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Casimir effect and creation of radiation in confined κ-deformed electrodynamics

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Abstract

We consider a κ-deformed electrodynamics in a sourceless situation and under boundary conditions dictated by the presence of two parallel conducting plates. Using the κ-deformed dispersion relation we compute the corresponding zero-point energy. The result is reduced to quadratures of elementary functions and has a real as well as an imaginary part due to the simultaneous effect of κ-deformation and boundary condition. The imaginary part exhibits a remarkable property of κ-deformed theories: the creation of radiation due to boundary conditions. The real part gives corrections to the Casimir effect due to the κ-deformation and is in agreement with previously known results. Real and imaginary parts also confirms a conjecture originated from a calculation of one-loop effective action for a massive scalar field.

The κ-deformed Poincaré algebra [1–6] is a quantum group (Hopf algebra) [7] related to the de Sitter and conformal algebras and its masslike deformation parameter κ may provide a natural regularization in the context of quantum field theory. The generators of the κ-deformed algebra are the four-momentum P^μ ($\mu = 0, 1, 2, 3$), the rotations J^i and boosts K^i ($i = 1, 2, 3$), the same as those appearing in the non-deformed algebra, but submitted to a set of deformed commutation relations. Obviously, these commutation relations reduce to the usual ones of the Poincaré algebra in the limit in which $\kappa \to \infty$, where the deformation is turned off.

In a recent Letter [8] it was proposed a new mechanism for the creation of matter and radiation based on the κ-deformed Poincaré algebra mentioned above. The essential idea is to consider a quantum field subjected to boundary conditions and whose spacetime symmetries are governed by the Poincaré quantum group. In Ref. [8] a massive scalar field and a Dirichlet boundary condition between two parallel planes were chosen. Using Schwinger's proper time method these authors computed the one-loop effective action and showed that the effective action developed an imaginary part, who was interpreted as a particle creation phenomenon. This particle creation phenomenon is a very surprising result that may have important applications, since it may provide a new mechanism for creation of matter and radiation for the early Universe and hence, it may be incorporated into some cosmological models with striking consequences.

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In this Letter, we shall obtain the phenomenon of creation of radiation from the sole hypothesis of boundary condition on κ-deformed electromagnetic field, that is, electromagnetic field described by a deformation in Maxwell equations which is compatible with the κ-deformed Poincaré algebra. The phenomenon of creation of radiation will appear as the imaginary part of the sum of zero modes of the κ-deformed electromagnetic field. On the other hand the real part of the sum gives the κ-deformed electromagnetic Casimir energy, which was previously obtained by Bowes and Jarvis [9]. The result of Bowes and Jarvis is reobtained by expanding the real part of our sum of modes in powers of κ^{-1}.

The most natural boundary condition on the κ-deformed electromagnetic field is to consider this field constrained by the presence of two parallel perfectly conducting plates. For our purposes, it suffices to know that the deformed algebra has a quadratic Casimir invariant given by $\mathbf{P}^2 - (2\kappa \sinh(P^0/2\kappa))^2$ [1]. It is a scalar that we represent by $-m^2$ where m is a real number that we call mass and labels the representations of the algebra. In this way the κ-deformation of the Poincaré algebra changes the mass-shell condition into $\mathbf{P}^2 - (2\kappa \sinh(P^0/2\kappa))^2 = -m^2$. It is important to notice that the κ-deformation also changes the four-momentum conservation law, as shown by Kosinski, Lukierski and Maslanka [16]. However, in our calculation starting from (7) it appears only the modification on the mass shell condition due to the κ-deformation. It is convenient to use as deformation parameter the positive number $q := (2\kappa)^{-1}$ in such a way that the usual Poincaré algebra is obtained as the limit of the deformed Poincaré algebra when $q \to 0$. In terms of the parameter q the quadratic Casimir invariant of the deformed algebra is then given by

$$\mathbf{P}^2 - \left[\frac{\sinh(qP^0)}{q}\right]^2 = -m^2. \tag{1}$$

In the limit $q \to 0$ we obtain the expected relation of the usual Poincaré algebra: $\mathbf{P}^2 - (P^0)^2 + m^2 = 0$. Also for convenience, we define on the space of smooth functions the linear operator

$$\partial_q := \frac{\sinh(iq\partial_0)}{iq} = \frac{\sin(q\partial_0)}{q}, \tag{2}$$

which reduces to a simple time derivative in the limit $q \to 0$.

