case study

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#### Abstract

With the promotion of the environmentally friendly transportation modes (the European Commission supports the freight transport operations in the rail sector), an increase in the diversification of the demand is observed. While most rail freight companies tend to apply fixed schedules, this approach is not effective turns out to be ineffective due to the need to meet the customer's specific requirements. The purpose of this paper is to present a case study of empty wagon flow planning over a medium term horizon and to discuss the opportunities of improvement of this plans by discrete optimization. In order to increase the utilization and availability of wagons, the planning procedure with a rolling horizon has to be implemented. Unfortunately, since the plan has to be updated ca. every 4 hours, this planning approach needs effective optimization tools. Our hybrid two-stage approach is designed to be implemented in such business environment. This formulation allows us to solve real life instances even for a 7-day time horizon.


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## 1. Introduction

The freight railway operator runs a number of trains in order to carry out its customer's orders. These trains need an appropriate type and number of wagons at geographically dispersed stations around Poland. Unfortunately, our experience shows that in most cases the rail freight companies operated in Poland plan their transport operation in the time horizon not longer than one day. Such a situation results from a complex optimization problem which arises in this area. In order to simplify the planning of rail cargo freights, a hierarchical approach is used. In general, the planning procedure is divided into 3 main parts:

- wagon flow planning (empty and loaded wagons),
- locomotive assignment planning,
- employee scheduling.

The operational planning procedure is initiated with the planning of the flow of wagons. While the flow of loaded wagons depends on the customer's orders, the flow of empty wagons can be used to decrease operational costs. Decision makers are aware of the relationship existing between the number of wagons used and the number of kilometers of empty wagons. The lower the number of wagons the higher the number of kilometers traveled by empty wagons. On the other hand, the lower the number of wagons, the bigger the number of problems with the wagon availability.

Similarly as in other areas of business planning, an important factor here is the uncertainty. In order to cope with varying parameters, the planning with the rolling horizon is widely used in this area. In such a situation, plans with a fixed planning horizon should be created in given intervals. This approach allows us to deal with uncertain issues, however it needs the support tools in the planning generation process (the planning procedure has to be executed at least once a day) and the system integration planning system as well as the freight execution system (optimization tools should consider as current values of parameters as possible). Our hybrid approach was designed to be used in such a business environment and is essential to plan the flow of empty wagons with a rolling horizon.

The paper is structured as follows. The description of the problem of the empty wagon flow planning is given in Section 2. In addition, this section contains a literature review and the main assumption of our proposal with a summarized contribution. In section 3 we discuss our two-stage hybrid approach in detail. Section 4 contains computational results. Finally, a summary is provided in Section 7.

## 2. The empty wagon flow problem

### 2.1. The empty wagon flow planning in practice

Rail freight transport carried out by PKP Cargo is assigned to one of two groups: block train traffic, single wagonload traffic. Block train transport orders are considered as the main ones and the plan must guarantee the maximization of the number of completed orders according to priorities. A block train traffic order is considered to be completed if the appropriate number of wagons of a given series is allocated and provided for the execution of a given order (at the right place and time). Single wagonload traffic orders may be fulfilled also as the train enhancement in block train transports (both loaded and empty). However, the train enhancement may take place only at the shunting station between 6:00 a.m. and 8:00 p.m. (ultimately, each shunting station may have different operation hours), and also if the maximum technical parameters are not exceeded (in particular, the length and gross weight of the train) and for loaded trains - if the enhanced train has a relevant low priority (priority $=3$ or 4). Furthermore, the time of waiting of any train for enhancement may not exceed the limit of 4 h (ultimately, a value of this parameter will vary for individual stations). A single wagonload traffic order is considered to be completed if the appropriate number of wagons of a given series is allocated and provided for the execution of a given order, and upon loading (the loading time of 8 h is assumed; a target value will vary for individual orders), these wagons are assigned to one or several trains for which the departure station of the first one is the same as the order provision station, and the arrival station of the last one is the same as the destination station of the order. If the single wagonload traffic order is executed by more than one train, it is necessary to make sure that the dates of departure of
subsequent trains are greater than the date of arrival of the preceding train. In the case of a "double loading operation", i.e. loading and unloading of wagons at the same station, it is necessary to make sure that time of provision of wagons used in the next order is at least 4 hours later than the time of release of wagons used in the preceding order.

