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On the Coulomb interaction between spherical and deformed nuclei

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Abstract

A method is presented to calculate the exact HI Coulomb potential between spherical and deformed nuclei in the framework of the double folding model. We used realistic density distributions taking the deformations of the target into account. We have compared between our calculations and one of the more recent analytical expressions based on assuming sharp surface of the interacting nuclei. We have found that the finite surface diffuseness affects strongly the HI Coulomb interaction in the inner region and has a smaller effect in the tail region. Moreover, neglecting non-linear higher order terms in the analytical expressions produces errors in the outer region of the Coulomb interaction.

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The interaction potential between two heavy ions, HI, is essential for understanding many physical aspects in the HI scattering processes. The HI optical model potential consists of two parts, the real and the imaginary parts. The real part includes the Coulomb interaction and it is important for calculating physical quantities as reaction, fission, fusion cross-section, and fusion barrier distribution.

In HI calculations, it is usually assumed that the Coulomb potential between the two interacting ions is equivalent to either the potential between a point charge and uniformly spherical charge distribution or the potential between two uniformly charged spheres with radii equal to the half density radii of the interacting ions [1]. On the other hand, many ions have a static deformed charge distribution in its equilibrium state. So, the deformation degree of freedom in charge distributions should be considered. Attempts to calculate the electrostatic Coulomb potential taking the deformation degree of freedom into account have been made by Wong [2] and Krappe [3]. A mathematical method has been given which allows finding analytical solutions for the Coulomb potential between spherical as well as between deformed nuclei with diffuse surfaces [3,5].

In many cases the physical HI observable quantities are sensitive to a small region in the tail of the nuclear potential where the Coulomb potential is expected to be model independent and generally is considered to be $V_C(R) = Z_P Z_t e^2/R$ plus a small correction due to deformation. This is often used in literature. On the other hand

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the angular distribution of some alpha-particle and light HI scattering reactions have been found to be sensitive to the real potential over the whole radial domain. Moreover, the fusion cross-section needs a correct nuclear and Coulomb potentials in the surface region and just before this region [6–8]. At high energy, the effects of the Coulomb force appear also. For example, the Coulomb effect on the pion production in heavy ion collisions where it is modifies the pion momentum and the available phase space [9,10]. These examples show the importance of accurate calculations of the electrostatic Coulomb force not only around the surface region but also in the inner region.

The more recent work deals with this problem, for deformed target, was done by Takigawa et al. [4] where they presented analytical expression of the Coulomb interaction between a spherical projectile and a deformed target which are valid for any separation distance between them, and remove various shortcomings in the standard formulas of point-sphere and two uniformly spheres interactions. Their formulae solve the cusp and discontinuity problems in the form factors of the Coulomb excitation and in their derivatives in other conventional models. This may help in many problems such as the system where the Coulomb radius is larger than the nuclear radius. In their work the Coulomb potential was not compared with that calculated from the double folding model with realistic density distributions. It is of interest to discuss if these available analytical expressions for the Coulomb potential are enough or it is necessary to derive a more realistic, folded, Coulomb potential in some cases? This is one of the aims for this Letter. We use realistic density distributions and the double folding model to derive the Coulomb HI potential for spherical-deformed nuclear pair. We compare our results with the results of the most recent work in this subject [4].

In the study of the effect of nuclear intrinsic degrees of freedom on the fusion reactions at sub-barrier energies using coupled channels method, the Coulomb coupling plays an important role. As a good approximation, for HI fusion reactions one can replace the angular momentum of the relative motion in each channel by the total angular momentum **J**, no-Coriolis approximation, [6,11,12]. The coupled channels equations then read

$$\left[-\frac{\hbar^2}{2\mu}\frac{d^2}{dR^2}+\frac{J(J+1)\hbar^2}{2\mu R^2}+V_n(R)+\frac{Z_P Z_T e^2}{R}+n\hbar\omega-E_{\text{c.m.}}\right]\psi_n(R)+\sum_m V_{nm}(R)\psi_m(R)=0, \tag{1}$$

V_{nm} is the coupling form factor, which in the collective model consists of Coulomb and nuclear components. The Coulomb part is given by

$$V^C_{nm}(R,\alpha)=\langle\psi_n|V(R,\alpha)-V^{(0)}(R)|\psi_m\rangle, \tag{2}$$

where $V(R,\alpha)$ is the total Coulomb interaction taking into account the ensemble of intrinsic coordinates, denoted by α, and $V^{(0)}(R)$ is the bare Coulomb interaction between two spherical nuclei.

