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## Transient flow simulation of municipal gas pipelines and networks using semi implicit finite volume method

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### Abstract

An efficient transient flow simulation for gas pipelines and networks is performed. The proposed transient flow simulation is based on the transfer function models and finite volume method. The equivalent transfer functions of the nonlinear governing equations are derived for different boundary condition types. To verify the accuracy of the proposed simulation, the results obtained are compared with those of the experiments. The effect of the flow inertia is considered in this simulation with discretisation by TVD scheme. The accuracy of the proposed method is discussed for two test cases. It is shown that the proposed simulation has a sufficient accuracy.

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*Keywords:* transient flow; natural gas; gas network; simulation; FVM

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### 1. Introduction

Nature gas pipelines and network are commonly applied in municipal gas transportation and distribution or area energy supply projects. To ensure the fluid flows as intended, it relies heavily on mathematical models and the simulations they enable. The analysis of flows in pipelines and networks

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has been studied by many workers. However, the steady state analysis of pipeline network is less applicable, and transient state is more often encountered. In a dynamic simulation model, the governing continuity and momentum equations which are a set of two non-linear first order hyperbolic partial differential equations with equation of state are commonly solved by an appropriate numerical method..

Numerical simulation of transient flow have been performed by using the method of characteristics<sup>[1,2]</sup>, the finite difference method<sup>[3]</sup> and the total variation diminishing (TVD) finite volume method<sup>[4,5]</sup>. And high-order accurate finite difference methods for the numerical solution of compressible fluid flow are presented by Abgrall et al.<sup>[6]</sup>, Cockburn<sup>[7]</sup> and L.M.C. Gato et al.<sup>[8]</sup>.

Tentis et al. applied an adaptive method of lines to simulate the transient gas flow in pipelines<sup>[9]</sup>. Ke and Tian analysed isothermal transient gas flow in the pipeline networks using the electrical models for the loops and nodes<sup>[10]</sup>. Recently, Gonzales et al. have used MATLAB-Simulink and prepared some S-functions to simulate transient flow in gas networks<sup>[11]</sup>. M. Behbahani-Nejad et.al also developed a MATLAB-Simulink library for transient flow simulation of gas pipelines and networks<sup>[12]</sup>.

The finite volume method is proposed based on fluid pressure, quantity of flow and density to simulate both fast and slow transients<sup>[13]</sup>. It is also discussed in accident condition of extra large flow dynamic analysis based on fluid pressure, velocity and temperature<sup>[14,15]</sup>. In these literatures, the finite volume method, FVM, shows reasonable results and high efficiency in prediction.

In the gas transportation projects, the pipelines can be several kilometres. In this circumstance, small changes of pipes and gas characteristics can lead to major differences of simulation results. However, that more factors are taken into consideration in model means more difficulties in calculation convergence and time consuming. And computing grids often need to be increased to get more accurate results. While applying the finite volume method, acceptable results can be get in more coarse grids and the results show apparent physical meaning

## **2. Mathematical model**

### *2.1. Pipe flow model*

The governing equations describing the general one-dimensional compressible flow through a pipeline under isothermal conditions are the conservation of mass (or continuity), momentum and an equation of state relating the pressure, velocity and the temperature. For a general pipe as shown in Fig. 1, these hyperbolic partial differential equations are (Kralik et al., 1998)

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} = 0 \quad (1)$$

$$\frac{\partial(\rho u)}{\partial t} + \frac{\partial(P + \rho u^2)}{\partial x} = -\frac{f \rho u |u|}{2d} - \rho g \sin \alpha \quad (2)$$

$$P = \rho Z R_g T \quad (3)$$

where  $\rho$  is the gas density,  $P$  is the pressure,  $u$  is the gas axial velocity,  $g$  is the gravitational acceleration,  $\alpha$  is the pipe inclination,  $D$  is the pipeline diameter,  $Z$  is

the gas compressibility factor, and  $f$  is the friction factor using the modified version of the Colebrook-White equation:

$$\frac{1}{\sqrt{f}} = -2 \log_{10} \left[ \frac{e}{3.7D} + \frac{2.825}{Re \sqrt{f}} \right] \quad (4)$$

There are several equations available to choose a compatible  $f$  in computation process, which can be used in different conditions in processing program.