The basic objects representing the transport network are stations and routes specified for them. Each route is defined by a starting station and an ending station. For each route, the train parameters (the maximum length and maximum gross weight) that can be provided for a given route are defined. It should be assumed that there are many routes between any two stations, in order to clearly indicate a selected route, the station referred to as the "through station" should be used additionally. At the planning phase, each route is assigned a distance in kilometers and travel time in minutes. It was assumed that the time of travel of an empty train between stations would be calculated according to the formula $4 \mathrm{~h}+$ route travel time with the speed of $20 \mathrm{~km} / \mathrm{h}+4 \mathrm{~h}$. The travel time at the planning stage is treated as an estimate and it should be assumed that upon obtaining an individual timetable or assigning an annual timetable this time will result directly from the timetable. Only the estimated times are taken into account in the reference problem.

From the set of stations, shunting stations at which it is possible to enhance or reduce a train, as well as to store empty wagons upon completion of a single wagonload transport order, waiting to be assigned to the next order, were singled out. Unless provided for in the timetable, upon the completion of single wagonload traffic orders, empty wagons must be transferred from the stations corresponding to the stations other than shunting stations within 24 hours (ultimately, this parameter will vary for individual stations) for the execution of further orders or to the selected shunting stations to await the assignment to an order in both single wagonload traffic and block train traffic.

The transport resources used to transport cargo are freight wagons. It is assumed that the wagons will be considered in the reference problem per series. For each wagon series, the following parameters are available: length including buffers (in mm ), construction weight - tare (in t ). Due to the fact that at the stage of planning of the movement of empty wagons, specific locomotives are not considered, it should be assumed that each train is 32 meters longer than the sum of the lengths of all its wagons. To each wagon series, a stock volume is related, i.e. a number of fully operational, empty wagons located at any given time (yyyy-mm-dd hh:mm) at a specific station. The stock volume applies to all stations in the transport network. At no station in the whole planning horizon, the stock of wagons may be smaller than the defined minimum stock. In the reference problem, 0 should be assumed for this parameter (ultimately, these values will be determined for each station separately). To determine the stock of wagons at individual stations in the whole planning horizon, it is necessary to consider the initial stock volume (i.e. the stock of wagons at the beginning of the planning horizon) and changes of the number of wagons resulting from the transport orders being executed and movements of trains with empty wagons. In addition, it is necessary to take into account the wagons arriving at the stations in the planning horizon as part of the trains that were in motion at the time corresponding to the beginning of the planning horizon, since these wagons are not included in the initial stock volume.

The information about transports to be carried out is included in the data on orders for wagons. For each order, a forwarding station, a destination station, a provision siding (optional), a target siding (optional), a number and series of required wagons, a date of provision, a date of release/execution are defined. For orders in the block train traffic, a release/execution date is the moment when wagons used to execute the order become available for the fulfillment of other orders, while for single wagonload transport orders, this date is interpreted as the date up to which the order should be executed. A priority (4-low, 3-average, 2-high, 1-highest) is assigned to each transport order. For orders with a higher priority, wagons must be allocated in the first place.

In addition, the length of the train may not exceed the maximum length of the train on a given route. Furthermore, the weight of the train must not exceed the maximum weight of the train on a given route, where the weight of the train is the weight of wagons (tare) plus the weight of the load (specified in an order), increased by the weight of the locomotive (the value assumed for the locomotive is 120 t ). Lastly, the number of wagons for each series at any station and in the whole planning horizon may not be negative.

### 2.2. Previous work

Several related papers can be found in the literature. The first papers where the distribution of empty freight cars was considered are the works by Feeney (1957) and Leddon et al. (1968). Unfortunately, these formulations are nondynamic, the supply and demand are known, the solution is calculated only for a current period, so finally the solving procedure uses a classical simplex approach.

Since the time dimension is very important in these practical decision making problems, the authors of the work published in the seventies took it into consideration. For example, the paper by White (1969) and White and Bomberault (1972) contains a special network algorithm which can be used to find a solution to the mentioned problem in a dynamic version. This problem was also solved by using other specific approaches. For example, Ratcliffe et al. (1984) proposed a hybrid approach based on a simulation-optimization methodology, Glickman and Sherali (1985) considered this problem as the network distribution of pooled empty freight cars over time.