In early studies it has been reported that the higher order Coulomb couplings are not important in heavy-ion fusion reactions [6,13,14]. Linear Coulomb coupling were used and still in the major literature. The study of effect of higher order Coulomb couplings was taking place in more recent work [15]. It is found that, considering the quadrupole deformation, the second order Coulomb coupling noticeably modifies the fusion barrier distribution for some interactions, for example, $^{32}\text{S}+{}^{168}\text{Er}$ interaction.

Krappe [3] had proposed a method to solve the six-dimensional Coulomb interaction integration not in the ordinary space but by means of Fourier transformations. This gives the Coulomb potential in the form of a multipole sum. In each term the interaction integral is evaluated in Fourier space utilizing the simple structure of the Fourier transform of the folded distribution. The result is a one-dimensional integral for each multipole term. It is valid also for overlapping charge distributions. Based on this method an expression for the Coulomb interaction between spherical-deformed nuclei is given in the form [4],

$$V\big(\vec{R},\alpha\big)=V^{(0)}(R)+V^{(1)}\big(\vec{R},\alpha\big)+V^{(2)}\big(\vec{R},\alpha\big)+\cdots, \tag{3}$$

where

$$V^{(0)}(R)=Z_P Z_T e^2 F^{(0)}(R) \tag{4}$$

is the bare Coulomb interaction,

$$V^{(1)}\big(\vec{R},\alpha\big) = Z_P Z_T e^2 \sum_{\lambda,\mu} F_\lambda^{(1)}(R) Y^*_{\lambda\mu}(\Omega_R)\alpha_{\lambda\mu} \tag{5}$$

is the linear Coulomb coupling, and

$$V^{(2)}\big(\vec{R},\alpha\big) = Z_P Z_T e^2 \sum_{\lambda_1,\mu_1,\lambda_2,\mu_2} \sum_{\lambda,\mu} F_\lambda^{(2)}(R) Y^*_{\lambda\mu}(\Omega_R)\alpha_{\lambda_1,\mu_1}\alpha_{\lambda_2,\mu_2} \int d\Omega\, Y_{\lambda\mu}\, Y^*_{\lambda_1\mu_1} Y^*_{\lambda_2\mu_2} \tag{6}$$

is the second order Coulomb coupling.

These expressions are done under the approximation of uniformally charged objects with a sharp surface for projectile and target and using expansion with respect to the deformation parameters.

The form factors are defined by

$$F^{(0)}(R) = \frac{18}{\pi}\int_0^\infty j_0(kR)\frac{j_1(kR_P)}{kR_P}\frac{j_1(kR_T)}{kR_T}\,dk, \tag{7}$$

$$F_\lambda^{(1)}(R) = \frac{18}{\pi}\int_0^\infty j_\lambda(kR)\frac{j_1(kR_P)}{kR_P} j_\lambda(kR_T)\,dk, \tag{8}$$

and

$$F_\lambda^{(2)}(R) = \frac{18}{\pi}\int_0^\infty j_\lambda(kR)\frac{j_1(kR_P)}{kR_P}\left\{ j_\lambda(kR_T) + \frac{kR_T}{2}\frac{dj_\lambda(kR_T)}{d(kR_T)}\right\} dk. \tag{9}$$

