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μ–τ symmetry and maximal CP violation

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Abstract

We argue the possibility that a real part of a flavor neutrino mass matrix only respects a μ–τ symmetry. This possibility is shown to be extended to more general case with a phase parameter θ, where the μ–τ symmetric part has a phase of $\theta/2$. This texture shows maximal CP violation and no Majorana CP violation.
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The present experimental data on neutrino oscillations [1,2] indicate the mixing angles [3] satisfying

$$0.70 < \sin^2 2\theta_\odot < 0.95, \qquad 0.92 < \sin^2 2\theta_{\rm atm}, \qquad \sin^2\theta_{\rm CHOOZ} < 0.05, \tag{1}$$

where $\theta_\odot$ is the solar neutrino mixing angle, $\theta_{\rm atm}$ is the atmospheric neutrino mixing angle and $\theta_{\rm CHOOZ}$ is for the mixing angle between ν_e and ν_τ. These mixing angles are identified with the mixings among three flavor neutrinos, ν_e, ν_μ and ν_τ, yielding three massive neutrinos, $\nu_{1,2,3}$: $\theta_{12} = \theta_\odot$, $\theta_{23} = \theta_{\rm atm}$ and $\theta_{13} = \theta_{\rm CHOOZ}$. These data seem to be consistent with the presence of a μ–τ symmetry [4–7] in the neutrino sector, which provides maximal atmospheric neutrino mixing with $\sin^2 2\theta_{23} = 1$ as well as $\sin\theta_{13} = 0$.

Although neutrinos gradually reveal their properties in various experiments since the historical Super-Kamiokande confirmation of neutrino oscillations [1], we expect to find yet unknown property related to CP violation [8]. The effect of the presence of a leptonic CP violation can be described by four phases in the PMNS neutrino mixing matrix, $U_{\rm PMNS}$ [9], to be denoted by one Dirac phase of δ and three Majorana phases of $\beta_{1,2,3}$ as $U_{\rm PMNS} = U_\nu K$ [10] with

$$U_\nu = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - s_{23}c_{12}s_{13}e^{i\delta} & c_{23}c_{12} - s_{23}s_{12}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{23}s_{12} - c_{23}c_{12}s_{13}e^{i\delta} & -s_{23}c_{12} - c_{23}s_{12}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}, \qquad K = {\rm diag}\left(e^{i\beta_1}, e^{i\beta_2}, e^{i\beta_3}\right), \tag{2}$$

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where $c_{ij} = \cos\theta_{ij}$ and $s_{ij} = \sin\theta_{ij}$ (i, j=1,2,3) and Majorana CP violation is specified by two combinations made of $\beta_{1,2,3}$ such as $\beta_i - \beta_3$ in place of β_i in K. To examine such effects of CP violation, there have been various discussions [11] including those on the possible textures of flavor neutrino masses [12–15].

In this Letter, we would like to focus on the role of the μ–τ symmetry in models with CP violation [14,15], which can be implemented by introducing complex flavor neutrino masses. The μ–τ symmetric texture gives $\sin\theta_{13} = 0$ as well as maximal atmospheric neutrino mixing characterized by $c_{23} = \sigma s_{23} = 1/\sqrt{2}$ ($\sigma = \pm 1$). Because of $\sin\theta_{13} = 0$, Dirac CP violation is absent in Eq. (2) and CP violation becomes of the Majorana type. Since the μ–τ symmetry is expected to be approximately realized, its breakdown is signaled by $\sin\theta_{13} \neq 0$. To have $\sin\theta_{13} \neq 0$, we discuss another implementation of the μ–τ symmetry such that the symmetry is only respected by the real part of M_ν. The discussion is based on more general case, where M_ν is controlled by one phase to be denoted by θ and the specific value of $\theta = 0$ yields the μ–τ symmetric real part. It turns out that Majorana CP violation is absent because all three Majorana phases are calculated to be $-\theta/4$ while Dirac CP violation becomes maximal.