The next important step was the introduction of a space-time network with a stochastic supply, demand and travel time. A description of this approach is given by Jordan and Turnquits (1983). An important contribution to this research area was given by Powell (1986). In this research, the empty wagon flow was considered parallel with loaded trips. This generalization was extended by some real-life assumption in a paper by Powell (1987).

Similarly, the wagon flow problem was considered parallel with a locomotive assignment problem. In the research by Cordeau et al. (2000) the Banders decomposition was used to find a solution to the mentioned problem. The heuristics approach was used in papers by Cordeau et al. (2001) and Lingaya et al. (2002) to find a solution to the real life problem at VIA Rail Canada.
There are only a few proposals which have been designed and implemented in real life. Empty wagons were improved by the German Federal Railways and the solution is provided in Weldinger et al. (1991). The second research was proposed by Holmberg et al. (1998). This work proposed the multicommodity network flow model with a train capacity, which was applied by the Swedish State Railways. In recent studies the empty car movement is is considered in Giacco ar al. (2014). Authors have proposed an integer programming formulation for integrating maintenance planning tasks in the railway rolling stock circulation problem.
In our work the using of rolling horizon in planing of rail transporation movements has been implemented. This approach is well know and has been considered in case of rail transportation system previously. As an example of this apporach works by Lai et al. (2008) and Meng and Zhou (2011) can be cited.

### 2.3. The proposed approach and contribution

The main objective of the proposed optimization method was the ability to effectively support the planning procedure with a rolling horizon. This approach has been chosen as primary by the decision makers. According to our observations, when a lot of parameters have an uncertain nature, the deterministic optimization model can be implemented and should be based on data collected in rail execution system (like trip scheduling, estimated arrival time). However, the elimination of the elements of uncertainty did not allow for the use of a simple optimization algorithm.

We decided to apply a hybrid approach where the two-stage procedure is implemented. Such an approach was successfully proposed for a complex optimization problem e.g. an inventory routing problem. In our proposal, in the first step, the set of trips (empty trains) is generated by using simple heuristics rules. Then, by using discrete optimization, the movement of empty wagons is planned and assigned to empty trains. This hybrid approach is implemented iteratively: in each step, the heuristics rule is applied and the solution is obtained by using the optimization model. Furthermore, in our case, a lot of real-life constraints, like multicommodity, return trains and single wagonload traffic, were taken into account.

The extended computational experiments on instances motivated by real-world operations performed by the biggest Polish cargo rail operator show that our formulations can be solved within a few minutes of runtime, which is good enough for real-world applications. Our experimental results show that real-world empty wagon flow problems can be solved and the planning with a rolling horizon can by effectively supported by optimization methods.

## 3. A hybrid approach to empty car flow problem solving

### 3.1. Procedure

The empty car flow problem with real life assumptions we deal with should be considered as a dynamic, complex and difficult-to-solve problem. In a typical application for a new order, someone/an operator is looking for free empty wagons in a nearby station and these wagons are transported to an appropriate station which is a source station for the order. There are usually many empty wagon trains ensuring the fulfillment of orders, however in the dynamic approach a wrong selection could have a consequence in the nearest time horizon. In this paper, in order to solve the empty car flow problem, a hybrid approach is applied. This approach consists of two stages. In the first phase, a heuristics method is used to prepare a set of empty trains. Then, the MIP optimization model is used to assign the movement of wagons to empty trains. The second part of the hybrid approach is intended to ensure that as many orders as possible are completed with as few empty trains as possible. In the optimization model, all constraints presented in point 2 must be satisfied.

In the proposed approach, the first heuristic method will generate empty wagon trains and the optimization model will apply, then generated trains not used will be removed and the next procedure will supplement the set of empty wagon trains, and the optimization model will apply again. The procedure will be repeated until a satisfactory solution is achieved.

### 3.2. Heuristic rules for train generation

There are a lot of different approaches to the problem of a set of empty train generation. It is possible to use a formal optimization method or a genetic algorithm, however we have proposed a set of heuristic rules. Each of these rules is based on the analysis of the problem assumptions and their application ensures the achievement of quite good results. To generate a set of empty trains in the proposed hybrid approach, the following 5 rules are considered.

