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Nonperturbative models of quark stars in $f(R)$ gravity

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ABSTRACT

Quark star models with realistic equation of state in nonperturbative $f(R)$ gravity are considered. The mass-radius relation for $f(R) = R + \alpha R^2$ model is obtained. Considering scalar curvature R as an independent function, one can find out, for each value of central density, the unique value of central curvature for which one has solutions with the required asymptotic $R \to 0$ for $r \to \infty$. In other words, one needs a fine-tuning for R to achieve quark stars in $f(R)$ gravity. We consider also the analogue description in corresponding scalar-tensor gravity. The fine-tuning on R is equivalent to the fine-tuning on the scalar field ϕ in this description. For distant observers, the gravitational mass of the star increases with increasing α ($\alpha > 0$) but the interpretation of this fact depends on frame where we work. Considering directly $f(R)$ gravity, one can say that increasing of mass occurs by the "gravitational sphere" outside the star with some "effective mass". On the other hand, in conformal scalar-tensor theory, we also have a dilaton sphere (or "disphere") outside the star but its contribution to gravitational mass for distant observer is negligible. We show that it is possible to discriminate modified theories of gravity from General Relativity due to the gravitational redshift of the thermal spectrum emerging from the surface of the star.

1. Introduction

The accelerated expansion of the Universe remains one of the puzzles of modern cosmology. Initially discovered by observations of distant standard candles [1–3], this acceleration is confirmed by several other observations such as microwave background radiation (CMBR) anisotropies [4], cosmic shear through gravitational weak lensing surveys [5] and data on Lyman alpha forest absorption lines [6]. Analysis of these observations shows that the required cosmological dynamics cannot be obtained by models where the universe contains only standard matter and radiation or, in some sense, canonical scalar fields.

One possible solution of this puzzle is that General Relativity should be modified. It is possible to obtain accelerated expansion in modified gravity without assuming dark energy as a new material field [7–15].

Another explanation considers the existence of a non-standard cosmic fluid with negative pressure consisting about 70% of the universe energy, which is not clustered in large scale structure. However, the nature of this dark energy fluid is unclear. According to the simplest hypothesis, the dark energy is nothing else but the Einstein Cosmological Constant. Despite some questions at fundamental level (for example the cosmological constant problem and problems with fine tuning [16]), the ΛCDM model [17], based on dark matter and cosmological Λ term, gives, in principle, a good agreement with observational data at present epoch.

It has been shown that modified gravity also could give adequate description of cosmological observations [17–20]. One can conclude therefore that cosmological observations only cannot witness in favor to modified gravity or ΛCDM model. We need new probes and testbeds at completely different scales.

Specifically, any theory of modified gravity should be tested at astrophysical level also. One can hope that strong field regimes of relativistic stars could discriminate between General Relativity and its possible extensions [21]. For example, some models of $f(R)$ gravity can be rejected since they do not allow the existence of

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stable star configurations [22–27,42]. On the other hand, the possibility of the emergence of new theoretical stellar structures constitutes a powerful signature for any Extended Gravity model [28,29]. For example, one can note that stability of stars in modified gravity can be achieved due to the so-called *Chameleon Mechanism* [30,31].

In $f(R)$ gravity models, new scalar degrees of freedom appear. The scalar curvature in General Relativity is defined by pressure and density inside the star but, in $f(R)$ gravity, scalar curvature must be considered into dynamics as an effective new scalar field.

The structure of compact stars in perturbative $f(R)$ gravity has been recently investigated in some papers [32–36]. In this approach, the scalar curvature R is defined by the Einstein field equations at zeroth order as a small parameter, i.e. $R \sim T$, where T is the trace of the energy–momentum tensor.

In this paper, we construct also self-consistent star models for $f(R) = R + \alpha R^2$ gravity. We consider the case of quark stars with very simple equation of state. These systems could be very useful both to constrain modified gravity and to consider stiff matter conditions in early phase transitions.

The main result (in comparison to perturbative approach) consists on the result that one can state that, although the gravitational mass of the star decreases with increasing α (as in the perturbative approach), outside the star a "gravitational sphere" emerges. In other words, we have that the gravitational mass of such objects (from the viewpoint of distant observers) increases with increasing α. This fact could constitute a new paradigm to probe such modified gravity at astrophysical scales.