Our complex flavor neutrino mass matrix of M_ν is parameterized by

$$M_\nu = \begin{pmatrix} M_{ee} & M_{e\mu} & M_{e\tau} \\ M_{e\mu} & M_{\mu\mu} & M_{\mu\tau} \\ M_{e\tau} & M_{\mu\tau} & M_{\tau\tau} \end{pmatrix}, \tag{3}$$

where $U^T_{\mathrm{PMNS}} M_\nu U_{\mathrm{PMNS}} = \mathrm{diag}(m_1, m_2, m_3)$.[1] The mixing angles have been calculated in the Appendix of Ref. [16] and satisfy

$$\sin 2\theta_{12}(\lambda_1 - \lambda_2) + 2\cos 2\theta_{12} X = 0, \tag{4}$$

$$\sin 2\theta_{13}\left(M_{ee}e^{-i\delta} - \lambda_3 e^{i\delta}\right) + 2\cos 2\theta_{13} Y = 0, \tag{5}$$

$$(M_{\tau\tau} - M_{\mu\mu})\sin 2\theta_{23} - 2M_{\mu\tau}\cos 2\theta_{23} = 2s_{13}e^{-i\delta}X, \tag{6}$$

and neutrino masses are given by

$$m_1 e^{-2i\beta_1} = \frac{\lambda_1 + \lambda_2}{2} - \frac{X}{\sin 2\theta_{12}}, \qquad m_2 e^{-2i\beta_2} = \frac{\lambda_1 + \lambda_2}{2} + \frac{X}{\sin 2\theta_{12}},$$
$$m_3 e^{-2i\beta_3} = \frac{c_{13}^2\lambda_3 - s_{13}^2 e^{-2i\delta} M_{ee}}{\cos 2\theta_{13}}, \tag{7}$$

where

$$\lambda_1 = c_{13}^2 M_{ee} - 2c_{13}s_{13}e^{i\delta}Y + s_{13}^2 e^{2i\delta}\lambda_3, \qquad \lambda_2 = c_{23}^2 M_{\mu\mu} + s_{23}^2 M_{\tau\tau} - 2s_{23}c_{23}M_{\mu\tau},$$
$$\lambda_3 = s_{23}^2 M_{\mu\mu} + c_{23}^2 M_{\tau\tau} + 2s_{23}c_{23}M_{\mu\tau}, \tag{8}$$

$$X = \frac{c_{23}M_{e\mu} - s_{23}M_{e\tau}}{c_{13}}, \qquad Y = s_{23}M_{e\mu} + c_{23}M_{e\tau}. \tag{9}$$

To clarify the importance of the μ–τ symmetry, which accommodates maximal atmospheric neutrino mixing and $\sin\theta_{13} = 0$, we first review what conditions are imposed by the requirement of $\sin\theta_{13} = 0$. From Eq. (5), we require that

$$Y = s_{23}M_{e\mu} + c_{23}M_{e\tau} = 0, \tag{10}$$

giving rise to $\tan\theta_{23} = -\mathrm{Re}(M_{e\tau})/\mathrm{Re}(M_{e\mu}) = -\mathrm{Im}(M_{e\tau})/\mathrm{Im}(M_{e\mu})$. Since $\sin\theta_{13} = 0$, Eq. (6) reads

$$(M_{\tau\tau} - M_{\mu\mu})\sin 2\theta_{23} = 2M_{\mu\tau}\cos 2\theta_{23}. \tag{11}$$

[1] It is understood that the charged leptons and neutrinos are rotated, if necessary, to give diagonal charged-current interactions and to define the flavor neutrinos of ν_e, ν_μ and ν_τ.

These are the well-known relations that determine θ_{23} if $\sin\theta_{13}=0$. Maximal atmospheric neutrino mixing arises if

$$M_{\tau\tau}=M_{\mu\mu}, \tag{12}$$

which in turn gives

$$M_{e\tau}=-\sigma M_{e\mu}. \tag{13}$$

The flavor neutrino masses satisfying Eqs. (12) and (13) suggest the presence of μ–τ symmetry in neutrino physics. The remaining mixing angle of θ_{12} satisfies

$$M_{\mu\mu}-\sigma M_{\mu\tau}=M_{ee}+\frac{2\sqrt{2}}{\tan 2\theta_{12}}M_{e\mu}, \tag{14}$$

which determines the definite correlation of the phases of the flavor neutrino masses.