Rule 1. An empty train on the opposite direction to the orders.
Rule 2. An empty train dedicated to a single wagonload order.
Rule 3. An empty train between a destination and shunting station.
Rule 4. An empty train to a station with lack of wagon.
Rule 5. An empty train on used routes.
The first heuristics rule was motivated by the observation that there are two sets of stations in the transportation network. The first one contains stations where the demand for empty wagons is higher than the supply, and the second one, an opposite situation is observed. In order to allow an appropriate flow of empty wagons, the set of empty wagons is extended by a train between the stations where the customer's orders have to be carried out but in the opposite direction.

The second rule generates empty load trains to allow single wagonload orders to be executed. In some cases, the single wagonload order cannot be fulfilled because of the lack of a train on a specific route. This rule allows us to guarantee that the feasible solution for this traffic will exist.

The third rule was implemented since the return trip of empty wagons had to be considered. On the station where the rail operator is not permitted to store wagons, there is a need to move wagons from this kind of station to a station where waiting for an order is accepted. Thanks to this rule, the set of empty trains is extended by a train from station to the nearest station capable of car storing.

The fourth rule is crucial for our formulation and can be executed only if the first feasible solution is determined. In order to solve the problem of missing wagons, the train between the station which has a sufficient level of empty wagons and the station where empty wagons are needed, is provided. Furthermore, the train is generated from the 5 closest station to the destination station, only in one appropriate time bucket and only when on the source station there are enough wagons to the end of the planning horizon.

The fifth rule is implemented in order to increase the capacity on the existing connection. If the flow should be bigger than the capacity of a single train, we have to add more trains on this connection.

### 3.3. An optimization model for the wagons to train assignment

To translate the reference problem into the optimization model, the definition of some data sets and parameters must be described. Let us assume $I$ is a set of customer orders and $J \in I$ is a set of single wagonload traffic orders. In network resources, there are: $R$ - a set of empty trains possible to run (these trains are given before the optimization procedure is started and they are generated by heuristic rules), $K$ - a set of wagon types; $N$ - a set of stations and $N_{j}^{T} \in N$ is a set of transfer stations for single wagonload traffic orders $j$. In the problem, the set of time points $T$ is considered. The time in the problem is modeled as discrete and the constraints have to be satisfied in each set of time points (day and time).

The set of parameters can be divided into 3 subsets associated with wagons and networks, orders and empty trains. In the first subset, there are: $w_{k, n}^{I}$ represents the number of wagons of $k$-series on the station $n$ at the beginning of the planning horizon; $w_{k, n}^{B}$ stands for a minimum number of wagons of $k$-series on the station $n$ (a safety level); $w_{k, n, t}^{S}$ is a number of wagons of $k$-series included in or excluded from the system on the station $n$ at a time point $t$; $w_{k, n, t}^{D}$ is a number of wagons of $k$-series arriving at the station $n$ at the time $t$ as a result of the movement of trains that were in motion at the time corresponding to the beginning of the planning horizon; $W$ - a maximum number of wagons which can be provided by one train (a technical parameter necessary to create a dependence between continuous and discrete variables); $L_{n, m}-$ a maximum length of the train on a route ( $n, m$ ); $G_{n, m}$ - a maximum weight of the train on a route $(n, m), D_{n, m}$ - a length of the route $(n, m) ; l_{k}$ - a length of the wagon $k$-series; $g_{k}^{T}$ - a weight of the wagon of $k$-series; $S(n)$ - a shunting station for the station $n$.

In the group of order parameters, there are: $w_{i}^{Q}$ - a number of wagons necessary to fulfill the entire order $i$; $w_{i}^{K}$ a series of wagons in the $i$ order; $w_{i}^{Z}$ - a type of a customer order ( $1-$ single wagonload traffic, 0 - block train traffic); $w_{i}^{Y}$ - a number of confirmed wagons before the optimization procedure; $g_{j}^{N}$ - a weight of the load in a single wagon in the $j$ - order, $s_{i}^{Z}$ - a time point (day and time) of the provision of wagons for the $i$ order, $f_{i}^{Z}$ - a time point (day and time) of the release of wagons with the $i$ order, $s_{i}^{N}$ - a source station for the $i$ order, $f_{i}^{N}$ - a destination station for the $i$ order and $p_{i}$ - a priority of the $i$ order (preferences).

