd}|$, we perform a comprehensive χ^2 fit to these experimental measurements of the partial decay rates and the products $f_+^\pi(q_i^2)|V_{cd}|$ listed in Tables 1, 2 and 3. The object function to be minimized in the fit is defined as

$$\chi^2 = \chi^2_R + \chi^2_P, \tag{3.1}$$

where χ^2_R is for both the decay rates extrapolated from measurements of decay branching fraction and the partial decay rates measured in different q^2 ranges, and χ^2_P corresponds to the products of $f_+^\pi(q_i^2)|V_{cd}|$ obtained from Belle's measurements of $f_+^\pi(q_i^2)$ and $f_+^\pi(0)|V_{cd}|$ measured at the BESIII experiment.

Taking into account the correlations between the measurements of the partial decay rates, the quantity χ^2_R is given by

$$\chi^2_R = \sum_{i=1}^{30}\sum_{j=1}^{30}(\Delta\Gamma_i^{ex} - \Delta\Gamma_i^{th})(\mathcal{C}_R^{-1})_{ij}(\Delta\Gamma_j^{ex} - \Delta\Gamma_j^{th}), \tag{3.2}$$

where $\Delta\Gamma^{ex}$ denotes the experimentally measured partial decay rate, $\Delta\Gamma^{th}$ is the theoretical expectation of the decay rate, and

$\mathcal{C}_R^{-1}$ is the inverse of the covariance matrix $\mathcal{C}_R$, which is a 30×30 matrix containing the correlations between the measured partial decay rates listed in Tables 1 and 2. The construction of C_R is discussed in Subsection 3.2. With the parametrization of the form factor, the theoretically predicted partial decay rate in a given q^2 bin is obtained by integrating Eq. (1.1) from the low boundary $q^2_{\rm low}$ to the up boundary $q^2_{\rm up}$ of the q^2 bin,

$$\Delta\Gamma^{\rm th} = \int_{q^2_{\rm low}}^{q^2_{\rm up}} X \frac{G_F^2}{24\pi^3}|V_{cd}|^2 \boldsymbol{p}^3 |f_+^\pi(q^2)|^2 dq^2. \tag{3.3}$$

In this analysis, we used several forms of the form-factor parameterizations which are discussed in Subsection 3.1.

The function $\chi^2_{\rm P}$ in Eq. (3.1) is defined as

$$\chi^2_{\rm P} = \sum_{i=1}^{11} \left(\frac{\tilde{f}_i^{\rm ex} - \tilde{f}_i^{\rm th}}{\sigma_i} \right)^2, \tag{3.4}$$

where $\tilde{f}_i^{\rm ex}$ is the measured product $f_+^\pi(q^2)|V_{cd}|$ at q_i^2 with the standard deviation σ_i, and $\tilde{f}_i^{\rm th}$ is the theoretical expectation of the product $f_+^\pi(q^2)|V_{cd}|$ at q_i^2.

3.1. Form-factor parameterizations

Several model dependent calculations of form factor are often used in analysis of experimental measurements of semileptonic D decays.

In general, the single pole model is the simplest approach to describe the q^2 dependent behavior of form factor. The single pole model is expressed as

$$f_+^\pi(q^2) = \frac{f_+^\pi(0)}{1 - q^2/m^2_{\rm pole}}, \tag{3.5}$$

where $f_+^\pi(0)$ is the value of form factor at $q^2 = 0$, $m_{\rm pole}$ is the pole mass which is predicted to be the mass of the D^{*+} meson for semileptonic $D \to \pi\ell^+\nu_\ell$ decays.

The so-called BK parameterization [15] is also widely used in Lattice QCD calculations and experimental studies of this decay. In the BK parameterization, the form factor of the semileptonic $D \to \pi\ell^+\nu_\ell$ decays is written as

$$f_+^\pi(q^2) = \frac{f_+^\pi(0)}{(1 - q^2/m^2_{D^{*+}})(1 - \alpha q^2/m^2_{D^{*+}})}, \tag{3.6}$$

where $m_{D^{*+}}$ is the mass of the D^{*+} meson, and α is a free parameter to be fitted. The value of α is assumed to be around 1.34 [16] for $D \to \pi\ell^+\nu_\ell$ in the BK parameterization.

The ISGW2 model [17] assumes

$$f_+^\pi(q^2) = f_+^\pi(q^2_{\rm max}) \left(1 + \frac{r^2}{12}(q^2_{\rm max} - q^2) \right)^{-2}, \tag{3.7}$$

where $q^2_{\rm max}$ is the kinematical limit of q^2, and r is the conventional radius of the meson. In this model, the prediction of r for $D \to \pi\ell^+\nu_\ell$ decays is 1.410 GeV^{-1} c^2 [16].

The most general parameterization of the form factor is the z-series expansion [18,19], which is based on analyticity and unitarity. In this parametrization, the variable q^2 is mapped to a new variable z through

$$z(q^2, t_0) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}} \tag{3.8}$$

with $t_\pm = (m_D \pm m_\pi)^2$ and $t_0 = t_+(1 - \sqrt{1 - t_-/t_+})$. The form factor is then expressed in terms of the new variable z as

$$f_+^\pi(q^2) = \frac{1}{P(q^2)\phi(q^2, t_0)} \sum_{k=0}^{\infty} a_k(t_0)[z(q^2, t_0)]^k, \tag{3.9}$$

where $P(q^2) = 1$ for $D \to \pi\ell^+\nu_\ell$, $\phi(q^2, t_0)$ is an arbitrary function, and $a_k(t_0)$ are real coefficients. In this analysis, the choice of $\phi(q^2, t_0)$ is taken to be

$$\phi(q^2, t_0) = \left(\frac{\pi m_c^2}{3} \right)^{\frac{1}{2}} \left(\frac{z(q^2, 0)}{-q^2} \right)^{\frac{5}{2}} \left(\frac{z(q^2, t_0)}{t_0 - q^2} \right)^{-\frac{1}{2}} \times \left(\frac{z(q^2, t_-)}{t_- - q^2} \right)^{-\frac{3}{4}} \frac{(t_+ - q^2)}{(t_+ - t_0)^{\frac{1}{4}}}, \tag{3.10}$$

where m_c is the mass of charm quark, which is taken to be 1.2 GeV/c^2. In practical use, one usually make a truncation on the above z-series. Actually, it is found that the current experimental data can be adequately described by only the first three terms in Eq. (3.9).