In place of Eqs. (10) and (11), using a Hermitian matrix of $\mathbf{M}=M_\nu^\dagger M_\nu$, we can find that $\tan\theta_{23}=-\operatorname{Re}(\mathbf{M}_{e\tau})/\operatorname{Re}(\mathbf{M}_{e\mu})=-\operatorname{Im}(\mathbf{M}_{e\tau})/\operatorname{Im}(\mathbf{M}_{e\mu})$, where $\mathbf{M}_{e\mu}=M_{ee}^*M_{e\mu}+M_{e\mu}^*M_{\mu\mu}+M_{e\tau}^*M_{\mu\tau}$ and $\mathbf{M}_{e\tau}=M_{ee}^*M_{e\tau}+M_{e\mu}^*M_{\mu\tau}+M_{e\tau}^*M_{\tau\tau}$. In addition, we have argued that $\tan\theta_{23}$ is directly determined by $\tan\theta_{23}=\operatorname{Im}(\mathbf{M}_{e\mu})/\operatorname{Im}(\mathbf{M}_{e\tau})$ satisfied in any models with complex neutrino masses irrespective of the values of $\sin\theta_{13}$ [17]. Both expressions of $\tan\theta_{23}$ are compatible if $(\operatorname{Im}(\mathbf{M}_{e\mu}))^2+(\operatorname{Im}(\mathbf{M}_{e\tau}))^2=0$, yielding $\operatorname{Im}(\mathbf{M}_{e\mu})=\operatorname{Im}(\mathbf{M}_{e\tau})=0$. Since the Dirac CP violation phase is absent for $\sin\theta_{13}=0$, $\mathbf{M}$ with the Majorana phases cancelled is necessarily real. In fact, we obtain that $\mathbf{M}_{e\mu}=c_{12}s_{12}c_{23}(m_2^2-m_1^2)$ and $\mathbf{M}_{e\tau}=-\tan\theta_{23}\mathbf{M}_{e\mu}$ which automatically satisfy $\operatorname{Im}(\mathbf{M}_{e\mu})=\operatorname{Im}(\mathbf{M}_{e\tau})=0$.

We next argue the implementation of the μ–τ symmetry based on the observation that it is sufficient for the symmetry to be respected by the real part of M_ν. From the discussions developed in Ref. [16], it can be extended to more general case, where the real and imaginary parts are, respectively, replaced by $(z+e^{i\theta}z^*)/2(\equiv z_+)$ and $(z-e^{i\theta}z^*)/2(\equiv z_-)$ for a complex number of z and the phase parameter of θ. It is useful to notice that $z_+=e^{i\theta/2}\operatorname{Re}(e^{-i\theta/2}z)$ and $z_-=ie^{i\theta/2}\operatorname{Im}(e^{-i\theta/2}z)$. The relevant mass matrix is provided by one of the textures found in Ref. [16]:

$$M_\nu=\begin{pmatrix} M_{ee} & M_{e\mu} & -\sigma e^{i\theta}M_{e\mu}^* \\ M_{e\mu} & M_{\mu\mu} & M_{\mu\tau} \\ -\sigma e^{i\theta}M_{e\mu}^* & M_{\mu\tau} & e^{i\theta}M_{\mu\mu}^* \end{pmatrix}, \tag{15}$$

where $M_{ee,\mu\tau}=e^{i\theta}M_{ee,\mu\tau}^*$, equivalently $(M_{ee,\mu\tau})_-=0$, is imposed. This texture gives

$$\tan 2\theta_{12}=2\sqrt{2}\frac{\cos 2\theta_{13}(M_{e\mu})_+}{c_{13}[(1-3s_{13}^2)(M_{\mu\mu})_+-c_{13}^2(\sigma(M_{\mu\tau})_++(M_{ee})_+)]}, \tag{16}$$

$$\tan 2\theta_{13}e^{i\delta}=2\sqrt{2}\frac{\sigma(M_{e\mu})_-}{(M_{\mu\mu})_++\sigma(M_{\mu\tau})_++(M_{ee})_+}. \tag{17}$$