In the last subset assigned to empty trains, there are four parameters: $s_{r}^{P}$ - a time point (day and time) of departure of the empty train r from the source station; $f_{r}^{P}$ - a time point (day and time) of arrival of the empty train r to the destination station; $s_{r}^{M}$ - the source station for the empty wagon train r and $f_{r}^{M}$ - the destination station for the empty wagon train $r$.

In the proposed formulation, there are 7 groups of decision variables:
$y_{i}^{W} \quad$ - a number of wagons to be confirmed (supplied) for the $i$ order;
$y_{i}^{B} \quad$ - a number of missing wagons for fulfillment of the $i$ order;
$y_{k, r}^{P} \quad$ - a number of empty wagons of $k$-series assigned to the empty train $r$;
$y_{j, r}^{R} \quad$ - a number of loaded wagons of the $j$ order assigned to the empty train $r$;
$y_{k, n, t}^{S} \quad$ - a number of missing wagons of $k$-series on the $n$ station in the point of time $t$ to complete orders;
$x_{r} \quad$ - indicator of launching the empty train $r$ (a binary variable: 1 empty train $r$ was launched, 0 otherwise);
$y_{i}^{Z} \quad$ - indicator of full fulfillment of the $i$ order, (a binary variable : $1-$ an order can be fulfilled, 0 otherwise).

In order to ensure the feasibility of the solution obtained, a set of constraints described below has to be considered. The number of wagons necessary to execute an order has to be connected with the number of wagons to be confirmed and with the number of missing wagons (1).

$$
\begin{equation*}
w_{i}^{Q}-y_{i}^{W}-y_{i}^{B}=0 \text { for each } i \in I \tag{1}
\end{equation*}
$$

The value of the variable $y_{i}^{B}$ allows us to calculate orders unfulfilled due to missing wagons.
The decision variable $y_{i}^{Z}$ (an indicator of a completed order) has to be connected with $y_{i}^{B}$ because it depends on the number of missing wagons (2)

$$
\begin{equation*}
y_{i}^{B} \leq W\left(1-y_{i}^{Z}\right) \text { for each } i \in I \tag{2}
\end{equation*}
$$

In addition, the number of wagons to confirm the order is less than or equal to the number of wagons necessary to fulfill the order and it is greater than or equal to the number of wagons confirmed earlier (3).

$$
\begin{equation*}
w_{i}^{Y} \leq y_{i}^{W} \leq w_{i}^{Q} \text { for each } i \in I \tag{3}
\end{equation*}
$$

The flow constraints (4) have to consider all types of wagon movement and it ensures that in every station and at all points of time the number of all series of wagons is greater than or equal to the safety level (a minimum number of wagons established by the company)

$$
\begin{equation*}
w_{k n}^{I}+\sum_{\left\{i: f_{i}^{N}=n, f_{i}^{Z}<t-1, w_{i}^{K}=k\right\}} y_{i}^{W}-\sum_{\left\{i: s_{i}^{N}=n, s_{i}^{Z}<t, w_{i}^{K}=k\right\}} y_{i}^{W}-\sum_{\left\{r: s_{r}^{M}=n, s_{r}^{P}<t\right\}} y_{k, r}^{P}+\sum_{\left\{r: f_{r}^{M}=n, f_{r}^{P}<t-1\right\}} y_{k, r}^{P}+\sum_{t^{\prime}<t} w_{k, n, t^{\prime}}^{S}+\sum_{t^{\prime}<t} w_{k, n, t^{\prime}}^{D}++y_{k, n, t}^{S} \geq w_{k, n}^{B} \tag{4}
\end{equation*}
$$

for each $n \in N, k \in K, t \in T$
The variable $y_{k, n, t}^{S}$ allows us to avoid a situation with no solution due to the lack of wagons on some stations. In the feasible solution, all variables $y_{k, n, t}^{S}$ should be equal to zero. The analysis of variables $y_{k, n, t}^{S}$ and $y_{i}^{B}$ allows for the problem with the wagon location to be traced.