Let us now consider the following equations:

$$\begin{aligned} &\nabla \cdot \mathbf{E} = 0, \qquad \nabla \times \mathbf{E} = -\partial_q \mathbf{B}, \\ &\nabla \cdot \mathbf{B} = 0, \qquad \nabla \times \mathbf{B} = \partial_q \mathbf{E}. \end{aligned} \tag{3}$$

The above equations lead to a modified wave equation, namely,

$$(\partial_q^2 - \nabla^2)\mathbf{E} = 0, \qquad (\partial_q^2 - \nabla^2)\mathbf{B} = 0. \tag{4}$$

Trying as solutions of the above equations the plane wave fields $\mathbf{E} = \mathbf{E}_o \exp(i(\mathbf{k}\cdot\mathbf{x} - \omega t))$ and $\mathbf{B} = \mathbf{B}_o \exp(i(\mathbf{k}\cdot\mathbf{x} - \omega t))$, we obtain the deformed dispersion relation:

$$\frac{\sinh(q\omega)}{q} = |\mathbf{k}|. \tag{5}$$

However, these plane wave fields are solutions of the deformed sourceless Maxwell's equations (3) only if $\mathbf{E}$ and $\mathbf{B}$ are orthogonal to $\mathbf{k}$, $\mathbf{k} \times \mathbf{E} = \mathbf{B}$. Besides, we have polarization states analogous to those in the non-deformed case. Note that in the limit $q \to 0$ all the above results for the deformed theory, given by Eqs. (3), (4) and (5), reduce to the well-known results for the non-deformed theory, as expected (usual Maxwell equations, usual wave equation and the dispersion relation $\omega = |\mathbf{k}|$, respectively). We call Eq. (3) the κ-deformed (sourceless) Maxwell equations in vacuum and the fields $\mathbf{E}$ and $\mathbf{B}$ that satisfy them we call the κ-deformed electromagnetic fields. Since (3) is obtained from the Maxwell equations by deforming only the time derivatives several results which are obtained for the usual electromagnetic field remain valid for the κ-deformed fields. In particular, the fields remain zero inside perfect conductors in equilibrium.

Now let us consider the vacuum of the quantized κ-deformed electromagnetic field confined between two perfect conductors in the form of two large parallel plates at a distance a apart. We consider the plates as squares of side ℓ with $\ell \gg a$. The original Casimir effect [10] is the shift of the vacuum energy of the electromagnetic field due to the presence of the conducting plates as described above. Nowadays the Casimir effect is understood in a much broader sense [11–15], but here we will take it in the original sense, except for the fact that our electromagnetic field is κ-deformed.

Let us then consider the sum of zero-point energies of the deformed Maxwell theory:

$$\mathcal{E}_q(a) = \sum_{\mathbf{k},\sigma} \frac{1}{2}\omega_{\mathbf{k}}, \tag{6}$$

where $\mathbf{k}$ is an allowed wave vector mode between the plates and σ stands for the polarization states. Due to (5) this sum is given by

$$\mathcal{E}_q(a) = \sum_{\mathbf{k}} \frac{1}{2q} \sinh^{-1}(kq). \tag{7}$$

To compute this sum it is easier to consider first the following regularized derivative:

$$\frac{\partial}{\partial q}(2q\mathcal{E}) = \sum_{\mathbf{k}} \frac{k\mathrm{e}^{-\epsilon\sqrt{1+(kq)^2}}}{\sqrt{1+(kq)^2}}, \tag{8}$$

where the regularization parameter ϵ will tend to the zero limit in due course. Taking the derivative with respect to ϵ, we get:

$$\mathcal{E}_q(a) = \frac{1}{2q}\int_{\epsilon}^{\infty} d\epsilon' \int_0^q \sum_{\mathbf{k}} k\mathrm{e}^{-\epsilon'\sqrt{1+(kq')^2}}\, dq'. \tag{9}$$