### 2.2. Model simplification

In order to make sure the reliability and high efficiency of the calculation process, it is possible to neglect some terms to make computation simple in some flow conditions. Firstly, in the momentum equation, the convective acceleration term  $\partial(\rho u^2)/\partial x$  is relatively small comparing to the other terms when the gas flow velocity is not close to velocity of sound; Secondly, the pipelines are considered laying at the same height and the gravity term  $\rho g \sin \alpha$  can be omitted.

In such assuming conditions and simplification, it can be derived to this dynamic model as following:

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} = 0 \quad (5)$$

$$\frac{\partial(\rho u)}{\partial t} + \frac{\partial P}{\partial x} + \frac{f \rho u |u|}{2d} = 0 \quad (6)$$

$$P = \rho Z R_g T \quad (7)$$

### 2.3. Compressibility factor $Z$

In high pressure flow, majority of real gases need the compressibility factor  $Z$  to correct. Here Dranchuk-Purvis-Robinson can be used as a suitable equation to get factor  $Z$ .

$$Z = 1 + \left( 0.31506 - \frac{1.0467}{T_r} - \frac{0.5783}{T_r^2} \right) \rho_r + \left( 0.5353 - \frac{0.6127}{T_r} - \frac{0.6185}{T_r^2} \right) \rho_r^2 \quad (8)$$

where,  $\rho_r = 0.27(P_r/ZT_r)$ , as natural gas specific density, no dimensional number.

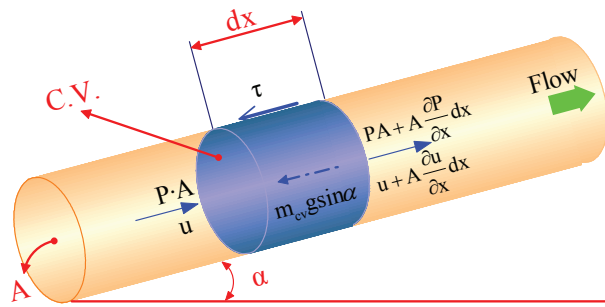


Fig.1 A control volume in a general gas pipeline.

### 3. Discretisation scheme

#### 3.1. Discretisation of continuity equation

When applying FVM to discretise the equation, a kind of staggered grid is adopted to avoid a ‘checker-board’ pressure field occurrence. The continuity equation use pressure node as center volume to discretise while the momentum equation use velocity node as center volume to discretise. The discretisation scheme of the continuity equation is shown in figure.2. And its discretisation equation is evaluated as

$$(\rho_P - \rho_P^0) \frac{\Delta x}{\Delta t} A_P + (\rho u)_e A_e - (\rho u)_w A_w = 0 \quad (9)$$

Where,  $\rho_P$  is density of control volume P node kg/m<sup>3</sup>,  $\Delta x$  and is length of control volume, and A is area of section of pipe.

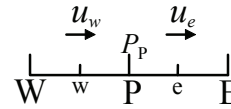


Fig.2 The scalar control volume used for the discretisation of the continuity equation

#### 3.2. Discretisation of momentum equation

When flow direction is from west to east, it is considered as  $u > 0$ ; On the contrary, it is considered as  $u < 0$ . The momentum equation scheme is shown as figure.3. as if  $u > 0$ . The flow term is discretised using TVD scheme. Equation (2) can be derived as