Now, when considering an axially symmetric quadrupole and hexadecapole deformations, β_2 and β_4, for the target nucleus and applying the angular momentum algebra we obtain the following expressions for the linear and the leading second order Coulomb couplings

$$V^{(1)}\big(\vec{R},\theta\big) = Z_P Z_T e^2\big[F_2^{(1)}(R)\beta_2 Y_{20}(\theta,0) + F_4^{(1)}(R)\beta_4 Y_{40}(\theta,0)\big], \tag{10a}$$

$$\begin{aligned} V^{(2)}\big(\vec{R},\theta\big) = Z_P Z_T e^2\Bigg\{&\left[F_2^{(2)}(R)\frac{1}{7}\sqrt{\frac{5}{\pi}}\,Y_{20}(\theta,0) + F_4^{(2)}(R)\frac{3}{7}\sqrt{\frac{1}{\pi}}\,Y_{40}(\theta,0)\right]\beta_2^2 \\ &+\left[F_2^{(2)}(R)\frac{6}{7}\sqrt{\frac{1}{\pi}}\,Y_{20}(\theta,0) + F_4^{(2)}(R)\frac{20}{77}\sqrt{\frac{5}{\pi}}\,Y_{40}(\theta,0)\right. \\ &\left.+ F_6^{(2)}(R)\frac{15}{11}\sqrt{\frac{5}{13\pi}}\,Y_{60}(\theta,0)\right]\beta_2\beta_4\Bigg\}. \end{aligned} \tag{10b}$$

In the double-folding model, the interaction Coulomb potential between spherical-deformed or deformed–deformed nuclei with separation distance $\vec{R}$ between their centers is given by

$$V_C\big(\vec{R}\big) = \int\!\!\int d\vec{r}_1\, d\vec{r}_2\, \frac{1}{|\vec{s}|}\rho_P\big(\vec{r}_1\big)\rho_T\big(\vec{r}_2\big), \tag{11}$$

where $\vec{S} = \vec{R} + \vec{r}_2 - \vec{r}_1$. ρ_P and ρ_T denote the nuclear charge distribution in the projectile and target nuclei normalized to the total charge $Z_p e$ and $Z_T e$, respectively.

The integral given by Eq. (11) is a six-dimensional integral that is too difficult to handle. We can simplify it by use of Fourier transformation. Restricting our derivation to be for spherical-deformed nuclear pair with the

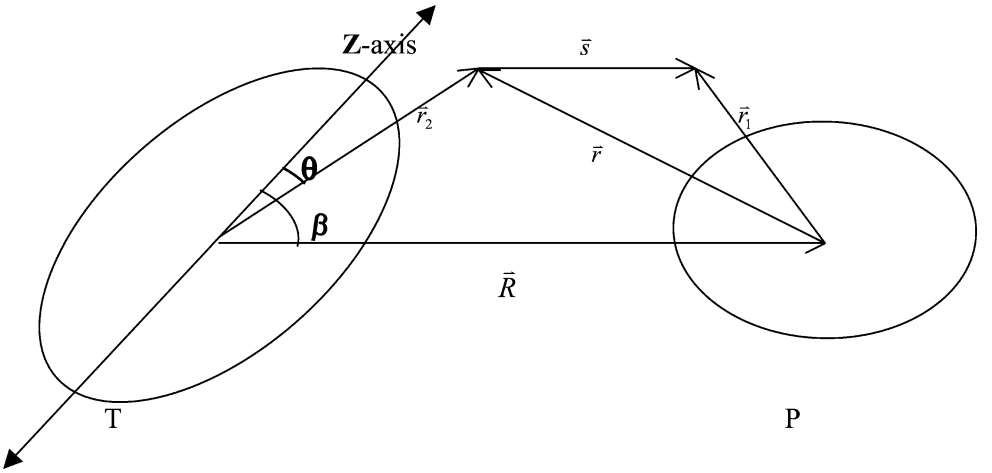


Fig. 1. A sketch of the interaction between two interacting deformed-spherical nuclei. The axis of symmetry of the deformed nucleus makes an angle β with the beam.

coordinates that shown in Fig. 1, we shall define $G(\vec{R},\beta,s)$ as

$$G\big(\vec{R},\beta,s\big)=\int \rho_T\big(\vec{R}+\vec{r}\big)\rho_P\big(\vec{r}+\vec{s}\big)\,d\vec{r} \tag{12}$$

where β is the orientation angle of the deformed nucleus (the angle between the symmetry axis of the target nucleus and the separation vector $\vec{R}$).