In this analysis we will fit the measured decay rates with the three-parameter z-series expansion. After optimizing the form factor parameters, we obtain the form for the three-parameter z-series expansion:

$$f_+^\pi(q^2) = \frac{f_+^\pi(0)P(0)\phi(0, t_0)(1 + \sum_{k=1}^{2} r_k[z(q^2, t_0)]^k)}{P(q^2)\phi(q^2, t_0)(1 + \sum_{k=1}^{2} r_k[z(0, t_0)]^k)}, \tag{3.11}$$

where $r_k \equiv a_k(t_0)/a_0(t_0)$ $(k = 1, 2)$.

3.2. Covariance matrix

It's a little complicated to compute the covariances of these 30 measurements of partial decay rates in different q^2 ranges and at different experiments. To be clear, we separate the correlations among these $\Delta\Gamma$ measurements into two case: the one associated with the experimental status of each independent experiment, and the other related to the external inputs of parameters such as the lifetime of the D meson.

The statistical uncertainties in the $\Delta\Gamma$ measurements from the same experiment are correlated to some extent, while these are independent for the measurements from different experiments. The systematic uncertainties from tracking, particle identification, etc. are usually independent between different experiments. In this analysis, we treat the systematic uncertainties except the ones from D lifetimes and branching fractions for $D \to Ke^+\nu_e$ as fully uncorrelated between the measurements performed at different experiments. We consider these below:

- The covariances of the $\Delta\Gamma$ measured at the same experiment are computed using the statistical errors, the systematic errors, and the correlation coefficients, which are presented in their original papers published. The statistical and systematic correlation matrices for the partial branching fractions of $D^0 \to \pi^-e^+\nu_e$ decay in ten q^2 bins measured at the BaBar experiment is listed in Table XV in Ref. [7]. The statistical and systematic correlation matrices for the partial decay rates of $D^0 \to \pi^-e^+\nu_e$ and $D^+ \to \pi^0e^+\nu_e$ decays measured at the CLEO-c experiment can be found in Tables XIV and XV in Ref. [10].
- For the measurements of $D^0 \to \pi^-e^+\nu_e$ decay, the lifetime of D^0 meson is used to obtain the partial decay rates in particular q^2 ranges. The systematic uncertainties due to imperfect knowledge of D^0 lifetime are fully correlated among all these

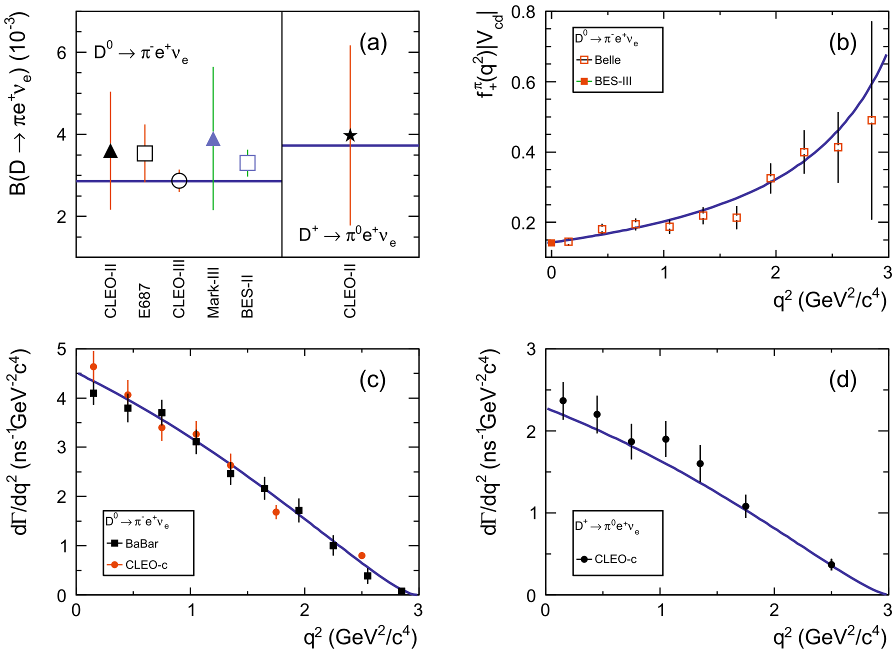

Fig. 1. (a) Comparisons of branching fraction measurements for $D \to \pi e^+ \nu_e$ decays, (b) the product $f_+^\pi(q^2)|V_{cd}|$ measured at the Belle and BESIII experiments, the differential decay rates as function of q^2 for (c) $D^0 \to \pi^- e^+ \nu_e$ measured at the BaBar and CLEO-c experiments, and (d) for $D^+ \to \pi^0 e^+ \nu_e$ measured at the CLEO-c experiment. The blue lines show the fit to these measurements using the z-series expansion for the form factor. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Table 4
Fitted parameters corresponding to different form-factor parameterizations and χ^2/d.o.f. of the fit.

Parameterization	$f_+^\pi(0)\vert V_{cd}\vert$	Shape parameters	χ^2/d.o.f.
Single pole	0.1447 ± 0.0015	$m_{\rm pole} = (1.905 \pm 0.016)$ GeV/c^2	27.3/39
BK	0.1429 ± 0.0017	$\alpha = 0.252 \pm 0.044$	25.5/39
ISGW2	0.1417 ± 0.0016	$r = (2.01 \pm 0.05)$ GeV$^{-1}c^2$	28.6/39
z-series expansion	0.1428 ± 0.0019	$r_1 = -1.95 \pm 0.33$	25.0/38
		$r_2 = -0.11 \pm 1.84$	

measurements of the partial rates of $D^0 \to \pi^- e^+ \nu_e$ decay. Similarly, the systematic uncertainties related to D^+ lifetime are fully correlated among all of the $\Delta\Gamma$ measurements for $D^+ \to \pi^0 e^+ \nu_e$ decay.

- An additional systematic uncertainty from the branching fraction for $D^0 \to K^- e^+ \nu_e$ decay is fully correlated between these relative measurements of $D^0 \to \pi^- e^+ \nu_e$ decay at the CLEO-II, E687 and CLEO-III experiments. Since we only use one relative measurement of $D^+ \to \pi^0 e^+ \nu_e$ decay which is from the CLEO-II experiment, there are no correlations due to the branching fraction for $D^+ \to \bar{K}^0 e^+ \nu_e$ between this measurement and other measurements.