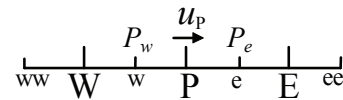


Figure.3 The scalar control volume used for the discretisation of the momentum equation

$$\left[ \frac{\rho P}{\Delta t} + F_e + \frac{f}{d} (\rho \tilde{u})_P \right] u_P = F_w u_w + \frac{(\rho u)_P^0}{\Delta t} + \frac{P_w - P_e}{\Delta x} + \frac{f \rho_P \tilde{u}_P^2}{2d} + S_u^{DC} - \rho_P g \sin \alpha \quad (10)$$

Where  $\tilde{u}$  is initial given value or last iterate value. Those discretised terms are given as following:

$$\frac{\partial(\rho u)}{\partial t} = \frac{(\rho u)_P}{\Delta t} - \frac{(\rho u)_P^0}{\Delta t} \quad (10.1)$$

$$\frac{\partial(\rho u^2)}{\partial t} = F_e u_e - F_w u_w = F_e u_P - F_w u_W - S_u^{DC} \quad (10.2)$$

$$\frac{\partial P}{\partial x} - \frac{f \rho u^2}{2d} - \rho g \sin \alpha = \frac{P_e - P_w}{\Delta x} - \frac{f(\rho \tilde{u})_P}{d} u_P + \frac{f(\rho \tilde{u})_P^2}{2d} \quad (10.3)$$

### 4. Numerical solutions

In gas transient flow using FVM scheme, the pressure-velocity equations are calculated sequentially. And the discretised variables are stored in different grid volume. The pressure and velocity field are continuously corrected during iteration. The equation (11) can be written as following

$$a_i u_i = \sum a_{nb} + (P_{i-1} - P_i) A_i + b_i \quad (12)$$

Assuming the correct pressure distribution is:

$$P = P^* + P' \quad (13)$$

And the correct velocity distribution is:

$$u = u^* + u' \quad (14)$$

From equations (12) to (14)

$$u'_i = \frac{\sum a_{nb} u'_{nb}}{a_i} + \frac{(p'_{i-1} - p'_i) A_i}{a_i} \quad (15)$$

Omit the first term of equation (15) and get

$$u_i = u_i^* + \frac{A_i}{a_i} (p'_{i-1} - p'_i) \quad (16)$$

With equation (9) and get

$$a_i p'_i = a_{i+1} p'_{i+1} + a_{i-1} p'_{i-1} + b'_i \quad (17)$$

Generally,  $b'_i$  indicate the remainder of the continuity equation. This term should tend to zero when the equations get into convergence. Therefore,  $b'_i$  is used as criteria for iteration ends. The coefficient factor as written as follow:

$$a_i = a_{i+1} + a_{i-1};$$

$$a_{i+1} = (\rho dA)_{i+1};$$

$$a_{i-1} = (\rho dA)_{i-1};$$

$$b'_i = (\rho_i^0 - \rho_i) \Delta x / \Delta t + (\rho u^* A)_i - (\rho u^* A)_{i+1}.$$

In this paper, the double precision PISO pattern of Simple Method was adopted to solve the gas pressure and velocity distribution in transient flow model.

## 5. Results and discussion

### 5.1. Test case 1: A step demand

The test case 1 is a straight pipe that was used by London Research Service (LRS) to demonstrate the versatility of their program (1974). Initially, the gas pipe is in a steady state condition as depicted in Table 2. Then, the demand is increased in a 50% step change remaining constant at this value during the rest of the simulation. During these conditions, the inlet pressure remains constant, while from the physical point of view, the outlet pressure must be fallen to a new stationary state. The initial conditions of high pressure case are shown in Table 1 in gas distribution networks.

Table 1 Geometrical and physical pipeline data for test case 1

Type	L (m)	D (m)	Q (MSCFH)	Pinlet (psig)
Case1	128747.5	0.4572	1500	350

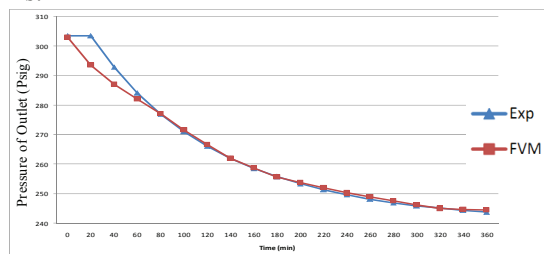


Fig.4. Outlet pressure with time for test case 1

The results show that the simulation has maximum relative error of 3.27% in the beginning of the process comparing to the experiment data of LRS. This phenomena means that the model reflects the

transient changes immediately after boundary condition suddenly changes, though the real experiments' results changes slowly.

