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Magneto-dipole radiation in the model of Hercules X-1

Ya.S. Lyakhova\textsuperscript{a,b,*}, G.S. Bisnovatyi-Kogan\textsuperscript{b,a}

\textsuperscript{a}National Research Nuclear University MEPhI (Moscow Engineering Physics Institute), Kashirskoe Shosse 31, Moscow, 115409, Russia
\textsuperscript{b}Space Research Institute of Russian Academy of Sciences, Profsoyuznaya 84/32, Moscow 117997, Russia

Abstract

We compare polarization properties of the cyclotron and relativistic dipole radiation of electrons moving in the magnetic field on a helix with ultra-relativistic longitudinal and non-relativistic transverse velocity components. The applicability of these models in the case of accretion onto a neutron star is discussed. The test, based on polarization observations is suggested, to distinguish between the cyclotron and relativistic dipole origin of features, observed in X-ray spectra of some X-ray sources, among which the Her X-1 is the most famous.

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1. Introduction

X-ray pulsar Hercules X-1 discovered in 1971 by the Uhuru satellite is one of the best studied X-ray source. Her X-1 is the first source in which X-ray spectrum the line feature in the 39-58 keV energy range was observed, which could not be identified with any chemical element, and was suggested to be a cyclotron line [13]. This feature was observed later [8] [14-16]. When this feature is interpreted as a cyclotron line, the magnetic field strength may be calculated from the non-relativistic formula

* Corresponding author. Tel.: +7-495-333-4588; fax: +7-495-333-4588.
E-mail address: yanalyakhova@gmail.com
where $\omega$ is the cycle frequency of the photons, identified with the frequency of the observed X-ray feature, $m_e$ is the mass of the electron, $c$ is the light speed. In this case the magnetic field strength should be of the order of $(3-5) \times 10^{12}$ Gs. But as large as this value comes into conflict with some theoretical reasonings among which the most important are consideration of the interrelation between radio and X-ray pulsars [4], and simulation of the pulse variability during the 35-days cycle in observations from the satellites ASTRON [12], Ginga and RXTE [6,10]. Obscuration of X-ray beams during the 35 day cycle is often used to explain the periodic X-ray high-low state transitions of Her X-1 during the accretion disk precession. If the obscuring material is the inner edge of the accretion disk, then the inner disk must be tilted out of the binary plane and be precessing to produce periodically varying obscuration. In such a situation, occultation of the neutron star would occur twice in each precession cycle, leading to the decline in flux, and termination of the main and short high states. This scheme was also extended [12] to explain pulse profile evolution with a reflection of the light on the off state by the inner edge of the accretion disk. The value of the dipole magnetic field of the neutron star, determining the radius of the inner edge, coinciding with the radius of the Alfven surface, was estimated in this model as $10^{10} \times 10^{11}$ Gs.

To solve the problem of discrepancy between this estimation and the value following from the cyclotron interpretation (1), it was suggested in [1], that the observed feature could be explained by the relativistic dipole radiation of electrons having strongly anisotropic distribution function, with ultra-relativistic motion along the magnetic field lines, and non-relativistic motion across it. Such distribution function is formed when the accretion flow into the magnetic pole of the neutron star is stopped in a non-collisional shock wave [3], and a rapid loss of transversal energy in the strong magnetic field leads to strongly anisotropic momentum distribution [2].

In this work we consider the problem of the observational choice between the above mentioned models by measuring the polarization of the radiation in this X-ray feature. The relativistic dipole and cyclotron radiation have different polarization properties, so such measurements could solve this long-standing problem. Such experiments could be performed on the Japanese satellite Astro-H which launch is planned for 2015 [17].