The next group of constraints is connected with trains. First of all, the assignment of loaded or empty wagons to the empty train $r$ is possible only if the empty train is launched (5).

$$
\begin{equation*}
\sum_{k \in K} y_{k, r}^{P}+\sum_{j \in J} y_{j, r}^{R} \leq W x_{r} \text { for each } r \in R . \tag{5}
\end{equation*}
$$

Furthermore, the length of the train $r$ must be less than or equal to the permissible length of the train on route $(n, m)$ - a constraint (6). The length of the train $r$ is a sum of the length of a locomotive, the length of empty wagons and the length of wagons in single wagonload traffic.

$$
\begin{equation*}
\sum_{k \in K} l_{k} y_{k, r}^{P}+\sum_{j \in J} l_{w_{j}^{K}} y_{j, r}^{R}+32 \leq L_{s_{r}^{M}, f_{r}^{M}} \text { for each } r \in R \tag{6}
\end{equation*}
$$

Moreover, the weight of the train may not exceed the maximum weight on route ( $n, m$ ) - the weight of the train is a sum of the weight of empty wagons and wagons with their load resulting from orders in single wagonload traffic (7).

$$
\begin{equation*}
\sum_{k \in K} g_{k}^{T} y_{k, r}^{P}+\sum_{j \in J}\left(g_{j}^{N}+g_{w_{j}^{K}}^{T}\right) y_{j, r}^{R} \leq G_{s_{r}^{K}, f_{r}^{M}} \text { for each } r \in R \tag{7}
\end{equation*}
$$

Finally, the set of constraints connected to single wagon traffic. The wagons operating in this traffic must depart from a destination or shunting station if the date of the order is included in the planning horizon (8).

$$
\begin{align*}
& \sum_{\left\{r: s_{r}^{M}=s_{j}^{N}, f_{r}^{M}=S\left(s_{j}^{N}\right) ; s_{r}^{P}>s_{j}^{Z} ; s_{j}^{N} \neq S\left(s_{j}^{N}\right) ; S\left(f_{j}^{N}\right) \neq S\left(s_{j}^{N}\right)\right\}} y_{j, r}^{R}+\sum_{\left\{r: s_{r}^{M}=s_{j}^{N}, f_{r}^{M}=S\left(f_{j}^{N}\right) ; s_{r}^{P}>s_{j}^{Z} ; f_{j}^{N} \neq S\left(f_{j}^{N}\right) ; S\left(f_{j}^{N}\right) \neq S\left(s_{j}^{N}\right)\right\} \quad \text { for each } j \in J}+\sum_{\left\{r: s_{r}^{M}=s_{j}^{N}, f_{r}^{M}=f_{j}^{N} ; s_{r}^{P}>s_{j}^{Z} ; S\left(f_{j}^{N}\right) \neq S\left(s_{j}^{N}\right)\right\}}=y_{j}^{W} \tag{8}
\end{align*}
$$

Executing wagons in single wagonload traffic must arrive at a destination or shunting station only if the date of the order is included in the planning horizon (9).

$$
\begin{equation*}
\sum_{\left\{r \cdot f_{r}^{M}=f_{j}^{N}, f_{r}^{P} \leq f_{j}^{2}\right\}} y_{j, r}^{R}=y_{j}^{W} \text { for each } j \in J \tag{9}
\end{equation*}
$$

The continuous movement of wagons in single wagonload traffic is ensured by a constraint (10). Wagons must leave the shunting station after they have arrived at it.

$$
\begin{equation*}
\sum_{\left\{r \cdot: f_{r}^{N}=n, f_{r}^{<}<t\right\}} y_{j, r}^{R}-\sum_{\left\{r: s_{r}^{n}=n, s_{r}^{p}<t\right\}} y_{j, r}^{R} \geq 0 \text { for each } j \in J, n \in N_{j}^{T} \backslash\left\{S\left(s_{j}^{N}\right) \neq s_{j}^{N}\right\}, t \in T \tag{10}
\end{equation*}
$$