Noting that the projectile nucleus is spherical and taking the Fourier transformation of ρ_P we get

$$\rho_p\big(\vec{r}+\vec{s}\big)=\int e^{i\vec{k}\cdot(\vec{r}+\vec{s})}\tilde{\rho}_p(k)\,d\vec{k} \tag{13}$$

and

$$\tilde{\rho}_p(\vec{k})=\frac{1}{(2\pi)^3}\int e^{-i\vec{k}\cdot\vec{x}}\rho_p\big(\vec{x}\big)\,d\vec{x}. \tag{14}$$

Expanding the plane wave $e^{-i\vec{k}\cdot\vec{x}}$ into its multipole components then substituting into Eq. (12) one gets

$$G\big(\vec{R},\beta,s\big)=\frac{1}{2\pi^2}\int d\vec{r}\,d\vec{k}\,x^2\,dx\,\rho_T\big(\vec{R}+\vec{s}\big)e^{-i\vec{k}\cdot\vec{s}}e^{-i\vec{k}\cdot\vec{r}}\rho_p(x)j_0(kx). \tag{15}$$

Inserting this into Eq. (11) then integrating over the solid angle of the vectors $\vec{s}$ and $\vec{k}$, $V_C(\vec{R},\beta)$ becomes

$$\mathrm{V}_C\big(\vec{R},\beta\big)=8\int_0^\infty\int_0^\infty s\,ds\,j_0(ks)k^2\,dk\int d\vec{r}\,\rho_T\big(\vec{R}+\vec{r}\big)j_0(kr)\int x^2\,dx\,j_0(kx)\rho_p(x) \tag{16}$$

the integration over s can be performed analytically.

The charge density distribution of the deformed nucleus is then assumed to be

$$\rho(r,\theta)=\frac{\rho_0}{1+e^{\frac{r-R(\theta)}{a(\theta)}}}, \tag{17}$$

where the half density radius of this Fermi distribution is given by

$$R(\theta)=R_0\big[1+\beta_2Y_{20}(\theta,0)+\beta_4Y_{40}(\theta,0)+\beta_6Y_{60}(\theta,0)+\cdots\big] \tag{18}$$

β_2, β_4 and β_6 are the quadruple, hexadecapole and hexacontatetrapole deformation parameters, respectively, and the angle θ is measured from the symmetry axis of the deformed nucleus. ρ_0 is determined from the condition

$$\int \rho(\vec{r})\, d\vec{r} = Ze. \tag{19}$$

The reaction ^{16}O–^{238}U is chosen as an example to show the deviation between the exact HI Coulomb potential between spherical and deformed nuclei given by Eq. (16) and the recent analytical expression derived in Ref. [4]. The values of radius $\boldsymbol{R}_0$, the diffuseness $\boldsymbol{a}$ and the deformation parameters for ^{238}U-nucleus are taken from Refs. [15,16]. The correction due to deformation dependent part of the Coulomb potential is defined in the present work as

$$V_C^{\text{corr}}(R,\beta) = V(R,\beta) - V^{(0)}(R), \tag{20}$$

where $V^{(0)}(R)$ is the bare Coulomb interaction between two spherical nuclei which has an analytical expression given by Eq. (4). We compare between the results of calculating this quantity using the folding model and the analytical expression in Ref. [4]. In the first case $V(R,\beta) = V_C(R,\beta)$ given by Eq. (16) while the approximate expression is obtained when $V_C^{\text{corr}}(R,\beta) = V^{(1)}(R,\beta) + V^{(2)}(R,\beta)$ given by Eq. (10). In Ref. [4] the authors expanded the Coulomb heavy-ion potential between deformed target and spherical projectile in terms of the deformation parameters of the deformed target. The first term of this expansion is the Coulomb potential between two uniformly charged spheres. The form factors, in Ref. [4], which depend on deformation parameters vanish at heavy-ion separation distance $R = 0$. In our estimation of the error in form factors derived in Ref. [4], we subtracted the spherical potential given by Eq. (4) from the folding potential calculated by Eq. (16). We expect that the quantity $V_C^{\text{corr}}(R,\beta)$ Calculated in the present Letter does not vanish at $R = 0$. This is mainly because the presence of surface diffuseness. Also the effect of deformation parameters on the Coulomb interaction at $R = 0$ decreases its value by small amount compared to that calculated for two charged spheres with sharp surfaces.