With these considerations mentioned above, we then construct a 30×30 covariance matrix $\mathcal{C}_{\rm R}$ which is necessary in the form factor fit. The covariance matrix $\mathcal{C}_{\rm R}$ can be obtained with the correlation matrix given in Appendix B.

3.3. Fits to experimental data

Four fits are applied to the experimental data with the form factor hypothesis of single pole model, BK model, ISGW2 model and z-series expansion. The fit to experimental data returns the normalization $f_+^\pi(0)|V_{cd}|$ and the shape parameters of the form factor which govern the behavior of form factor in high q^2 range.

The numerical results of the fit corresponding to each form of the form-factor parameterization are summarized in Table 4, where the errors are from the fits. As an example, Fig. 1 presents the result of the fit in the case of using the form-factor parameterization of z-series expansion. In Fig. 1(a), we compared the measured branching fractions of $D \to \pi e^+ \nu_e$ decays from different experiments. Fig. 1(b) depicts the measurements of $f_+^\pi(q^2)|V_{cd}|$ at different q^2 from the Belle and BESIII experiments. Fig. 1(c) and (d) show the measured differential decay rates for $D^0 \to \pi^- e^+ \nu_e$ and $D^+ \to \pi^0 e^+ \nu_e$, respectively. In these figures, the lines show the best fit to these measurements of $D \to \pi e^+ \nu_e$ decays.

To check the fit quality and also the isospin invariance, the experimentally measured decay branching fractions and/or partial rates are mapped into the product $f_+^\pi(q_i^2)|V_{cd}|$ via

$$f_+^\pi(0)|V_{cd}| = \sqrt{\frac{B}{\tau_D}\frac{1}{XN}} \tag{3.12}$$

and

$$f_+^\pi(q_i^2)|V_{cd}| = \sqrt{\left(\frac{d\Gamma}{dq^2}\right)_i \frac{24\pi^3}{XG_F^2 \boldsymbol{p}_i'^3}}, \tag{3.13}$$

where B denotes the measured branching fraction, the differential decay rate $(d\Gamma/dq^2)_i$ is obtained by dividing measured decay rate in q^2 bin i by the corresponding bin size. The normalization N is given by

$$N = \frac{G_F^2}{24\pi^3|f_+^\pi(0)|^2} \int_0^{q^2_{\rm max}} \boldsymbol{p}^3 |f_+^\pi(q^2)|^2 dq^2. \tag{3.14}$$

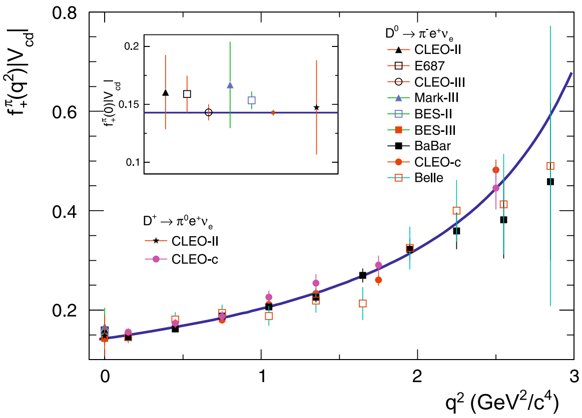

Fig. 2. The product $f_+^\pi(q^2)|V_{cd}|$ measured at different experiments as a function of q^2. The blue curve represents the z-series expansion fit to these $f_+^\pi(q^2)|V_{cd}|$. The insert plot shows the comparison of the products $f_+^\pi(0)|V_{cd}|$ which are obtained using the branching fractions measured at different experiments. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

The effective $\boldsymbol{p}_i'^3$ in q^2 bin i is given by

$$\boldsymbol{p}_i'^3 = \frac{\int_{q^2_{\rm low}}^{q^2_{\rm up}} \boldsymbol{p}^3 |f_+^\pi(q^2)|^2 dq^2}{|f_+^\pi(q_i^2)|^2 (q^2_{\rm up} - q^2_{\rm low})}. \quad (3.15)$$

To calculate the integral in Eqs. (3.14) and (3.15), we use the shape parameters of the form factor, which is obtained from the z-series expansion fit to the data.

Fig. 2 shows the product $f_+^\pi(q^2)|V_{cd}|$ as a function of q^2, where the blue curve corresponds to the best z-series expansion fit to the experimental data. In this fit, seven measurements of $f_+^\pi(0)|V_{cd}|$ locate at $q^2 = 0$, which overlap each other. To be clear, these $f_+^\pi(0)|V_{cd}|$ translated from the decay branching fractions measured at different experiments are also displayed in the insert plot in Fig. 2.

4. Results

In this analysis we choose the results from the fit using z-series expansion as our primary results and use this extracted $f_+^\pi(0)|V_{cd}|$ from the fit to determine the form factor $f_+^\pi(0)$ or extract the magnitude of the CKM matrix element V_{cd}. To be conservative, we take the spread of the values of the $f_+^\pi(0)|V_{cd}|$ from these four different fits as the systematic uncertainty of the extracted $f_+^\pi(0)|V_{cd}|$. In this case we obtain

$$f_+^\pi(0)|V_{cd}| = 0.1428 \pm 0.0019^{+0.0019}_{-0.0011}. \quad (4.1)$$

4.1. Form factor $f_+^\pi(0)$

Dividing the value of $f_+^\pi(0)|V_{cd}|$ shown in Eq. (4.1) by the $|V_{cd}| = 0.2253 \pm 0.0008$ (assuming $|V_{cd}| = |V_{us}| = 0.2253 \pm 0.0008$ directly extracted from semileptonic K decays and leptonic K^+ decays [2]) yields the form factor

$$f_+^\pi(0) = 0.634^{+0.012}_{-0.010} \pm 0.002, \quad (4.2)$$

where the first uncertainty is from the combined uncertainties in measured $f_+^\pi(0)|V_{cd}|$ given in Eq. (4.1), and the second is due to the uncertainty in the $|V_{us}|$. The result of the form factor determined in this analysis is compared with the theoretical calculations of the form factor from the LQCD [20–22] and from QCD