The range of $\mathbf{k}$ in the sum can be taken as $\mathbf{k}_\parallel + \hat{\mathbf{z}}k_z$, where $\mathbf{k}_\parallel \in \mathbb{R}^2$ and $k_z = \pi n/a$ ($n \in \mathbb{Z}$). To calculate the sum we use the principle of the argument (essentially the Cauchy's integral [17]), which states that if ϕ is analytic inside and on the contour $\mathcal{C}$ and f is analytic inside, except for a finite number of poles, and on the contour $\mathcal{C}$, then

$$\sum_n \phi(k_n) - \sum_p \phi(k_p) = \frac{1}{2\pi i}\oint_{\mathcal{C}} \phi(z)\frac{d\log(f(z))}{dz}\, dz, \tag{10}$$

where k_n are the zeros and k_p are the poles of f inside $\mathcal{C}$. In the present case we take $f(z) = \sin(az)$ and $\mathcal{C}$ to enclose the whole complex plane outside the branch cuts of the square-root to obtain for the sum in (9) the expression:

$$\sum_{n\in\mathbb{Z}} \sqrt{k_\parallel^2 + k_n^2}\, \mathrm{e}^{-\epsilon'\sqrt{1+q'^2(k_\parallel^2+k_n^2)}} = 4\frac{1}{2\pi i}\int_{\infty}^{k_\parallel} \sqrt{k_\parallel^2 - y^2}\, \mathrm{e}^{-\epsilon'\sqrt{1+q'^2(k_\parallel^2-y^2)}} \times \frac{d\log(\sin(iay))}{d(iy)}\, d(iy). \tag{11}$$

Substituting this result in (9) and integrating in ϵ' we arrive at

$$\begin{aligned}\mathcal{E}_q(a) = \frac{\ell^2}{2q\pi^2}\int_0^q dq' \int_0^\infty dk_\parallel \times k_\parallel \Bigg\{ &\int_{k_\parallel}^{\sqrt{k_\parallel^2+(q')^{-2}}} \frac{\sqrt{y^2-k_\parallel^2}}{\sqrt{1+q'^2(k_\parallel^2-y^2)}} \times \mathrm{e}^{-\epsilon\sqrt{1+q'^2(k_\parallel^2-y^2)}}\frac{d\log(\sinh(ay))}{dy}\, dy \\ &- i\int_{\sqrt{k_\parallel^2+(q')^{-2}}}^{\infty} \frac{\sqrt{y^2-k_\parallel^2}}{\sqrt{-1-q'^2(k_\parallel^2-y^2)}} \times \mathrm{e}^{-i\epsilon\sqrt{-1-q'^2(k_\parallel^2-y^2)}}\frac{d\log(\sinh(ay))}{dy}\, dy\Bigg\}.\end{aligned} \tag{12}$$

To perform the integration in $k_\parallel$ and q' we have to change the limits of integration. After a lengthy, but straightforward calculation, we obtain:

$$\begin{aligned}\frac{\mathcal{E}_q(a)}{a\ell^2} = -\frac{1}{(2\pi a)^2}\sum_{n=1}^{\infty}\frac{1}{n^2} \times \Bigg\{ &\int_0^{1/q}\left(y+\frac{1}{2an}\right)\frac{\mathrm{e}^{-2any}}{\sqrt{1-(qy)^2}}\, dy \\ &+ i\int_{1/q}^{\infty}\left(y+\frac{1}{2an}\right)\frac{\mathrm{e}^{-2any}}{\sqrt{(qy)^2-1}}\, dy\Bigg\},\end{aligned} \tag{13}$$