The paper is organized as follows. In Section 2, we present the field equations for $f(R)$ gravity. For spherically symmetric solutions of these equations, we obtain the modified Tolman–Oppenheimer–Volkov (TOV) equations. In Section 3, we give the description of the models in the corresponding scalar-tensor theory. In Section 4, the quark star models are presented. Discussion and conclusions are reported in Section 5.

2. Modified TOV equations in $f(R)$ gravity

The action for $f(R)$ gravity (in units where $G = c = 1$) can be written in the form:

$$S = \frac{1}{16\pi} \int d^4x \sqrt{-g} f(R) + S_{\text{matter}}, \tag{1}$$

where g is determinant of the metric $g_{\mu\nu}$ and S_{matter} is the action of the standard perfect fluid matter. Therefore the Hilbert–Einstein action is replaced by a generic function $f(R)$ of the Ricci scalar R.

For solutions describing stellar objects, one can assume that metric is spherically symmetric with two independent functions of radial coordinate, that is:

$$ds^2 = -e^{2\psi} dt^2 + e^{2\lambda} dr^2 + r^2 d\Omega^2. \tag{2}$$

Varying the action with respect to $g_{\mu\nu}$ gives the field equations for metric functions:

$$f'(R) G_{\mu\nu} - \frac{1}{2}\big(f(R) - f'(R)R\big) g_{\mu\nu} - (\nabla_\mu \nabla_\nu - g_{\mu\nu}\Box) f'(R) = 8\pi T_{\mu\nu}. \tag{3}$$

Here $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}$ is the Einstein tensor, $f'(R) = df(R)/dR$ is the derivative of $f(R)$ with respect to the scalar curvature and $T_{\mu\nu}$ is the energy–momentum tensor. For a perfect fluid, we have $T_{\mu\nu} = \mathrm{diag}(e^{2\psi}\rho, e^{2\lambda} p, r^2 p, r^2 \sin^2\theta p)$, where ρ is the matter density and p is the pressure.

The components of the field equations are nothing else but the Tolman–Oppenheimer–Volkov equations for $f(R)$ gravity:

$$\frac{f'(R)}{r^2} \frac{d}{dr}\big[r\big(1 - e^{-2\lambda}\big)\big] = 8\pi\rho + \frac{1}{2}\big(f'(R)R - f(R)\big) + e^{-2\lambda}\left[\left(\frac{2}{r} - \frac{d\lambda}{dr}\right)\frac{df'(R)}{dr} + \frac{d^2 f'(R)}{dr^2}\right] \tag{4}$$

$$\frac{f'(R)}{r}\left[2e^{-2\lambda}\frac{d\psi}{dr} - \frac{1}{r}\big(1 - e^{-2\lambda}\big)\right] = 8\pi p + \frac{1}{2}\big(f'(R)R - f(R)\big) + e^{-2\lambda}\left(\frac{2}{r} + \frac{d\psi}{dr}\right)\frac{df'(R)}{dr} \tag{5}$$

From the conservation equations for the energy–momentum tensor, $\nabla^\mu T_{\mu\nu} = 0$, the hydrostatic condition equilibrium follows:

$$\frac{dp}{dr} = -(\rho + p)\frac{d\psi}{dr}. \tag{6}$$

For $f(R) = R$ these equations reduce to the ordinary TOV equations of General Relativity. In $f(R)$ gravity, the scalar curvature is dynamical variable and the equation for R can be obtained by taking into account the trace of field equations (3). We have

$$3\Box f'(R) + f'(R)R - 2f(R) = -8\pi(\rho - 3p), \quad \text{where } e^{2\lambda}\Box = -e^{2\lambda - 2\psi}\frac{\partial^2}{\partial t^2} + \left(\frac{2}{r} + \frac{d\psi}{dr} - \frac{d\lambda}{dr}\right)\frac{\partial}{\partial r} + \frac{\partial^2}{\partial r^2}. \tag{7}$$

It is a Klein–Gordon-like equation. For $f(R) = R$ this equation is reduced to the equality

$$R = 8\pi(\rho - 3p). \tag{8}$$

Inside the star Eqs. (4)–(7) can be solved numerically for a given Equation of State (EoS) $p = f(\rho)$ and initial conditions $\lambda(0) = 0$, $R(0) = R_c$, $R'(0) = 0$ and $\rho(0) = \rho_c$.