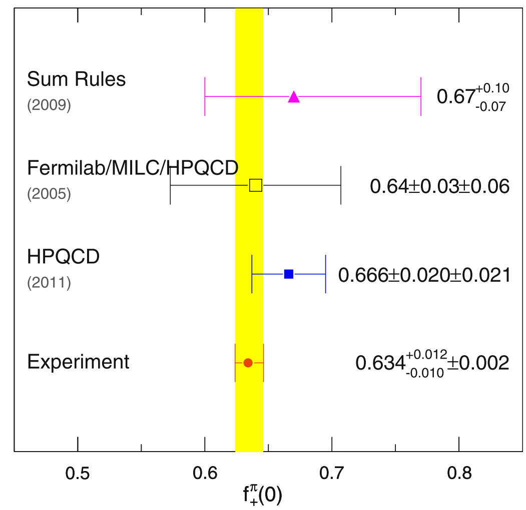

Fig. 3. Comparison of our determined form factor from experimental measurements with the theoretical calculations of the form factor.

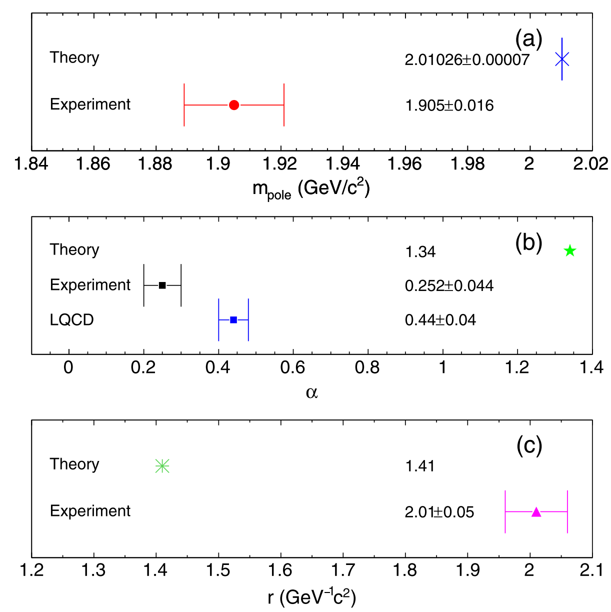

Fig. 4. Comparisons of the form factor parameters determined from experimental measurements and the theoretical expectations: (a) the pole mass $m_{\rm ploe}$ in single pole model, (b) α in the BK model, and (c) r in the ISGW2 model.

light-cone sum rules [23] in Fig. 3. Our result of the form factor determined by analyzing all existing experimental measurements of these decays and considering uncertainties in different form-factor parametrizations in the fits is consistent within error with these values predicted by theory, but is with higher precision than the most accurate value of the form factor, $f_+^\pi(0)_{\rm LQCD} = 0.666 \pm 0.020 \pm 0.021$ calculated in LQCD [21], by a factor of 2.3.

4.2. Parameters of form factor

When these shape parameters of the form-factor parameterization are left free in the fit, the form-factor parametrizations of the single pole model, BK model, the ISGW2 model, and the z-series expansion model are all capable of describing the experimental data with almost identical χ^2 probability. However, for the physical interpretation of the shape parameters in the single pole model, BK model, the ISGW2 model, the values of the parameters obtained from the fits are largely deviated from those expected values by these models. This indicates that the experimental data do not support the physical interpretation of the shape parameters in these parametrizations. Fig. 4(a), (b) and (c) show the comparisons between the measured values and the theoretically expected values for the pole mass $m_{\rm pole}$ in single pole model, α in BK model, and r in ISGW2 model, where the theoretically expected values for α and r are quoted from Ref. [16]. These measured parameters do not agree with the values predicted by these form factor models.

Our determined $\alpha = 0.252 \pm 0.044$ from this comprehensive analysis is 3.2σ smaller than $\alpha^{\rm LQCD} = 0.44 \pm 0.04$ calculated in LQCD [22]. Fig. 4(b) shows this comparison.

4.3. Check LQCD approach to charm sector

To check the consistency of the SM and test the LQCD approach to the charm sector, we compare the ratio of the semileptonic D decay form factor $f_+^\pi(0)$ and D^+ decay constant f_{D^+} from both the measurements and the LQCD calculations of these quantities. In fact, the ratio of $f_+^\pi(0)/f_{D^+}$ implicitly contains some possible NP effects involved in either the semileptonic D decays or leptonic D^+ decays, or both of them. So comparing this ratio measured from the experiments to that calculated in LQCD would also present some information for understanding whether or not some NP effect involve in these semileptonic D decays and leptonic D^+ decays.

With the most accurate $f_+^\pi(0)_{\rm LQCD} = 0.666 \pm 0.020 \pm 0.021$ calculated in LQCD [21] and most accurate $f_{D^+} = (212.6 \pm 0.4^{+1.0}_{-1.2})$ MeV calculated in LQCD [24], we obtain

$$[f_+^\pi(0)/f_{D^+}]^{\rm LQCD} = (3.13 \pm 0.14)\ {\rm GeV}^{-1}. \tag{4.3}$$

From our determined $f_+^\pi(0)$ given in Eq. (4.2) and f_{D^+} given in Eq. (A.5) (see Appendix A), we find

$$[f_+^\pi(0)/f_{D^+}]^{\rm exp} = (3.11 \pm 0.09)\ {\rm GeV}^{-1}. \tag{4.4}$$

To get the corresponding dimensionless quantities, we multiply the mass of D^+ meson, $m_{D^+} = 1869.61 \pm 0.10$ MeV [2], to both Eq. (4.3) and Eq. (4.4), and obtain

$$[m_{D^+} f_+^\pi(0)/f_{D^+}]^{\rm LQCD} = 5.85 \pm 0.26, \tag{4.5}$$

and

$$[m_{D^+} f_+^\pi(0)/f_{D^+}]^{\rm exp} = 5.81 \pm 0.17. \tag{4.6}$$

These two ratios from LQCD calculations and experimental measurements are in excellent agreement within error.