2. Polarization of the cyclotron radiation

The cyclotron radiation is produced during a motion of non-relativistic electrons across a magnetic field direction. It is radiated in the form of the line with the energy $\hbar \omega_B$, with the cyclotron frequency

$$\omega_B = \frac{eB}{m_e c} \tag{2}$$

The electron is moving along the Larmor circle with the radius

$$R_L = \frac{mv_{\perp 0}}{eB} \tag{3}$$

where the total electron velocity $v_{\perp 0}$ is supposed to be situated in the plane perpendicular to the direction of the magnetic field. The electron is radiating also on the harmonic frequency $\omega_{nB} = n\omega_B$. At $v_{\perp} \ll c$, the strength of the harmonic lines is rapidly decreasing with the number $n$. If also $|\omega| \ll c$, the change of cyclotron frequency due to Doppler shifting may be neglected, and only the gravitational redshift in the gravitational field of the neutron star
(not present in (1)) should be taken into account for the magnetic field evaluation. Taking into account only the radiation on the first harmonic of the cyclotron frequency, we have it’s differential power as [11]

$$\frac{dW_0(\mathcal{G})}{d\Omega} = \frac{e^2 \omega_0^2 \nu_0^2}{8\pi c^2} \left(1 + \cos^2(\mathcal{G}_0)\right) \delta\left(\omega - \omega_0\right) \text{erg} \cdot \text{s} \cdot \text{sterad} \cdot \text{Hz}$$

(4)

and the total power is:

$$W_{\text{tot}} = \frac{2e^2 \omega_0^2 \nu_0^2 \text{erg}}{3c^2}$$

(5)

Expressions for the degrees of linear and circular polarization, respectively, are [7]:

$$\rho_{l0} = \frac{1 - \cos^2 \mathcal{G}_0}{1 + \cos^2 \mathcal{G}_0}, \rho_{c0} = \frac{2 \cos \mathcal{G}_0}{1 + \cos^2 \mathcal{G}_0}$$

(6)

$$\rho_{l0}^2 + \rho_{c0}^2 = 1$$

(7)

The cyclotron radiation of a single electron is totally polarized, inducing the last equality. The cyclotron radiation along the direction of the magnetic field is fully circularly polarized and in the plane perpendicular to the magnetic field it’s fully linearly polarized. We shall use the subscript "0" for the frame, connected with the plane of the Larmor circle where \( v_{l0} = 0 \). Angular distribution of the power - total, linearly, and circularly polarized are presented in Fig. 1

a)

![Angular distribution of the Cyclotron radiation](image)

b)

![Polarization degrees vs. angle](image)

Fig. 1: (a) Angular distribution of the Cyclotron radiation; (b) polarization degrees vs. angle
3. Polarization of the Relativistic Dipole Radiation

Let’s consider an electron in the magnetic field, with the following values of the velocity components in the laboratory frame

\[ v_\parallel \simeq c, \gamma_\parallel = \frac{1}{\sqrt{1 - \frac{v_\parallel^2}{c^2}}} \gg 1, v << c, \sqrt{1 - \frac{v_\parallel^2}{c^2}} = \frac{c}{\gamma_\parallel} \]  

(8)

The trajectory of the electron is helical, with the helix step significantly larger than it’s radius (see Fig. 2). The radiation provided by such system is called Relativistic Dipole (RDR). The properties of RDR have been considered in detail in [7]. The calculations of the angular distributions of the power and both types of polarization in RDR in the laboratory frame, where the electron is moving along the magnetic field to the observer with the velocity \( v_0 \), may be calculated by making Lorentz transformation in (4), (6), (7). The angle \( \theta_0 \) and the velocity \( v_{\perp 0} \) in the Larmor circle frame are connected with the angle \( \theta \) and the velocity \( v_{\perp} \) in the laboratory frame as

\[ \sin \theta_0 = \frac{\sin \theta \sqrt{1 - \beta_\parallel^2}}{1 - \beta_\parallel \cos \theta}, \cos \theta_0 = \frac{\cos \theta - \beta_\parallel}{1 - \beta_\parallel \cos \theta}, v_{\perp 0} = \gamma v_{\perp} \]  

(9)