Fig. 2 shows $V_C^{\text{corr}}(R,\beta)$ calculated in the present Letter and that calculated in Ref. [4] for three values of the orientation angle, β, of the deformed target. Two values of the quadrupole deformation parameters were assumed and both the linear and quadratic terms in β_2 are taken into consideration. In Fig. 2 the hexadecapole deformation did not consider.

Fig. 3 is the same as Fig 2 but a positive value for the hexadecapole deformation parameter is assumed to be present. The figure presents a comparison of our calculations and that of Ref. [4] when the term contains $\beta_2\beta_4$ is included in the analytical formula Eq. (10). Fig. 4 is the same as Fig. 3 except that β_4 has opposite sign.

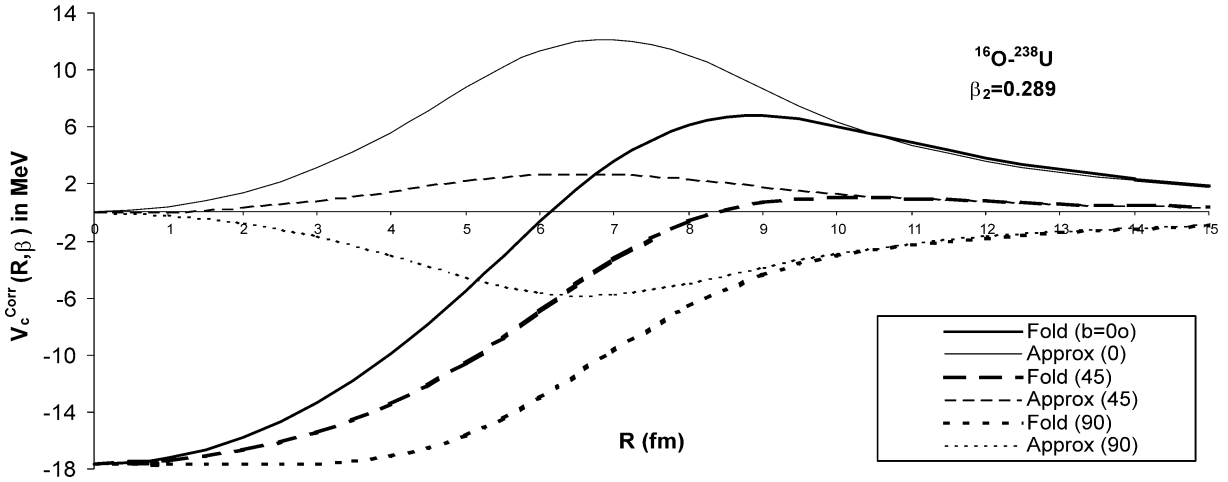
We define the percentage error in the analytical formula by

$$\chi = \frac{[V_C(R,\beta) - V^{(0)}(R)] - [V^{(1)}(R,\beta) + V^{(2)}(R,\beta)]}{V_C(R,\beta) - V^{(0)}(R)} \times 100. \tag{21}$$