4.4. CKM matrix element $|V_{cd}|$

Using the product $f_+^\pi(0)|V_{cd}| = 0.1428 \pm 0.0019^{+0.0019}_{-0.0011}$ obtained from the comprehensive z-series expansion fit and considering uncertainties in measured value of $f_+^\pi(0)|V_{cd}|$ due to different form-factor parametrizations in the fits in conjunction with the form factor $f_+^\pi(0)_{\rm LQCD} = 0.666 \pm 0.020 \pm 0.021$ [21] calculated in LQCD for the $D \to \pi$ transition, we extract the magnitude of the CKM matrix element V_{cd} from all existing measurements of semileptonic D decays to be

$$|V_{cd}|^{D\to\pi e^+\nu_e} = 0.2144^{+0.0040}_{-0.0033} \pm 0.0093, \tag{4.7}$$

where the first error is from both the uncertainties in experimental measurements and the uncertainties in the value of $f_+^\pi(0)|V_{cd}|$ due to different form-factor parametrizations, and the second uncertainty corresponds to the accuracy of the form factor $f_+^\pi(0)$ calculated in LQCD. The precision of this extracted $|V_{cd}|^{D\to\pi e^+\nu_e}$ is 1.5 times better than the PDG'2014 $|V_{cd}|^{D\to\pi e^+\nu_e}_{\rm PDG'2014} = 0.220 \pm 0.006 \pm 0.010$ [2] extracted from the average of the CLEO-c [10] and Belle [11] measurements of $D \to \pi \ell^+ \nu_\ell$ decays in conjunction with $f_+^\pi(0)_{\rm LQCD} = 0.666 \pm 0.020 \pm 0.021$ [21]. This big progress in improvement of the experimental accuracy of $|V_{cd}|^{D\to\pi e^+\nu_e}$ is mainly due to the recent BESIII measurement [13,14] and BaBar measurement [7] of $D^0 \to \pi^- e^+ \nu_e$ decay. This value of $|V_{cd}|$ can be compared to another value

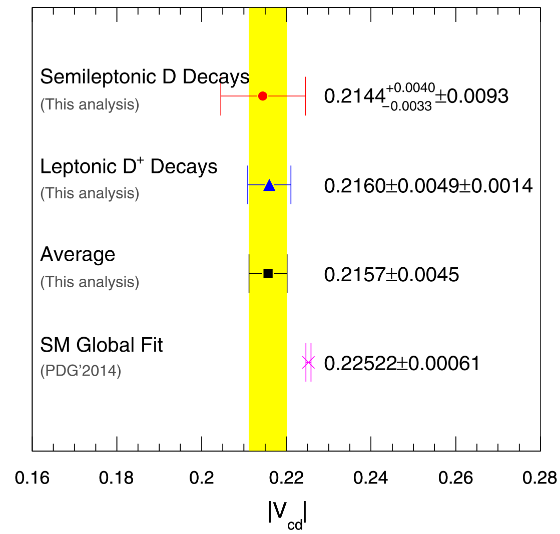

Fig. 5. Comparison of $|V_{cd}|$ extracted from semileptonic D decays in this analysis with the one extracted from leptonic D^+ decays and along with the one from the global SM fit.

$$|V_{cd}|^{D^+\to\mu^+\nu_\mu} = 0.2160 \pm 0.0049 \pm 0.0014 \tag{4.8}$$

extracted from both the BESIII and CLEO-c's measurements of leptonic D^+ decays (see Appendix A).

In a generic scenario of NP, the semileptonic D decay and leptonic D^+ decay rates are modified differently. So the ratio of $|V_{cd}|^{D\to\pi e^+\nu_e}$ and $|V_{cd}|^{D^+\to\mu^+\nu_\mu}$ could provide a valuable tool for understanding whether or not some NP effects involve in these decays. From these two extracted values of $|V_{cd}|$, we find

$$\frac{|V_{cd}|^{D\to\pi e^+\nu_e}}{|V_{cd}|^{D^+\to\mu^+\nu_\mu}} = 0.993^{+0.052}_{-0.051}, \tag{4.9}$$

where the error is mainly dominated by the uncertainty of $f_+^\pi(0)$ calculated in LQCD. To improve the experimental sensitivity of NP effect which may involve in these two kinds of decays, it is requited not only to improve experimental precision on measurements of these decays, but also to improve precision of $f_+^\pi(0)$ calculated in LQCD. At present, at 5% level of accuracy no such NP effects involved in these two kinds of decays is observed at this stage.

As no evidence of such NP effect is observed in these two kinds of decays in the generic scenario of NP point of view, we can safely combined these two values of $|V_{cd}|$ given in Eq. (4.7) and Eq. (4.8) to get the average of the magnitude of the CKM matrix element V_{cd}. Averaging these two values of $|V_{cd}|$ extracted from semileptonic D and leptonic D^+ decays yields

$$|V_{cd}| = 0.2157 \pm 0.0045. \tag{4.10}$$

Fig. 5 shows a comparison of the value of $|V_{cd}|$ which is determined with the $|V_{cd}|^{D\to\pi e^+\nu_e}$ obtained in this analysis together with the $|V_{cd}|^{D^+\to\mu^+\nu_\mu}$ extracted from leptonic D^+ decays, and the value from the global SM fit [2].

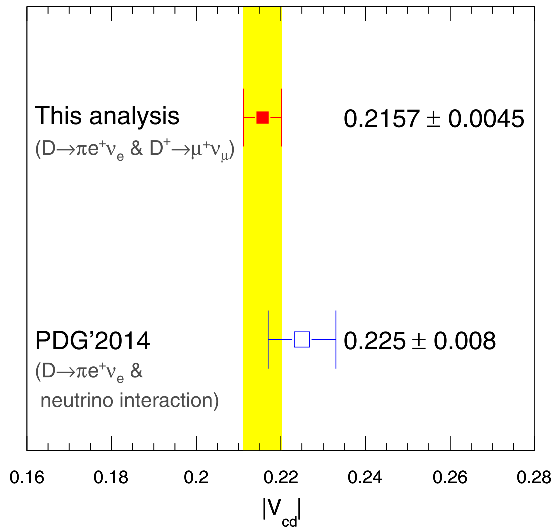

Fig. 6. Comparison of $|V_{cd}|$ extracted from semileptonic D decays and leptonic D^+ decays in this analysis along with the PDG'2014 value.