Outside the star, the solution is defined by Eqs. (4), (5), (7) where one needs to put $\rho = p = 0$. We have to use the junction conditions on the surface of the star, that is ($r = r_s$):

$$\lambda_{in}(r_s) = \lambda_{out}(r_s), \qquad R_{in}(r_s) = R_{out}(r_s), \qquad R'_{in}(r_s) = R'_{out}(r_s).$$

A mass parameter $m(r)$ can be defined according to the relation:

$$e^{-2\lambda} = 1 - \frac{2m}{r}. \tag{9}$$

The asymptotic flatness requirement gives the constraints on the scalar curvature and the mass parameter:

$$\lim_{r\to\infty} R(r) = 0, \qquad \lim_{r\to\infty} m(r) = \text{const}.$$

For the following considerations, it is convenient to use dimensional variables $m \to mM_\odot$ and $r \to r_g r$ where $r_g = GM_\odot/c^2$. This assumption allows to refer to typical stellar structures.

3. The scalar-tensor gravity picture

Let us consider the analogue modified gravity description in terms of scalar-tensor gravity (see [37,38]). For $f(R)$ gravity one can construct the equivalent Brans–Dicke-like theory where the action for the gravitational sector is:

$$S_g = \frac{1}{16\pi}\int d^4x \sqrt{-g}\big(\Phi R - U(\Phi)\big). \tag{10}$$

Here $\Phi = df(R)/dR$ is gravitational scalar and $U(\Phi) = Rf'(R) - f(R)$ is potential. It is worth noticing that standard Brans–Dicke shows a kinetic term instead of a potential like in this case.

Using the conformal transformation $\tilde{g}_{\mu\nu} = \Phi g_{\mu\nu}$, one can write the action in the Einstein frame as

$$S_g = \frac{1}{16\pi G} \int d^4x\sqrt{-\tilde{g}}\left(\tilde{R} - 2\tilde{g}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - 2V(\phi)\right), \qquad (11)$$

where $\phi = \sqrt{3}\ln\Phi/2$. In such a frame, the potential becomes $V(\phi) = \Phi^{-2}(\phi)U(\Phi(\phi))/2$. It is convenient to choose the space-time metric in a form that formally coincided with (2). We can redefine the functions ψ and λ as

$$d\tilde{s}^2 = \Phi ds^2 = -e^{2\tilde{\psi}}dt^2 + e^{2\tilde{\lambda}}d\tilde{r}^2 + \tilde{r}^2 d\Omega^2. \qquad (12)$$

In Eq. (12) $\tilde{r}^2 = \Phi r^2$, $e^{2\tilde{\psi}} = \Phi e^{2\psi}$ and from equality

$$\Phi e^{2\lambda}dr^2 = e^{2\tilde{\lambda}}d\tilde{r}^2$$

it follows that

$$e^{-2\lambda} = e^{-2\tilde{\lambda}}\left(1 - \tilde{r}\phi'(\tilde{r})/\sqrt{3}\right)^2.$$

Therefore the mass parameter $m(\tilde{r})$ can be obtained from $\tilde{m}(\tilde{r})$ as

$$m(\tilde{r}) = \frac{\tilde{r}}{2}\left(1 - \left(1 - \frac{2\tilde{m}}{\tilde{r}}\right)\left(1 - \tilde{r}\phi'(\tilde{r})/\sqrt{3}\right)^2\right)e^{-\phi/\sqrt{3}}. \qquad (13)$$

The resulting equations for metric functions $\tilde{\lambda}$ and $\tilde{\psi}$ (tildes in the following will be omitted for simplicity):

$$\frac{1}{r^2}\frac{d}{dr}\left[r\left(1 - e^{-2\lambda}\right)\right] = 8\pi e^{-4\phi/\sqrt{3}}\rho + e^{-2\lambda}\left(\frac{d\phi}{dr}\right)^2 + V(\phi), \qquad (14)$$

$$\frac{1}{r}\left[2e^{-2\lambda}\frac{d\psi}{dr} - \frac{1}{r}\left(1 - e^{-2\lambda}\right)\right] = 8\pi e^{-4\phi/\sqrt{3}}p + e^{-2\lambda}\left(\frac{d\phi}{dr}\right)^2 - V(\phi). \qquad (15)$$

The hydrostatic equilibrium condition can be rewritten as

$$\frac{dp}{dr} = -(\rho + p)\left(\frac{d\psi}{dr} - \frac{1}{\sqrt{3}}\frac{d\phi}{dr}\right). \qquad (16)$$

Finally the last field equation for the scalar field is equivalent to Eq. (7) in $f(R)$ theory:

$$\Box\phi - \frac{1}{2}\frac{dV(\phi)}{d\phi} = -\frac{4\pi}{\sqrt{3}}e^{-4\phi/\sqrt{3}}(\rho - 3p). \qquad (17)$$