where spurious terms relative to self energy of the Casimir plates and uniform energy density of the vacuum were subtracted. This is our main result. This energy presents a real and an imaginary part. The real part agrees with κ-deformed electromagnetic Casimir energy previously obtained by Bowes and Jarvis [9], as it can be verified by an expansion in q/a. In the limit $q \to 0$ the imaginary part disappears and the real part reduces to the usual Casimir energy [10], as expected. Let us consider the imaginary part which exhibits a remarkable property of κ-deformation but whose interpretation is not obvious. In Schwinger's formalism the vacuum persistence amplitude between a time interval $t_2 - t_1 > 0$ is given by $\langle 0, t_2 | 0, t_1 \rangle = \exp(i\mathcal{W})$, where $\mathcal{W}$ is the Schwinger's effective action. In the case of one-loop QED, $\mathcal{W}$ may present an imaginary part due, for example, by the presence of an external eletromagnetic field [18]. In this case, due to the vacuum fluctuations of the charged quantum fields, the virtual electron–positron pairs would absorb energy from the external field and would appear as real pairs. The phenomenon of pair production would justify the existence of an imaginary part. Besides, in this formalism $\mathcal{E} = -\mathcal{W}/(t_2 - t_1)$ [19]. From Eq. (13), this implies that $|\langle 0, t_2 | 0, t_1 \rangle|^2 = \exp(-2(t_2 - t_1)|\Im(\mathcal{E}_q(a))|)$, which clearly means that here, in virtue of symmetries given by the κ-deformed algebra taken simultaneously with boundary conditions for the field in consideration, particle creation is a natural process.

Our calculation can also be compared with a previous one made in the literature [8] for the neutral massive scalar field. As mentioned before, in this reference, the authors obtained the Schwinger's effective action for this field subjected to Dirichlet boundary conditions at two parallel planes, also in the context of a κ-deformed Poincaré algebra. An imaginary part was also encountered for the effective action. In fact, it can be shown after some mathematical manipulations that in the limit of zero mass, and apart from a factor of 2, the real and imaginary parts quoted in Eq. (13) are in complete agreement with those obtained in Ref. [8]. This fact suggests that the κ-deformed effective action computed a la Schwinger [8] is equivalent to Eq. (6) via $\mathcal{E} = -\mathcal{W}/(t_2 - t_1)$. A definite confirmation of this equivalence, though not obvious in the deformed theory, can be explicitly checked following the same lines of thought as those found in Ref. [20].

The interpretation of the real and the imaginary parts of (13) as a Casimir energy and energy of created excitations, respectively, is just heuristic and a rigorous interpretation must be based on a Hamiltonian formulation of the problem (the energy–momentum tensor of the deformed theory and many of the mathematical details of the present letter will be published elsewhere).

The dependence of the κ-deformed electromagnetic Casimir energy on the parameter κ relates the hypothesis of a κ-deformed spacetime symmetry to the observed electromagnetic Casimir effect [9], which has been measured with greater and greater accuracy in the last few years [21–25]. Hence, this effect could be used to set experimental lower bounds on κ and check the possibility of a κ-deformed spacetime symmetry. However, since Lorentz invariance is experimentally verified with very high precision, the bounds from the above mentioned Casimir experiments will hardly add new restriction on κ. Eventhough, the κ-dependent correction to the Casimir effect are of interest in quantum field theory as a matter of first principles and will be further investigated elsewhere. The contribution up to first-order in $q := 1/2\kappa$ is given by (for the contributions in all orders see [9]):

$$\Re\left(\frac{\mathcal{E}_q(a)}{a\ell^2}\right) = -\frac{\pi^2}{720}\frac{1}{a^4}\left\{1 + \frac{\pi^2}{21}\left(\frac{q}{a}\right)^2 + \mathcal{O}\big((q/a)^4\big)\right\}. \tag{14}$$

Regarding the particle creation phenomenon described here, it is worth emphasizing that it may be of some relevance in the early Universe. Suppose, for instance, that during the very beginning of its existence, the Universe space–time symmetries were governed by a κ-deformed Poincaré algebra. Besides, at this stage, it would also experience boundary conditions due to its tiny dimensions. As a consequence, particle creation (including radiation) would take place due to the simultaneous effect of boundary conditions and deformation. Of course, if we want to take this possibility more seriously, we must generalize the concept of energy (as it happened many times in the history of science: for instance, when heat was included into the energy conservation law and so on). A new quantity, some kind of a sum of the usual energy of the particles plus some function of the deformation, through the parameter κ, will be conserved in such a way that as the deformation diminishes, the energy content of the particles enhances. Let us notice that this possibility of non-conservation of energy relies on a global character of the theory, the boundary conditions, while the more fundamental non-conservation of four-momentum obtained by Kosinski, Lukierski and Maslanka [16] is a general property of κ-deformed theories. Finally, in a situation where there is no deformation any more, the energy, as we understand it nowadays, would be a conserved quantity. Though quite speculative, this idea provides a new scenario for particle creation in the early Universe.

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