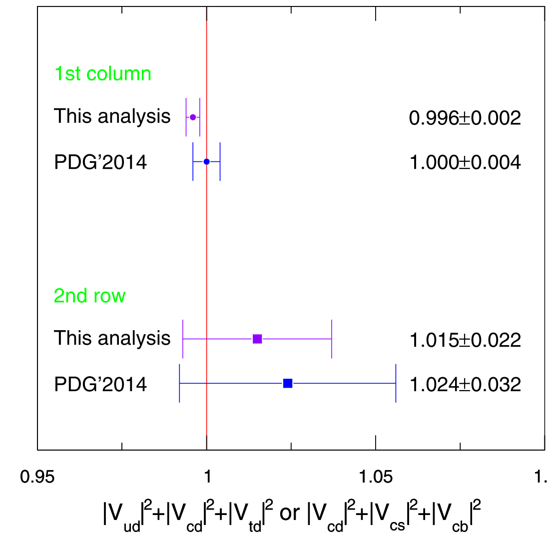

Fig. 7. Unitarity checks for the first column and second row of the CKM matrix.

Fig. 6 shows a comparison of our extracted $|V_{cd}|$ from all existing measurements of $D \to \pi e^+ \nu_e$ and from both the BESIII and CLEO-c's measurements of $D^+ \to \mu^+ \nu_\mu$ decays along with the PDG'2014 value of the $|V_{cd}|$ determined with CLEO-c and Belle's measurements of $D \to \pi \ell^+ \nu_\ell$ decays and neutrino interactions [2]. Our extracted $|V_{cd}| = 0.2157 \pm 0.0045$ is in good agreement within error with the PDG'2014 value $|V_{cd}|_{\text{PDG'2014}} = 0.225 \pm 0.008$, but improves the precision of the PDG'2014 value by over 70%.

The average value of $|V_{cd}| = 0.2157 \pm 0.0045$ deviating from the value of $|V_{cd}| = 0.22522 \pm 0.00061$ obtained from SM global fit by 2.1 standard deviations may arise from three possibilities: (1) some NP effects involved in both the semileptonic D decays and leptonic D^+ decays, which reduce both of these decay rates; (2) overestimated both decay form factor $f_+^\pi(0)$ and decay constant f_{D^+} in LQCD; (3) some NP effects involved in other decays for which related measurements are used in the SM global fit. Any of these would modify these decay rates resulting in shift of the average of $|V_{cd}|$ extracted from both the semileptonic D and leptonic D^+ decays, or shift of the $|V_{cd}|$ obtained from the SM global fit.

4.5. Unitarity checks

Using the newly extracted $|V_{cd}| = 0.2157 \pm 0.0045$, the PDG'2014 values $|V_{ud}| = 0.97425 \pm 0.00022$ and $|V_{td}| = (8.4 \pm 0.6) \times 10^{-3}$ [2], we check the first column unitarity of the CKM matrix, which is

$$|V_{ud}|^2 + |V_{cd}|^2 + |V_{td}|^2 = 0.996 \pm 0.002. \tag{4.11}$$

Using these newly extracted $|V_{cd}| = 0.2157 \pm 0.0045$, the value $|V_{cs}| = 0.983 \pm 0.011$ which is recently extracted from semileptonic D decays and leptonic D_s^+ decays [25], and the PDG'2014 value $|V_{cb}| = (41.1 \pm 1.3) \times 10^{-3}$ [2], we find

$$|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 = 1.015 \pm 0.022 \tag{4.12}$$

for the second row of the CKM matrix. The unitarity check results for the first column and the second row of the CKM matrix are shown in Fig. 7 together with the unitarity checks given in PDG'2014 [2]. The newly determined $|V_{cd}|$ and $|V_{cs}|$ give more stringent checks of the CKM matrix unitarity compared to those in PDG'2014.

The sum of the squared matrix element in the first column of the CKM matrix deviates from the unitarity by

$$|V_{ud}|^2 + |V_{cd}|^2 + |V_{td}|^2 - 1 = -0.004 \pm 0.002,$$

which is 2σ deviations from the unitarity.

5. Summary

By globally analyzing all existing branching fractions of the $D \to \pi e^+ \nu_e$ decays measured at earlier experiments and products $f_+^\pi(q^2)|V_{cd}|$ measured at the Belle and the BESIII experiments as well as the partial decay rates in q^2 bins measured at the BaBar and CLEO-c experiments together, we obtain the most precise product

$$f_+^\pi(0)|V_{cd}| = 0.1428 \pm 0.0019^{+0.0019}_{-0.0010}.$$

From this product we determined the form factor

$$f_+^\pi(0) = 0.634^{+0.012}_{-0.010} \pm 0.002,$$

which is in good agreement within error with LQCD calculations of the form factor, but with more precision than the most accurate LQCD calculation of the form factor by 2.3 factors.

We determine the dimensionless ratio of the product of D^+ mass and semileptonic form factor $f_+^\pi(0)$ over D^+ decay constant f_{D^+} to be

$$[m_{D^+} f_+^\pi(0)/f_{D^+}]^{\text{exp}} = 5.81 \pm 0.17$$

from all existing experimental measurements, which is in excellent agreement within error with the ratio

$$[m_{D^+} f_+^\pi(0)/f_{D^+}]^{\text{LQCD}} = 5.85 \pm 0.26$$

from LQCD calculations.

Alternately, with the most precise $D \to \pi e^+ \nu_e$ decay form factor calculated in LQCD, we obtain

$$|V_{cd}|^{D \to \pi e^+ \nu_e} = 0.2144^{+0.0040}_{-0.0033} \pm 0.0093,$$

where the error is still dominated by the uncertainties in LQCD calculation of the semileptonic $D \to \pi$ form factor. This extracted $|V_{cd}|$ is consistent within 1.1σ with $|V_{cd}| = 0.22522 \pm 0.00061$ from the global SM fit. Combining this $|V_{cd}|^{D \to \pi e^+ \nu_e}$ together with

$$|V_{cd}|^{D^+ \to \mu^+ \nu_\mu} = 0.2160 \pm 0.0049 \pm 0.0014$$

extracted from leptonic D^+ decays together, we find

$|V_{cd}| = 0.2157 \pm 0.0045$.