The first two equations in fact coincide with the ordinary TOV equations with redefined matter density and pressure where also the energy density and the pressure of the scalar field ϕ are included. It is worth noticing that the potential $V(\phi)$ can be written in explicit form only for simple $f(R)$ models. For example for $f(R) = R + \alpha R^2$, one obtains that

$$V(\phi) = \frac{1}{8\alpha}\left(1 - e^{-2\phi/\sqrt{3}}\right)^2. \qquad (18)$$

Considering simple power models like $f(R) = R + \alpha R^m$, we have

$$V(\Phi) = D\Phi^{-2}(\Phi - 1)^{\frac{m}{m-1}}, \quad \text{with } D = \frac{m-1}{2m^{\frac{m}{m-1}}}\alpha^{\frac{1}{1-m}}, \quad \Phi = e^{2\phi/\sqrt{3}}. \qquad (19)$$

These considerations can be applied to quark star models by assuming suitable EoS.

4. Quark star models in $f(R) = R + \alpha R^2$ gravity

A quark star is a self-gravitating system consisting of deconfined u, d and s quarks and electrons [39]. Deconfined quarks form color superconductor system, leading to a softer equation of state in comparison with the standard hadron matter.

In the frame of the so-called *MIT bag model* one can obtain a simple equation of state for quark matter:

$$p = c(\rho - 4B), \qquad (20)$$

where B is the bag constant. The value of parameter c depends on the chosen mass of strange quark m_s and QCD coupling constant. For $m_s = 0$, the parameter is $c = 1/3$ as for radiation. For more realistic model with $m_s = 250$ MeV, we have $c = 0.28$. The value of B lies in the interval $0.98 < B < 1.52$ in units of $B_0 = 60$ MeV/fm^3 [40].

The solution of Eqs. (4)–(7) can be achieved by using a perturbative approach (see [32,33] for details). In this perturbative approach, the curvature scalar cannot considered as an additional degree of freedom since its value is fixed by the relation (8). Perturbative calculations show that gravitational mass of star decreases with increasing α for the $f(R) = R + \alpha R^2$ model.

Let us consider now the system (4)–(7) for $f(R) = R + \alpha R^2$ model assuming that R is an independent function. One has to note that authors of [41] estimated the upper limit for α as $\sim 5 \times 10^{15}$ cm$^2 = 2.3 \times 10^5 r_g^2$ from binary pulsar data.

We find that for each value of central density, we have the solution with required asymptotic $R \to 0$ at $r \to \infty$ only for a unique value of $R(0)$. In the scalar-tensor description, this uniqueness of $R(0)$ is equivalent to a fine-tuning conditions for the scalar field ϕ at the center of star for a given density.

It is interesting to note that the gravitational mass of star M (by calculating as asymptotic mass value the parameter $m(r)$) increases with increasing α. At first glance this contradiction (in comparison with results coming from perturbative approach) shows that perturbative approach is inadequate to deal with such problems in $f(R)$ gravity. However a detailed investigation leads to the conclusion that perturbative approach is neither inadequate nor incomplete. One can say that the increasing of mass occurs on the “gravitational sphere” outside the star as some “effective mass”. Without this “sphere” the gravitational mass of star decreases (of course we cannot actually distinguish the star from this sphere but this interpretation has a right to exist). In the framework of the perturbative approach, one cannot account for the existence of such a sphere because the Schwarzschild solution outside the star is assumed.

In the conformal frame of the corresponding scalar-tensor theory, we have also the so-called dilaton sphere (“disphere”) outside the star but its contribution to gravitational mass for distant observer is negligible.

The mass-radius and the mass-central density diagram for quark stars with realistic EoS ($c = 0.28$, $B = 60$ MeV/fm^3) are represented in Fig. 1. The radius of star increases in comparison with General Relativity. The star configurations with maximal mass correspond to larger central densities (see Table 1). We consider also an EoS with $c = 0.28$ and $B = 60$ MeV/fm^3 (Table 2). In Fig. 2 we plot the dependence of mass parameter $m(r_s)$ (the value of mass parameter on the surface of star) against the radius and the central density. In Fig. 3 the dependence $\Delta M = M - m(r_s)$ from central density is presented.