Finally, wagons located on the station which is not a shunting station must be transferred from this station within 3 time slots (12 hours) - a constraint (11). The value of the variable $y_{n, k, t}^{D}$ identifies the number of wagons to be transferred

$$
\begin{align*}
& \sum_{\left\{i: f_{i}^{N}=n, f_{i}^{Z}>t-1, f_{i}^{Z} \leq t, w_{i}^{K}=k\right\}} y_{i}^{W}-\sum_{\left\{i: s_{i}^{N}=n, s_{i}^{Z}>t-1, s_{i}^{Z} \leq t+3, w_{i}^{K}=k\right\}} y_{i}^{W}+\sum_{\left\{i: f_{i}^{N}=n, f_{i}^{Z}>t-1, f_{i}^{Z} \leq t+2, w_{i}^{K}=k\right\}} y_{i}^{W}-\sum_{\left\{i: s_{i}^{N}=n, s_{i}^{Z}>t-1, s_{i}^{Z} \leq t+2, w_{i}^{K}=k\right\}} y_{i}^{W} \\
& \sum_{\left\{r: f_{r}^{M}=n, f_{r}^{P}>t-1, f_{r}^{P} \leq t\right\}} y_{k, r}^{P}-\sum_{\left\{r: s_{r}^{M}=n, s_{r}^{P}>t-1, s_{r}^{P} \leq t+3\right\}} y_{k, r}^{P}+\sum_{\left\{r: f_{r}^{M}=n, f_{r}^{P}>t-1, f_{r}^{P} \leq t+2\right\}} y_{k, r}^{P}-\sum_{\left\{r: s_{r}^{M}=n, s_{r}^{P}>t-1, s_{r}^{P} \leq t+2\right\}} y_{k, r}^{P} \sum_{\left\{t^{\prime}>t, t^{\prime} \leq t+3\right\}} y_{n, k, t^{\prime}}^{D}+y_{n, k, t}^{D} \geq 0  \tag{11}\\
& \text { for each } k \in K, n \in N \backslash\{S(n)=n\}, t \in T
\end{align*}
$$

The objective function (12) consist of two main parts.

$$
\begin{equation*}
\sum_{i} p_{i} y_{i}^{Z}-\left(\alpha \sum_{r} x_{r}+\beta \sum_{k, n, t} y_{k, n, t}^{w}\right) \rightarrow \max \tag{12}
\end{equation*}
$$

The first element concerns on maximization of number executed orders with taking in the account the priority of the orders. The second part consist of two technical elements which should be minimal these are number of empty trains lunched and number of missing wagons on each station. In order to guarantee the feasibility of solution the parameters $\alpha$ and $\beta$ are used. The value of these parameters are set during optimization procedure.

## 4. Computational results

### 4.1. The test case

The presented algorithm was applied to find a solution to the real life problem with 745 orders ( 185 single wagonload orders therein). There are 1043 stations in the network and only 152 of them are shunting stations. There are 41 types of wagons in the orders. We considered a 3-day time period started on 2014-07-07 at 00:00:00 and ended on 2014-07-10 at 00:00:00. In this period, we considered 4 time slots so we had 25 points of time.

The flow of empty wagons should take into account also the following conditions. The number of wagons for each series at any station and in the whole planning horizon may not be negative. The length of the train may not exceed the maximum length of the train on a given road - due to the lack of such data, we assumed the length of the train to be 600 m . Similarly, the weight of the train must not exceed the maximum weight of the train on a given route. The weight of the train is the weight of wagons (tare) plus the weight of the load (specified in an order), increased by the weight of the locomotive (the value assumed for the locomotive is $120 t$ ). The maximum weight per connection assumed for our problem is $3,000 \mathrm{t}$. For this case, the most important is to ensure the fulfillment of as many orders as possible. The problem of the number of kilometers or the number of necessary trains is considered, however it is not crucial. The planning horizon covers the 3 -day period considered. In the optimization mode, we have two binary variables, two integer variable and the rest of them are positive continuous.

### 4.2. Experiments

The presented approach was used to find a solution to a test instance. We considered only one instance due to the limited availability of real data. Two problem solving strategies were taken into consideration.

The first strategy was tested in the Experiment 1. This approach started with the generation of an empty train set by using the first rule and the optimization model was solved. In the next step, the empty trains not used in the obtained solution were removed, the next heuristic rule of the empty train generation was applied, and the optimization model was solved. The first iteration was finished when the last heuristic rule was applied and the model was solved. These iterations were repeated iteratively until the solution was improved. In the presented case, the solution improvement was not observed after 3 iterations. The results obtained in the Experiment 1 are presented in Table 1.