This newly extracted $|V_{cd}|$ improves the accuracy of the PDG'2014 determination of $|V_{cd}|_{\text{PDG'2014}} = 0.225 \pm 0.008$ by over 70%, and is the most precisely extracted $|V_{cd}|$ from all existing measurements of semileptonic D decays and from both the BESIII and CLEO-c's measurements of leptonic D^+ decays up to date. This newly extracted $|V_{cd}|$ deviates from the $|V_{cd}| = 0.22522 \pm 0.00061$ obtained from SM global fit by 2.1 standard deviations.

Combining the most precise $|V_{cd}|$ extracted in this work together with other updated $|V_{ud}|$ and $|V_{td}|$ given in PDG'2014, we find that the sum of the squared CKM matrix element in the first column deviates from unitarity by 2σ.

Acknowledgements

This work is supported in part by the Ministry of Science and Technology of the People's Republic of China under Contract No. 2009CB825204; National Natural Science Foundation of China (NSFC) under Contacts No. 10935007 and No. 11305180.

Appendix A. Extraction of $|V_{cd}|$ from leptonic D^+ decays

In this appendix, we present the determination of $|V_{cd}|$ by analyzing the existing measurements of leptonic $D^+ \to \mu^+\nu_\mu$ decays.

In SM of particle physics, the branching fraction for $D^+ \to \mu^+\nu_\mu$ decay is given by

$$B(D^+ \to \mu^+\nu_\mu) = \frac{G_F^2}{8\pi}\tau_{D^+} m_\mu^2 m_{D^+}\left(1 - \frac{m_\mu^2}{m_{D^+}^2}\right)^2 \times f_{D^+}^2 |V_{cd}|^2, \tag{A.1}$$

where τ_{D^+} is the lifetime of D^+ meson, m_μ is the mass of muon and m_{D^+} is the mass of D^+ meson. The parameter f_{D^+} is the decay constant, which is associated with the strong interaction effects between the two initial-state quarks.

In 2008, the CLEO-c Collaboration accumulated $460\,055 \pm 787$ D^- tags by analyzing 818 pb^{-1} data taken at 3.773 GeV and selecting D^- mesons from 6 hadronic decay modes of the D^- meson. They observed 149.7 ± 12.0 signal events for $D^+ \to \mu^+\nu_\mu$ decays in the system recoiling against these D^- tags, and measured the branching fraction $B(D^+ \to \mu^+\nu_\mu) = (3.82 \pm 0.32 \pm 0.09) \times 10^{-4}$ [26].

In 2014, the BESIII Collaboration investigated the $D^+ \to \mu^+\nu_\mu$ decays by analyzing 2.92 fb^{-1} data taken at 3.773 GeV. From 9 hadronic decay modes of D^- meson, the BESIII Collaboration accumulated $1\,703\,054 \pm 3405$ D^- tags. In this D^- tag sample they observed 409.0 ± 21.2 signal events for $D^+ \to \mu^+\nu_\mu$ decays and measured the branching fraction $B(D^+ \to \mu^+\nu_\mu) = (3.71 \pm 0.19 \pm 0.06) \times 10^{-4}$ [27].

Averaging these two branching fractions, we obtain

$$B(D^+ \to \mu^+\nu_\mu) = (3.74 \pm 0.17) \times 10^{-4}, \tag{A.2}$$

where the error is the combined statistical and systematic errors together.

Inserting the values $m_\mu = (105.6583715 \pm 0.0000035)$ MeV/c^2, $m_{D^+} = (1869.61 \pm 0.10)$ MeV/c^2, and $\tau_{D^+} = (1040 \pm 7) \times 10^{-15}$ s, from PDG'2014 [2] and the average value of branching fraction given in Eq. (A.2) into Eq. (A.1), the product of the decay constant and the magnitude of CKM matrix element V_{cd} is determined to be

$$f_{D^+}|V_{cd}| = (45.92 \pm 1.04 \pm 0.15)\ \text{MeV}, \tag{A.3}$$

where the first error is from the statistical and systematic uncertainties in the measured branching fractions, and the second error is due to the uncertainties in the masses of muon and D^+ meson, the lifetime of D^+ meson.

Dividing the product $f_{D^+}|V_{cd}|$ by the value $f_{D^+} = (212.6 \pm 0.4^{+1.0}_{-1.2})$ MeV which is the newest and most precise value of decay constant calculated in LQCD with $N_f = 2+1+1$ quark flavors [24], we obtain

$$|V_{cd}|^{D^+\to\mu^+\nu_\mu} = 0.2160 \pm 0.0049 \pm 0.0014, \tag{A.4}$$

where the first error is from the statistical and systematic uncertainties in the measured branching fractions, and the second error is mainly due to the uncertainties in the lifetime of D^+ meson, and the f_{D^+} calculated in LQCD.

Alternatively, using the most precise determination of $|V_{us}| = 0.2253 \pm 0.0008$ from semileptonic and leptonic K decays and the relation $|V_{cd}| = |V_{us}|$ from unitarity of the CKM matrix [2], we determine

$$f_{D^+} = (203.8 \pm 4.6 \pm 1.0)\ \text{MeV}, \tag{A.5}$$

which is the most precisely determined D^+ decay constant based on the branching fractions for $D^+ \to \mu^+\nu_\mu$ decays measured at both the BESIII and CLEO-c experiments.

Appendix B. Correlation matrix

The correlation matrix for the 30 measurements of the decay rates is shown in Table 5.

Table 5

The correlation matrix for the measurements of partial decay rates given in Table 1 and Table 2. The indices in the table are matched to the indices in the second column of Table 1 and Table 2. This matrix is symmetric relative to the diagonal. The element in ith row and jth column gives the correlation coefficient (ρ_{ij}) of the ith and jth measurements of the decay rates. The covariance matrix element is then given by $(\mathcal{C}_R)_{ij} = \rho_{ij}\sigma_i\sigma_j$, where σ_i is the error of the partial decay rate measurement.