The mass parameter profile $m(r)$ for star configurations with maximal mass is represented in Fig. 4. One can see that the radius of gravitational sphere increases with growing α. The mass parameter $\tilde{m}$ reaches the value close to the maximal on the surface of

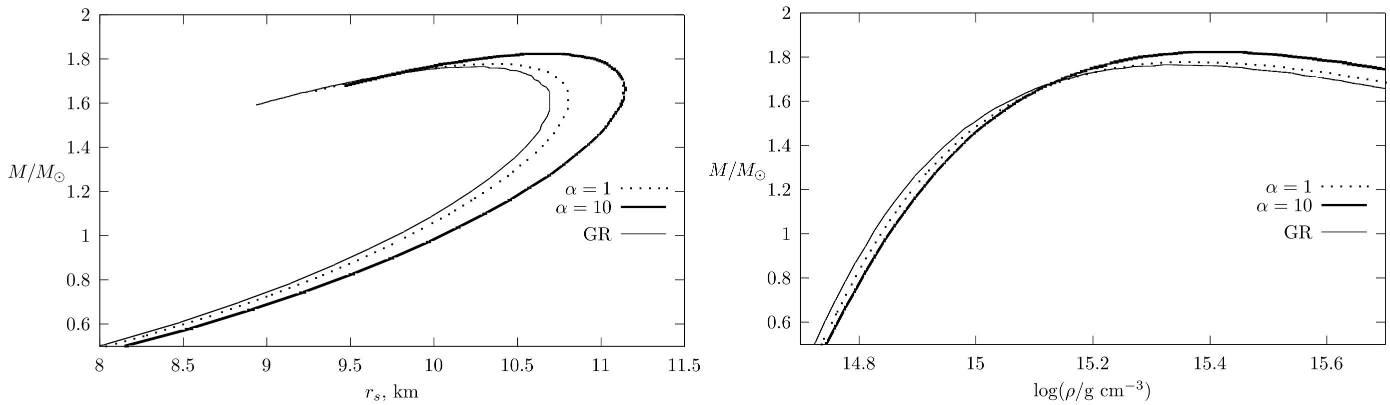

Fig. 1. The mass-radius (left panel) and mass-central density (right panel) diagram in model $f(R) = R + \alpha R^2$ and in GR for quark stars with $B = 60$ MeV/fm^3 and $c = 0.28$.

Table 1
Quark star properties using the simple model (20) with $c = 0.28$ and $B = 60$ MeV/fm^3 for $f(R) = R + \alpha R^2$ gravity.

α, r_g^2	M_{max}, $M_\odot$	$m(r_s)$, $M_\odot$	r_s, km	ρ_c, 10^{15} g/cm^3	R_c, $10^{-3} r_g^{-2}$
0	1.764	1.764	10.26	2.17	28.62
1	1.778	1.649	10.38	2.26	19.34
10	1.832	1.552	10.68	2.54	5.78

Table 2
Quark star properties using the simple model (20) with $c = 0.31$ and $B = 60$ MeV/fm^3 for $f(R) = R + \alpha R^2$ gravity.

α, r_g^2	M_{max}, $M_\odot$	$m(r_s)$, $M_\odot$	r_s, km	ρ_c, 10^{15} g/cm^3	R_c, $10^{-3} r_g^{-2}$
0	1.883	1.883	10.54	2.09	22.01
1	1.901	1.772	10.61	2.26	19.34
10	1.966	1.681	10.92	2.54	5.38

the star. In Fig. 5 the scalar curvature and the scalar field (in terms of scalar-tensor description) as functions of radial coordinate are given.

One has to note that the initial condition for R in the center correlates with the value $R^{(0)} = 8\pi(\rho - 3p)$, i.e. the scalar curvature in General Relativity. For $\alpha \to 0$, the scalar curvature $R(0) \to R^{(0)}$ as it is expected.

One can see that the deviation of the mass-radius relation from General Relativity, in principle, is not so large. Taking into account that there are no precise radius measurements for stars, one cannot hope that the mass-radius relation gives argument in favor (or not) of modified gravity.

However, in principle, it is possible to discriminate modified theories of gravity from General Relativity due to the redshift of the surface atomic lines. The gravitational redshift z of thermal spectrum detected at infinity can be calculated as

$$z = e^{-\psi} - 1. \qquad (21)$$

In General Relativity we have simply

$$z(r_s) = \frac{1}{\sqrt{1 - 2M/r_s}} - 1.$$

In the case of modified gravity, we have another dependence on the surface redshift from gravitational mass. Calculations give the following results. In General Relativity for maximal mass we have $z(r_s) = 0.424$ ($c = 0.28$ and $B = 60$ MeV/fm^3). In the case of quadratic gravity with the surface redshift for star with maximal mass, it is 0.431 ($\alpha = 1$) and 0.458 ($\alpha = 10$).