As discussed in Ref. [16], these expressions yield real values of $\tan 2\theta_{12,13}$ because of the property that $z'_+/z_+=\operatorname{Re}(e^{-i\theta/2}z')/\operatorname{Re}(e^{-i\theta/2}z)$ and $z'_-/z_+=i\operatorname{Im}(e^{-i\theta/2}z')/\operatorname{Re}(e^{-i\theta/2}z)$ for any complex values of z and z'. As a result, $\delta=\pm\pi/2$ is derived and M_ν gives maximal CP violation.

A texture with the Dirac CP violation phase related to the μ–τ symmetric texture is obtained by decomposing z and $e^{i\theta}z^*$ into z_+ and z_- and turns out to be $M_\nu=M_{+\nu}+M_{-\nu}$ with

$$M_{+\nu}=\begin{pmatrix} (M_{ee})_+ & (M_{e\mu})_+ & -\sigma(M_{e\mu})_+ \\ (M_{e\mu})_+ & (M_{\mu\mu})_+ & (M_{\mu\tau})_+ \\ -\sigma(M_{e\mu})_+ & (M_{\mu\tau})_+ & (M_{\mu\mu})_+ \end{pmatrix}=e^{i\theta/2}\operatorname{Re}\big(e^{-i\theta/2}M_\nu\big),$$
$$M_{-\nu}=\begin{pmatrix} 0 & (M_{e\mu})_- & \sigma(M_{e\mu})_- \\ (M_{e\mu})_- & (M_{\mu\mu})_- & 0 \\ \sigma(M_{e\mu})_- & 0 & -(M_{\mu\mu})_- \end{pmatrix}=ie^{i\theta/2}\operatorname{Im}\big(e^{-i\theta/2}M_\nu\big), \tag{18}$$

which shows that $M_{+\nu}$ has a phase $\theta/2$ modulo π while $M_{-\nu}$ has a phase $(\theta+\pi)/2$ modulo π. The μ–τ symmetry exists in $M_{+\nu}$ because Eqs. (12) and (13) are satisfied but is explicitly broken by $M_{-\nu}$. Therefore, this texture shows "incomplete" μ–τ symmetry [15]. Since $M_{+\nu}$ does not contribute to $\sin\theta_{13}$, $\sin\theta_{13}$ should be proportional to the flavor neutrino masses in $M_{-\nu}$. In fact, it is proportional to $(M_{e\mu})_-$ in Eq. (17). To speak of the Majorana phases, we have to determine neutrino masses, which can be computed from Eq. (7) and are given by

$$
\begin{aligned}
m_1 e^{-2i\beta_1} &= (M_{\mu\mu})_+ - \sigma(M_{\mu\tau})_+ - \frac{1+\cos 2\theta_{12}}{\sin 2\theta_{12}} \frac{\sqrt{2}(M_{e\mu})_+}{c_{13}},\\
m_2 e^{-2i\beta_2} &= (M_{\mu\mu})_+ - \sigma(M_{\mu\tau})_+ + \frac{1-\cos 2\theta_{12}}{\sin 2\theta_{12}} \frac{\sqrt{2}(M_{e\mu})_+}{c_{13}},\\
m_3 e^{-2i\beta_3} &= \frac{c_{13}^2((M_{\mu\mu})_+ + \sigma(M_{\mu\tau})_+) + s_{13}^2(M_{ee})_+}{\cos 2\theta_{13}}.
\end{aligned}
\tag{19}
$$

Since $z_+ = e^{i\theta/2}\,\mathrm{Re}(e^{-i\theta/2}z)$, the texture gives three Majorana phases calculated to be: $\beta_{1,2,3} = -\theta/4$ modulo $\pi/2$. The common phase does not induce Majorana CP violation. This result reflects the fact that the source of the Majorana phases is the phase of M_ν in Eq. (18) equal to $\theta/2$, which can be rotated away by redefining appropriate fields. The remaining imaginary part $\mathrm{Im}(e^{-i\theta/2}M_\nu)$ supplies the Dirac phase δ. Therefore, our proposed mass matrix becomes $\mathrm{Re}(e^{-i\theta/2}M_\nu) + i\,\mathrm{Im}(e^{-i\theta/2}M_\nu)$, which is equivalent to M_ν with $\theta = 0$. No CP violating Majorana phases exist in our mass matrix.