Table 1 The results of the experiment 1.

| 754 orders | After 1st <br> iteration | After 2nd <br> iteration | After 3rd <br> iteration |
| :--- | :---: | :---: | :---: |
| \% of completed orders | 98.4 | 98.9 | 99.1 |
| \% of completed single <br> wagonload orders | 98.9 | 99.5 | 99.5 |
| Number of trains 259 266 | 258 |  |  |
| Distance traveled $[\mathrm{km}]$ | 28,417 | 27,989 | 26,194 |

As it was mentioned previously, we obtained a finished solution after 3 iterations. Our solution ensures the acceptance of $99.1 \%$ of the orders. The reason why a wagon was missing was the lack of a wagon of a given series in the whole system, which is a common issue. During the solution process, the optimization model was solved 15 times which results in the solution time of about 20 minutes.

The second strategy of finding a solution we considered is implemented as follows. In the initial iteration, the first heuristics rule was applied and the optimal solution was calculated. The next iteration consists of applying the rules from 2 to 5 and solving the optimization model only once after a set of empty trains was extended. These iterations were repeated iteratively until the solution was improved. As previously, the results we got after each iteration are presented in Table 2.

Table 2 The results of the experiment 2.

| 754 orders | After initial <br> iteration | After 1st <br> iteration | After 2nd <br> iteration | After 3rd <br> iteration | After 4th <br> iteration | After 5th <br> iteration |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| \% of completed orders | 86.3 | 98.7 | 99.1 | 99.2 | 99.5 | 99.5 |
| \% of completed single | 89.2 | 99.5 | 99.5 | 99.5 | 100 | 100 |
| wagon orders | 185 | 263 | 273 | 306 | 311 | 308 |
| Number of trains | 44,061 | 29,859 | 27,781 | 29,927 | 30,299 | 29,793 |
| Distance traveled $[\mathrm{km}]$ |  |  |  |  |  |  |

In order to finish the optimization process, a 6 iterations were needed. If the main optimization criterion is taken into consideration, the obtained result is better than in the previous case. The solution time is only 4 minutes, which is better than 20 minutes of the Experiment 1.

However, we have to point at some interesting findings we get after the solution comparison. While the second solution lets us complete only $0.4 \%$ (4) more customer's orders, the distance traveled in kilometers and the number of launched trains are substantially higher. Unfortunately, such situations could occur if the wagons needed to complete one order are geographically dispersed on 15 stations, about 15 trains are needed to move these wagons to an appropriate station. The high number of trains results in a long distance we have to travel in order to fulfill an order. This issue suggests that in order to find an optimal solution, the cost parameters should be used. Although the proposed algorithm is able to take into account the cost parameters, decision makers are not able to deliver values of these parameters.

The problem was solved by CPLEX implemented in the AIMMS software. The computation was carried out on a PC with Intel Core Duo $2,66 \mathrm{GHz}$ processor and 8 GB internal memory. In order to speed up the solution process, the value of the parameter - mixed integer gap - was set to $5 \%$. The solution time of the optimization model does
not exceed 80 seconds (including the model preparation) so on average, the process of finding the solution in the experiment 1 lasted about 20 minutes, while for the experiment 2 only 4 minutes. The time needed to generate the empty trains in heuristics was negligible. Our proposal can be applied for even a 7 day-planning horizon.

## 5. Summary

The presented empty wagon flow order is an important problem for all freight railway operators. Obtaining a good solution that ensures the fulfillment of most of customer orders allows for achieving not only a higher revenue but first of all the reduction of costs by decreasing a number of empty wagon orders completed. This in turn could lead to reducing the number of wagons and locomotives. The application of the presented hybrid approach results in obtaining a satisfactory solution within a short time with the use of the standard optimization software.

The hybrid approach proposed in the paper seems to be very a good solution to the empty wagon flow problem. In both concerned cases, the application of the heuristic algorithm of generating empty trains in connection with the optimization model allowed a satisfactory solution to be obtained.

In both cases, after the first heuristics we achieved over $99 \%$ fulfillment of the orders. The biggest improvement in the number of executed orders was observed after the second heuristics. This heuristics concerns single wagonload orders, so the number of executed single wagonload orders also increased mostly in this iteration. The next iteration improved the solution but this increase was rather inconsiderable. As the improvement of the solution is small in the third and next iteration, it is possible to stop the algorithm after the third iteration, but the time necessary to solve another model is short enough to repeat the iteration a few times again.

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