Of course the measurement of $z(r_s)$ can constrain the theories of gravity only in the case where the mass of the star is measured with high precision (for example by precise measurements by binary systems dynamics). Another requirement for discriminating between General Relativity and modified gravity is that one should know the realistic equation of state in extreme details. This information is not available with today facilities but could be acquired by forthcoming experiments like the *Large Observatory For X-ray Timing* (LOFT) whose one of the main scientific goals is to select reliable equations of state for compact objects in strong gravity regimes [43].

Another difficulty is that present accuracy on redshift measurements is not sufficient yet for constraining gravity in strong regime. In future, one can hope that the increasing number of good quality data on the thermal emission with mass measurements could

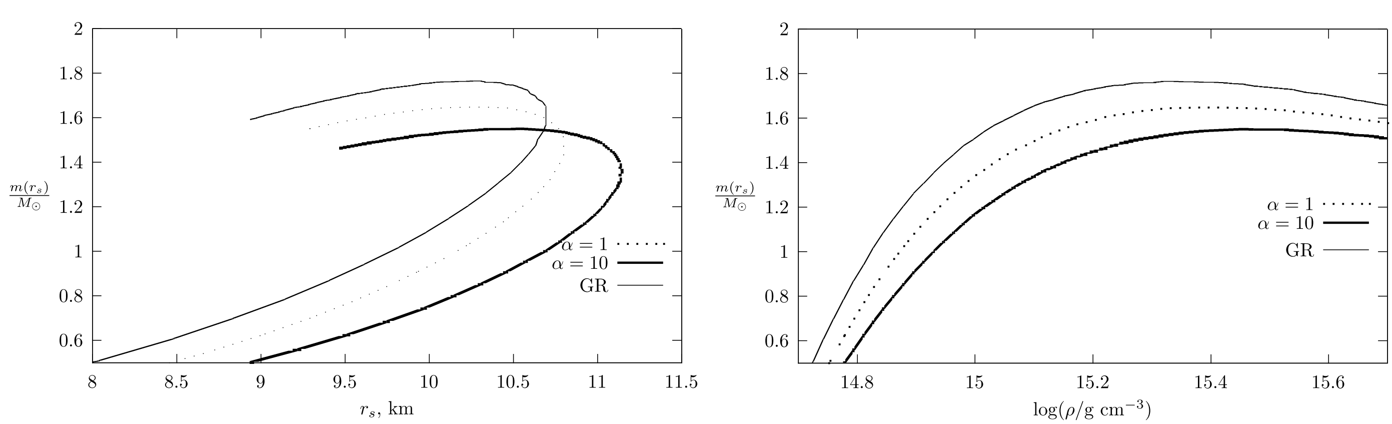

Fig. 2. The $m(r_s)$-radius diagram (left panel) and $m(r_s)$-central density (right panel) diagram in model $f(R) = R + \alpha R^2$ for quark stars with $B = 60$ MeV/fm^3 and $c = 0.28$.

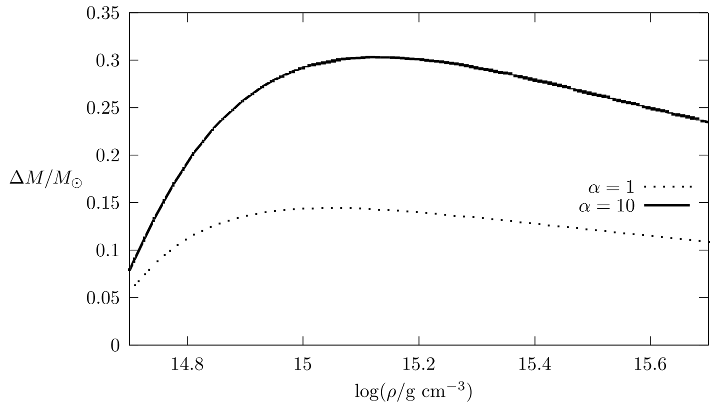

Fig. 3. The dependence $\Delta M = M - m(r_s)$ from central density in model $f(R) = R + \alpha R^2$ for quark stars with $B = 60$ MeV/fm^3 and $c = 0.28$.

