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Corrigendum

Corrigendum to “An equivalence of categories for graded modules over monomial algebras and path algebras of quivers” [J. Algebra 353 (1) (2012) 249–260]

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A B S T R A C T

Our published paper contains an incorrect statement of a result due to Artin and Zhang. This corrigendum gives the correct statement of their result and includes a new result that allows us to use the correct version of Artin and Zhang’s Theorem to prove our main theorem. Thus the main theorem of our published paper is correct as stated but its proof must be modified.

1. The error

1.1. We retain the notation and definitions in our published paper [2].

1.2. Proposition 2.1 in [2] is stated incorrectly. It should be replaced by the following statement.

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Proposition 1.1. *(See [1, Prop. 2.5].) Let A and B be $\mathbb{N}$-graded k-algebras such that $\dim_k A_i < \infty$ and $\dim_k B_i < \infty$ for all i. Let $\phi : A \to B$ be a homomorphism of graded k-algebras. If $\ker\phi$ and $\operatorname{coker}\phi$ belong to $\mathsf{Fdim}\, A$, then $- \otimes_A B$ induces an equivalence of categories*

$$\mathsf{QGr}\, A \to \frac{\mathsf{Gr}\, B}{\mathsf{T}_A}$$

where

$$\mathsf{T}_A = \{M \in \mathsf{Gr}\, B \mid M_A \in \mathsf{Fdim}\, A\}.$$

1.3. As in our published paper, A is a finitely presented connected monomial algebra and kQ is the path algebra of its Ufnarovskii graph, Q. Proposition 3.3 in [2] proved the existence of a homomorphism $\bar{f} : A \to kQ$ of graded k-algebras and [2, Prop. 4.1] showed that $\ker\bar{f}$ and $\operatorname{coker}\bar{f}$ belong to $\mathsf{Fdim}\, A$. Therefore Proposition 1.1 implies that $- \otimes_A kQ$ induces an equivalence of categories

$$\mathsf{QGr}\, A \to \frac{\mathsf{Gr}\, kQ}{\mathsf{T}_A}.$$

Thus to prove our main theorem, [2, Thm. 4.2] and [2, Thm. 1.1], which says that $- \otimes_A kQ$ induces an equivalence of categories

$$\mathsf{QGr}\, A \equiv \mathsf{QGr}\, kQ = \frac{\mathsf{Gr}\, kQ}{\mathsf{Fdim}\, kQ},$$

we must prove that $\mathsf{Fdim}\, kQ = \mathsf{T}_A$. We do this in Proposition 2.3 below.

2. Corrected proof

2.1. The algebra A has a distinguished set of generators called *letters* and its relations are generated by a finite set of words in those letters. The vertices in Q are certain words, the arrows in Q are also words, and the arrow corresponding to a word w is labeled by the first letter of w. The details are in [2, Sect. 3.3].

Lemma 2.1. *The arrows in Q have the following properties.*

(1) *Different arrows ending at the same vertex have different labels.*
(2) *Different arrows having the same label end at different vertices.*

Proof. (1) Let a and a' be different arrows ending at the vertex v. By definition, there are words w and w' such that $a = a_w$ and $a' = a_{w'}$, and letters x and x' such that $w = xv$ and $w' = x'v$. But $a \neq a'$ so $w \neq w'$ and therefore $x \neq x'$. But a is labeled x and a' is labeled x'.

(2) This is obviously equivalent to (1). □

2.2. The homomorphism $\bar{f} : A \to kQ$ is defined as follows: if x is one of the letters generating A, then

$$\bar{f}(x) := \begin{cases} \text{the sum of all arrows labeled } x, \\ 0 \quad \text{if there are no arrows labeled } x. \end{cases}$$

Lemma 2.2. *Let $\bar{f} : A \to kQ$ be the homomorphism above and write $A_n(kQ)$ for the right ideal of kQ generated by $\bar{f}(A_n)$. For all $n \geqslant 0$,*

$$A_n(kQ) = kQ_{\geqslant n}.$$

Proof. Write $B = kQ$.

We will prove that $A_n B_0 = B_n$ for all $n \geqslant 0$. This is certainly true for $n = 0$.

We will now show that $A_1 B_0 = B_1$. To prove this, let a be an arrow in Q that begins at vertex u and ends at vertex v. Then there are letters x and y such that $a = a_w$ and $w = uy = xv$. The arrow a is therefore labeled x and $\bar{f}(x) = \cdots + a + \cdots$. By Lemma 2.1, a is the *only* arrow labeled x that ends at v; hence, if e_v is the trivial path at vertex v, then $f(x)e_v = ae_v = a$. Hence $a \in A_1 B_0$. Thus $B_1 \subset A_1 B_0$. It is clear that $A_1 B_0 \subset B_1$ so this completes the proof that $A_1 B_0 = B_1$.

We now argue by induction on n. If $A_{n-1} B_0 = B_{n-1}$, then

$$A_n B_0 = (A_1)^n B_0 = A_1 (A_1)^{n-1} B_0 = A_1 B_{n-1} = A_1 B_0 B_{n-1} = B_1 B_{n-1} = B_n.$$

This completes the proof that $A_n B_0 = B_n$ for all $n \geqslant 0$. It follows that $A_n B = B_{\geqslant n}$. □

Proposition 2.3. $\mathsf{T}_A = \mathsf{Fdim}\, kQ$.

Proof. Every kQ-module is an A-module so a right kQ-module that is the sum of its finite dimensional kQ-submodules is also the sum of its finite dimensional A-submodules. Therefore $\mathsf{Fdim}\, kQ \subset \mathsf{T}_A$.

To prove the reverse inclusion, let $M \in \mathsf{Gr}\, kQ$ and suppose M is in T_A; i.e., M is the sum of its finite dimensional A-submodules. Let m be a homogeneous element in M. Then $\dim_k(mA) < \infty$ so $mA_{\geqslant n} = 0$ for $n \gg 0$. In particular, $mA_n = 0$ so

$$m(kQ_{\geqslant n}) = mA_n(kQ) = 0.$$

Hence $m(kQ)$ is isomorphic to a quotient of $kQ/kQ_{\geqslant n}$ and therefore finite dimensional. In particular, m belongs to the sum of the finite dimensional kQ-submodules of M. Hence $M \in \mathsf{Fdim}\, kQ$.

Thus, $\mathsf{T}_A \subset \mathsf{Fdim}\, kQ$ and the claimed equality follows. □

This completes the proof of [2, Thm. 4.2] and [2, Thm. 1.1].

References

[1] M. Artin, J.J. Zhang, Non-commutative projective schemes, Adv. Math. 109 (1994) 228–287.

[2] Cody Holdaway, S. Paul Smith, An equivalence of categories for graded modules over monomial algebras and path algebras of quivers, J. Algebra 353 (1) (2012) 249–260, doi:10.1016/j.jalgebra.2011.11.033.