5.2. Test Case 2: A gas network

A typical network which has been studied by Ke and Ti is considered and simulated with the proposed approach. Figure. 5 shows a schematic of this network. The geometrical data of the network is introduced in Table 2 and the gas demand at the nodes 2 and 3 are illustrated in Fig. 5. The flow is supplied in the network via node 1 which is maintained at a constant pressure of 50 bars. The gas specific gravity is approximately 0.6, the operational temperature is 278 K, and the friction factor is considered to be constant and equal to 0.003.

Table 2 Pipe geometrical data for the considered network

Gas pipe ID	From node	To node	Diameter (m)	Length (km)
1	1	3	0.6	80
2	1	2	0.6	90
3	2	3	0.6	100

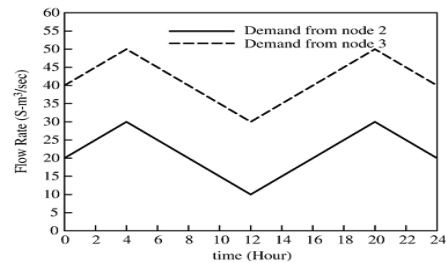


Fig.5. Demands versus time for nodes 2 and 3 of the network considered in test case 2.

The proposed simulation results for outlet pressures in nodes 2 and 3 are compared with those obtained by Experiments respectively in Fig. 6 and Fig. 7. As is shown in these figures, a good agreement is observed although some differences exist at the sharp points.

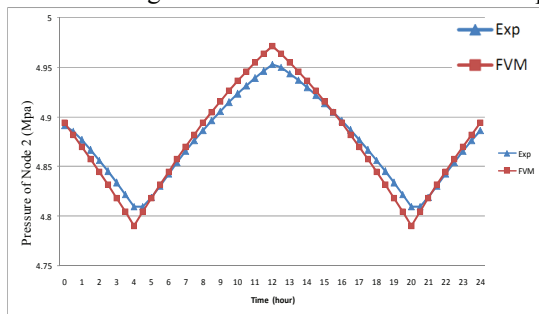


Fig. 6 Outlet pressure results for nodes 2 of the gas network considered in test case 2

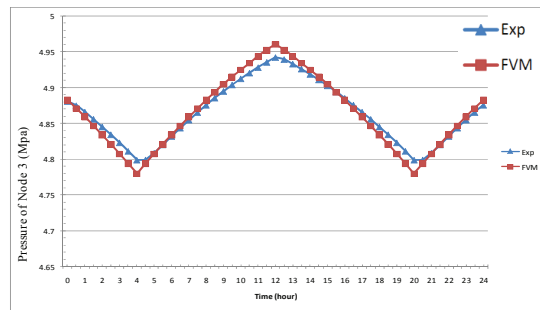


Fig.7 Outlet pressure results for nodes 3 of the gas network considered in test case 2

The results show that the simulation has maximum relative error of 1.15% in the curve point of the process comparing to the experiment data of LRS. In this pipeline network, the simulation results predicted are acceptable.

6. Conclusions

An implicit finite volume method for the prediction of transient flows in pipe networks has been presented in this paper. The method can retain the convective acceleration term in the momentum equation in condition of gas velocity value. This term has a significant impact on the accuracy in the case

of high speed flows. For consideration of the changes of gas density along with the collapse of pressure and temperature, this method is especially suitable for long distance transportation and distribution in city gas engineering. The simulation results show the advantages of the present method in both fast and slow transients with acceptable accuracy and stability.

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