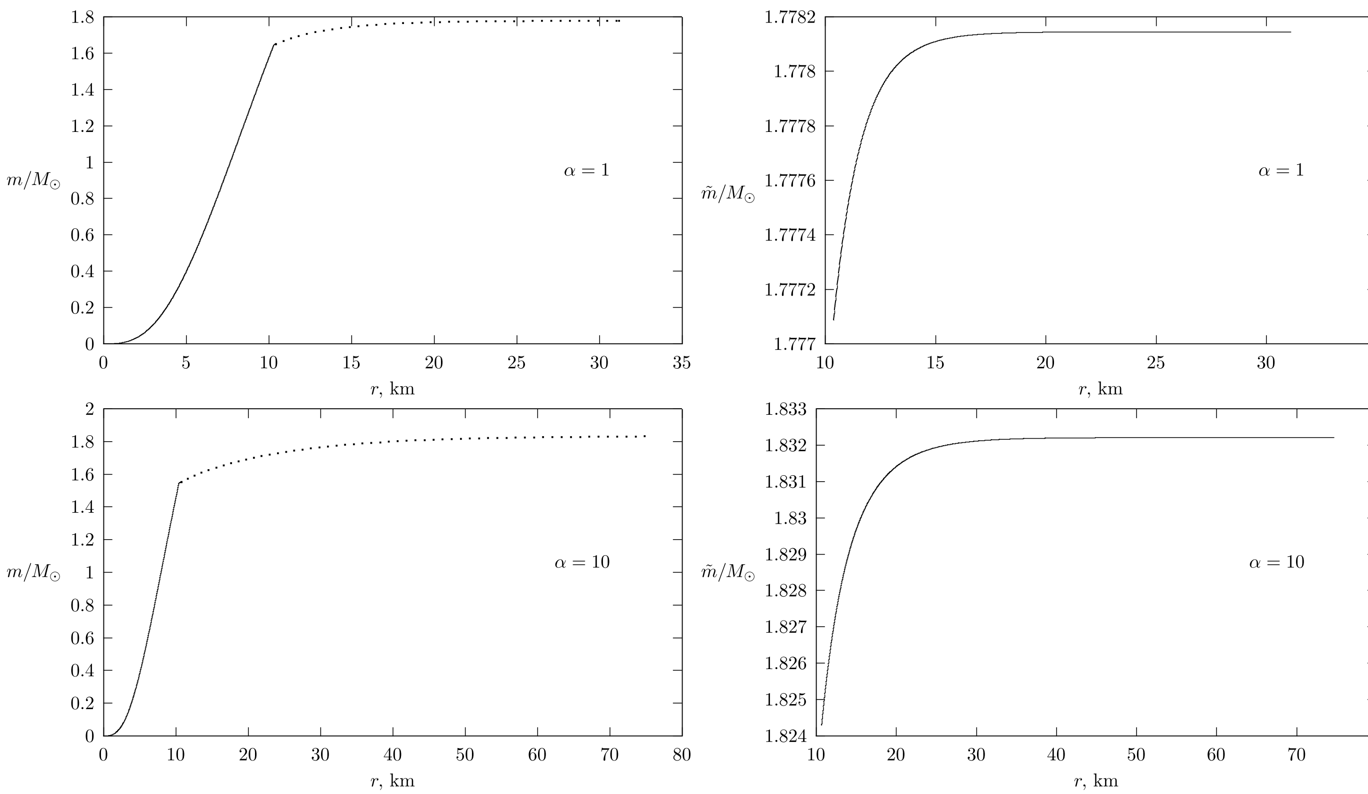

Fig. 4. The mass parameter $m(r)$ profile inside (bold curves) and outside the star (dotted curves) (left panel) in model $f(R) = R + \alpha R^2$ for stars configuration with maximal mass ($B = 60$ MeV/fm^3 and $c = 0.28$). For comparison we give the dependence $\tilde{m}(r)$ outside the star in corresponding scalar-tensor theory (right panel).

help to distinguish General Relativity from models of $f(R)$ gravity. Again LOFT experiment could be useful for this task.

In this framework, the question about possible instabilities during the star formation arises. First of all, we have to note that the scalar curvature inside the matter sphere is smaller in comparison with General Relativity for $\alpha > 0$ for the considered models. It is also known that adding the higher derivative term ($\sim R^\beta$, $1 < \beta \leqslant 2$) to the standard Hilbert–Einstein action could cure the singularity (for details see [42]).