Fig. 2: The RDR case trajectory: electron moves on helix along the magnetic field; \( h \) is the helix step.
Expressions for the linear and circular polarization degrees in the laboratory frame are obtained from (6), (7), with account of (9), as

$$\rho_{t0} = \frac{1 - \left( \frac{\cos \vartheta - \beta \parallel}{1 - \beta \parallel \cos \vartheta} \right)^2}{1 + \left( \frac{\cos \vartheta - \beta \parallel}{1 - \beta \parallel \cos \vartheta} \right)^2}, \quad \rho_{\epsilon 0} = \frac{2 \cos \vartheta - \beta \parallel}{1 - \beta \parallel \cos \vartheta}$$

(10)

It follows from (4) that RDR radiation is emitted in a small angle ($\theta \leq 1/\gamma_\parallel$), along the magnetic field direction. It is convenient therefore [5], to introduce a variable

$$\psi = \gamma_\parallel \vartheta$$

(11)

Then for small $\theta$, and large $\gamma_\parallel$ we have the following expansions

$$\beta_\parallel \approx 1 - \frac{1}{2\gamma_\parallel^2} - \frac{1}{8\gamma_\parallel^4}, \cos \vartheta \approx 1 - \frac{\vartheta^2}{2} + \frac{\vartheta^4}{24}, \cos^2 \vartheta \approx 1 - \vartheta^2 + \frac{\vartheta^4}{3}$$

(12)

which with account of (11) lead to expressions

$$1 - \beta_\parallel \cos \vartheta \approx \frac{1}{2\gamma_\parallel^2} \left( 1 + \psi^2 \right), \quad \cos \vartheta - \beta_\parallel \approx \frac{1}{2\gamma_\parallel^2} \left( 1 - \psi^2 \right)$$

(13)

Angular dependencies of the polarization degrees in the laboratory frame from (10), with account of (11) are written as [7]:

$$\rho_t = 2 \frac{\psi^2}{1 + \psi^4}$$

(14)

$$\rho_\epsilon = \frac{1 - \psi^4}{1 + \psi^4}$$

(15)

The relative graphics of angular dependencies for different values of $\gamma_\parallel$ are presented on Figs 3 and 4.

The differential power of the radiation in the unity of the solid angle $\Omega$, with $d\Omega = \sin \Theta d\Theta d\phi$, time $t$, and frequency $\omega$ is obtained from (4), with account of (9), and relations
\[
d\Omega_0 = d\Omega \frac{d\cos \vartheta}{d\cos \vartheta} = d\Omega \frac{1 - \beta^2}{(1 - \beta \cos \vartheta)^2}
\]

We have then from (4), using (9), the expression for the differential power in the laboratory frame as (see [7])

\[
\frac{d^2 W}{d\Omega d\omega} = \frac{e^2 \omega_0^2 \nu^2}{8\pi c^2 \gamma} \left( 1 + \left( \frac{\cos \vartheta - \beta}{1 - \beta \cos \vartheta} \right)^2 \right) \left( 1 - \beta^2 \right)^{3/2} \delta \left( \omega - \frac{\omega_0}{\gamma \left( 1 - \beta \cos \vartheta \right)} \right) \text{erg s \cdot sterad \cdot Hz}
\]

Fig. 3: (a) Angular distribution of the RDR; (b) polarization degrees. Lorentz-parameter \( \gamma_l = 1.1 \).
Fig. 4: (a) Angular distribution of the RDR; (b) polarization degrees. Lorentz-parameter $\gamma=10$.

4. Summary and discussion

The analysis shows that two radiative regimes, Cyclotron and Relativistic Dipole ones, are characterized by different angle dependencies of polarization degrees (Figs. 2b and 3b). It’s natural to assume that the maximum of radiation coincides with the direction to the neutron star pole. In this case linear polarization degree of the Cyclotron radiation almost absents whereas for the Relativistic Dipole radiative regime it’s significant. It’s possible to use this difference in polarimetric investigations of X-ray sources with such satellites as Astro-H, X-Calibur, etc. [17].

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