The simplest choice of $\theta = 0$ provides the case where the real part of M_ν respects the μ–τ symmetry. This texture has been discussed in Refs. [13,17], which takes the form of

$$
M_\nu^{\mu-\tau} = \mathrm{Re}\begin{pmatrix} M_{ee} & M_{e\mu} & -\sigma M_{e\mu} \\ M_{e\mu} & M_{\mu\mu} & M_{\mu\tau} \\ -\sigma M_{e\mu} & M_{\mu\tau} & M_{\mu\mu} \end{pmatrix} + i\,\mathrm{Im}\begin{pmatrix} 0 & M_{e\mu} & \sigma M_{e\mu} \\ M_{e\mu} & M_{\mu\mu} & 0 \\ \sigma M_{e\mu} & 0 & -M_{\mu\mu} \end{pmatrix},
\tag{20}
$$

where the real part is the well-known μ–τ symmetric texture as expected while the imaginary part breaks it.[2] The mixing angles of $\theta_{12,13}$ are given by

$$
\begin{aligned}
\tan 2\theta_{12} &\approx 2\sqrt{2}\frac{\mathrm{Re}(M_{e\mu})}{\mathrm{Re}(M_{\mu\mu}) - \sigma\,\mathrm{Re}(M_{\mu\tau}) - \mathrm{Re}(M_{ee})},\\
\tan 2\theta_{13} e^{i\delta} &= 2\sqrt{2}\sigma\frac{i\,\mathrm{Im}(M_{e\mu})}{\mathrm{Re}(M_{\mu\mu}) + \sigma\,\mathrm{Re}(M_{\mu\tau}) + \mathrm{Re}(M_{ee})},
\end{aligned}
\tag{21}
$$

from Eqs. (16) and (17). The expression of $\tan 2\theta_{12}$ is obtained by taking the approximation $\sin^2\theta_{13} \approx 0$. The maximal CP violation by $e^{i\delta} = \pm i$ is explicitly obtained.

Summarizing our discussions, we have advocated to use the possibility that the real part of M_ν only respects the μ–τ symmetry. This possibility is extended to the more general case of $M_\nu = M_{+\nu} + M_{-\nu}$ in Eq. (18), where $M_{+\nu}$ serves as a μ–τ symmetric texture and the symmetry-breaking term of $M_{-\nu}$ acts as a source of $\sin\theta_{13} \neq 0$. The consistency of the texture is given by the property that particular combinations of z, z^* and $e^{i\theta}$ become real or pure imaginary. This property ensures the appearance of real values of $\theta_{12,13}$ while the real value of θ_{23} arises from $\tan\theta_{23} = \mathrm{Im}(\mathbf{M}_{e\mu})/\mathrm{Im}(\mathbf{M}_{e\tau})$. It should be noted that θ_{23} is not determined by $\tan\theta_{23} = -\mathrm{Re}(\mathbf{M}_{e\tau})/\mathrm{Re}(\mathbf{M}_{e\mu})$ as in the μ–τ symmetric texture because the Dirac CP violation phase is now active. It turns out that $M_\nu = e^{i\theta/2}[\mathrm{Re}(e^{-i\theta/2}M_\nu) + i\,\mathrm{Im}(e^{-i\theta/2}M_\nu)]$, which gives no intrinsic Majorana CP violation while the Dirac CP violation becomes maximal.

[2] In this context, another solution is to abandon to have $\sin\theta_{13} = 0$ in $\mathrm{Re}(M_\nu^{\mu-\tau})$, which is realized by $M_{e\tau} = \sigma M_{e\mu}$ instead of $M_{e\tau} = -\sigma M_{e\mu}$ in Eq. (20), and CP violation ceases to be maximal [16]. To discuss μ–τ symmetry in this type of texture is out of the present scope.

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