For the quadratic gravity, it is easy to see this fact by using the scalar-tensor description. Eq. (17) for the scalar field can be rewritten as:

$$\Box\phi - \frac{dV_{eff}}{d\phi} = 0, \quad V_{eff} = \frac{1}{2}V + \pi e^{-4\phi/\sqrt{3}}(\rho - 3p). \tag{22}$$

The effective squared mass of the scalar field is defined as

$$m^2_{eff} = \frac{d^2 V_{eff}}{d\phi^2} = \frac{1}{2}\frac{d^2 V}{d\phi^2} + \frac{16\pi}{3} e^{-4\phi/\sqrt{3}}(\rho - 3p). \tag{23}$$

The first term for the model with R-squared term is positive. For quark stars, the second term is also positive (because $p < \rho/3$). Therefore effective mass has a real value. It is well-known that in this case the solution corresponds to the minimum of potential (this minimum corresponds to some value of curvature). For radial modes of the perturbations, we have decaying solutions (see [26]). Therefore the considered gravity model (with the quark equation of state) could become free of curvature singularity.

5. Conclusion

We have considered realistic quark star models in nonperturbative $f(R)$ gravity and obtained the parameters of stars in $f(R) = R + \alpha R^2$ model. The key issue of such a nonperturbative approach

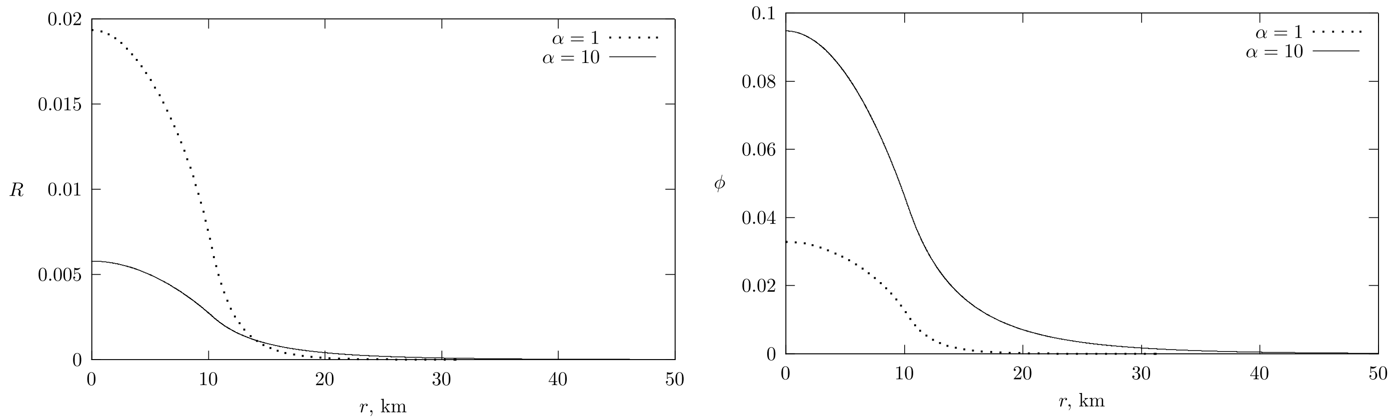

Fig. 5. The scalar curvature $R(r)$ (left panel) in model $f(R) = R + \alpha R^2$ for stars configuration with maximal mass ($B = 60$ MeV/fm^3 and $c = 0.28$). On right panel we depict the corresponding profile of scalar field ϕ in corresponding scalar-tensor theory.

is that one needs to consider the scalar curvature as an independent function. The shooting method of solution gives that there is a unique value of the curvature at the center of the star where solution has the required asymptotic behavior. This fine-tuning for R at the center of the star is equivalent to the fine-tuning of the scalar field ϕ in the corresponding scalar-tensor theory. For a distant observer, the gravitational mass of the star increases with increasing α ($\alpha > 0$). One can say that the increasing of the mass occurs on the "gravitational sphere" outside the star with some "effective mass". In the corresponding conformally transformed scalar-tensor theory, we have also the dilaton sphere (or the disphere) outside the star but its contribution to the gravitational mass for distant observer is negligible.

The considered approach can be applied for the analysis of structure of neutron stars in modified gravity. The calculations show that for realistic hyperon EoS we have, in principle, the same effects as for quark stars.

Although the deviation of mass-radius relation from General Relativity is sufficiently small, it is possible to discriminate modified theory of gravity from General Relativity due to the redshift of the surface atomic lines. In $f(R) = R + \alpha R^2$ gravity the surface redshift grows with the increasing of the parameter α.

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