ρ_{ij}	$\Delta\Gamma_1$	$\Delta\Gamma_2$	$\Delta\Gamma_3$	$\Delta\Gamma_4$	$\Delta\Gamma_5$	$\Delta\Gamma_6$	$\Delta\Gamma_7$	$\Delta\Gamma_8$	$\Delta\Gamma_9$	$\Delta\Gamma_{10}$	$\Delta\Gamma_{11}$	$\Delta\Gamma_{12}$	$\Delta\Gamma_{13}$	$\Delta\Gamma_{14}$	$\Delta\Gamma_{15}$
$\Delta\Gamma_1$	1.000														
$\Delta\Gamma_2$	0.003	1.000													
$\Delta\Gamma_3$	0.006	0.011	1.000												
$\Delta\Gamma_4$	0.001	0.001	0.002	1.000											
$\Delta\Gamma_5$	0.000	0.001	0.002	−0.090	1.000										
$\Delta\Gamma_6$	0.000	0.001	0.002	0.030	0.060	1.000									
$\Delta\Gamma_7$	0.000	0.001	0.002	0.179	−0.128	0.007	1.000								
$\Delta\Gamma_8$	0.000	0.001	0.001	0.118	0.156	−0.090	0.092	1.000							
$\Delta\Gamma_9$	0.000	0.001	0.001	0.068	0.121	0.137	−0.124	0.170	1.000						
$\Delta\Gamma_{10}$	0.000	0.000	0.001	0.020	0.045	0.124	0.053	−0.064	0.416	1.000					
$\Delta\Gamma_{11}$	0.000	0.000	0.001	0.010	0.051	0.094	0.074	0.060	0.109	0.604	1.000				
$\Delta\Gamma_{12}$	0.000	0.000	0.000	0.035	0.077	0.078	0.050	0.131	0.051	0.081	0.568	1.000			
$\Delta\Gamma_{13}$	0.000	0.000	0.000	0.041	0.082	0.067	0.031	0.126	0.037	−0.071	0.303	0.924	1.000		
$\Delta\Gamma_{14}$	0.000	0.000	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	
$\Delta\Gamma_{15}$	0.000	0.000	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000

(continued on next page)

Table 5 (*continued*)

ρ_{ij}	$\Delta\Gamma_1$	$\Delta\Gamma_2$	$\Delta\Gamma_3$	$\Delta\Gamma_4$	$\Delta\Gamma_5$	$\Delta\Gamma_6$	$\Delta\Gamma_7$	$\Delta\Gamma_8$	$\Delta\Gamma_9$	$\Delta\Gamma_{10}$	$\Delta\Gamma_{11}$	$\Delta\Gamma_{12}$	$\Delta\Gamma_{13}$	$\Delta\Gamma_{14}$	$\Delta\Gamma_{15}$
$\Delta\Gamma_{16}$	0.000	0.001	0.002	0.003	0.002	0.003	0.002	0.002	0.002	0.001	0.001	0.000	0.000	0.000	0.000
$\Delta\Gamma_{17}$	0.000	0.001	0.002	0.003	0.002	0.003	0.002	0.002	0.002	0.001	0.001	0.000	0.000	0.000	0.000
$\Delta\Gamma_{18}$	0.000	0.001	0.002	0.003	0.002	0.002	0.002	0.002	0.002	0.001	0.001	0.000	0.000	0.000	0.000
$\Delta\Gamma_{19}$	0.000	0.001	0.002	0.003	0.002	0.002	0.002	0.002	0.002	0.001	0.001	0.000	0.000	0.000	0.000
$\Delta\Gamma_{20}$	0.000	0.001	0.002	0.003	0.002	0.002	0.002	0.002	0.001	0.001	0.001	0.000	0.000	0.000	0.000
$\Delta\Gamma_{21}$	0.000	0.001	0.002	0.003	0.002	0.002	0.002	0.002	0.001	0.001	0.001	0.000	0.000	0.000	0.000
$\Delta\Gamma_{22}$	0.000	0.001	0.002	0.003	0.002	0.002	0.002	0.002	0.001	0.001	0.001	0.000	0.000	0.000	0.000
$\Delta\Gamma_{23}$	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
$\Delta\Gamma_{24}$	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
$\Delta\Gamma_{25}$	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
$\Delta\Gamma_{26}$	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
$\Delta\Gamma_{27}$	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
$\Delta\Gamma_{28}$	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
$\Delta\Gamma_{29}$	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
$\Delta\Gamma_{30}$	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

ρ_{ij}	$\Delta\Gamma_{16}$	$\Delta\Gamma_{17}$	$\Delta\Gamma_{18}$	$\Delta\Gamma_{19}$	$\Delta\Gamma_{20}$	$\Delta\Gamma_{21}$	$\Delta\Gamma_{22}$	$\Delta\Gamma_{23}$	$\Delta\Gamma_{24}$	$\Delta\Gamma_{25}$	$\Delta\Gamma_{26}$	$\Delta\Gamma_{27}$	$\Delta\Gamma_{28}$	$\Delta\Gamma_{29}$	$\Delta\Gamma_{30}$
$\Delta\Gamma_{16}$	1.000														
$\Delta\Gamma_{17}$	−0.026	1.000													
$\Delta\Gamma_{18}$	0.020	−0.035	1.000												
$\Delta\Gamma_{19}$	0.020	0.023	−0.035	1.000											
$\Delta\Gamma_{20}$	0.024	0.018	0.020	−0.042	1.000										
$\Delta\Gamma_{21}$	0.011	0.020	0.022	0.020	−0.030	1.000									
$\Delta\Gamma_{22}$	0.013	0.018	0.020	0.018	0.014	−0.010	1.000								
$\Delta\Gamma_{23}$	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000							
$\Delta\Gamma_{24}$	0.014	0.013	0.012	0.012	0.010	0.010	0.009	0.001	1.000						
$\Delta\Gamma_{25}$	0.013	0.012	0.011	0.011	0.010	0.009	0.008	0.001	−0.041	1.000					
$\Delta\Gamma_{26}$	0.012	0.011	0.010	0.010	0.009	0.009	0.007	0.001	0.022	−0.095	1.000				
$\Delta\Gamma_{27}$	0.011	0.011	0.010	0.010	0.009	0.009	0.008	0.001	0.043	0.041	−0.128	1.000			
$\Delta\Gamma_{28}$	0.009	0.009	0.008	0.008	0.007	0.007	0.007	0.001	0.062	0.034	0.068	−0.104	1.000		
$\Delta\Gamma_{29}$	0.009	0.009	0.008	0.008	0.007	0.008	0.007	0.001	0.025	0.035	0.038	0.039	−0.047	1.000	
$\Delta\Gamma_{30}$	0.005	0.005	0.004	0.005	0.004	0.005	0.005	0.000	−0.026	−0.008	0.042	−0.002	0.002	−0.053	1.000

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