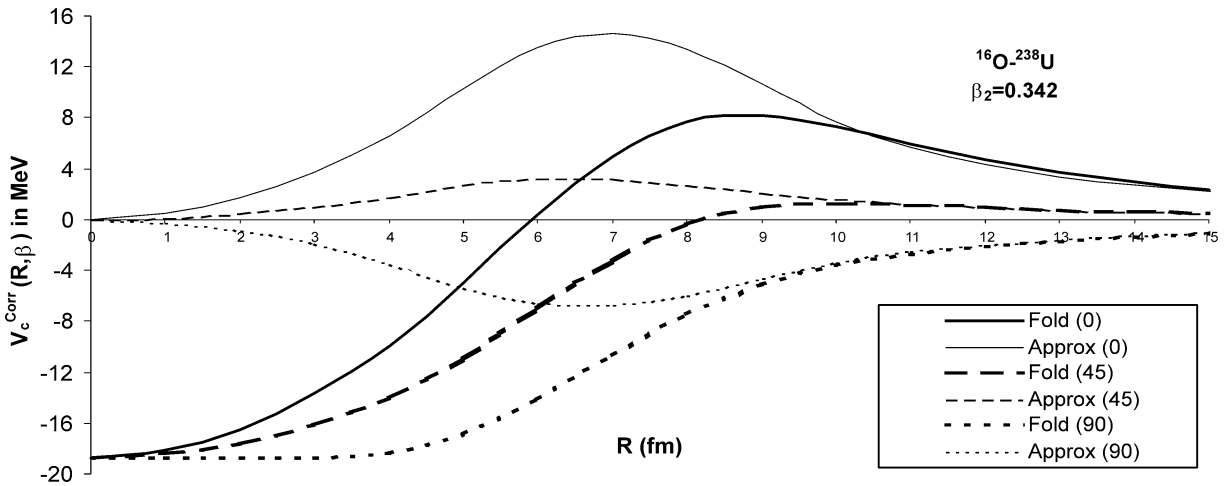
The value for χ is presented on Table 1 for three different orientation angles, β, and for several values of the separation distance R. The table presents χ for $\beta_2 = 0.289$ and the three values of $\beta_4 = 0, \pm 0.087$. In all calculations the quadratic term containing $\beta_2\beta_4$ is included.

Concerning the internal region of the Coulomb HI interaction, the figures and the table described above show a large difference between the folding model prediction and the analytical formula in Ref. [4]. Also they show some deviation between the two approaches in the physically significant surface and tail regions. We found that the inclusion of the $\beta_2\beta_4$ term do not improve the error in some cases. A significant difference between the folding model, considered in the present work, and the different approaches for calculating the Coulomb potential studied in Ref. [4] is that all the form factors of these approaches have the same value for $R \geqslant 9$ fm while our calculations have deviations compared to them in the range of $R \approx 9$ fm.

As pointed out in Ref. [4], both the point sphere and sphere–sphere models differ significantly in the internal region of the Coulomb potential compared to the more accurate analytical expression derived in Ref. [4]. The folding model predictions differ in the same region than the best analytical formula. This difference in $V_C^{\text{corr}}(R,\beta)$



(a)



(b)

Fig. 2. (a) Comparison between $V_C^{\text{corr}}(R,\beta)$ calculated in the present Letter, bold lines, and that calculated in Ref. [4], regular lines, for three values of the orientation angle, $\beta = 0^\circ$, 45° and 90°, of the deformed target with $\beta_2 = 0.289$. (b) Comparison between $V_C^{\text{corr}}(R,\beta)$ calculated in the present Letter, bold lines, and that calculated in Ref. [4], regular lines, for three values of the orientation angle, $\beta = 0^\circ$, 45° and 90°, of the deformed target with $\beta_2 = 0.342$.

can be more than 250% depending on the orientation angle and the deformation parameters. For $R = 11$ fm, the table shows that there exist maximum error of about 13% between our calculations and the results of Ref. [4]. This difference becomes too small as R increases to the extreme tail region.

The difference between the present calculations based on the folding model and the analytical expressions derived in Ref. [4] can be explained in the following way. In Ref. [4], the interacting nuclei were assumed uniformly charged objects with sharp surfaces. This assumption is the main reason for the large deviation, in the internal region, between the present calculations and those in Ref. [4]. Fig. 5 shows $V_C^{\text{corr}}(R,\beta)$ (for $\beta = 0^\circ$ and $\beta = 90^\circ$) derived from Eq. (16) when both the projectile and target nuclei have diffuseness parameters $\boldsymbol{a} = 0.01$ fm. This very small value of diffuseness parameter produces almost uniformly charged objects. In this case the difference

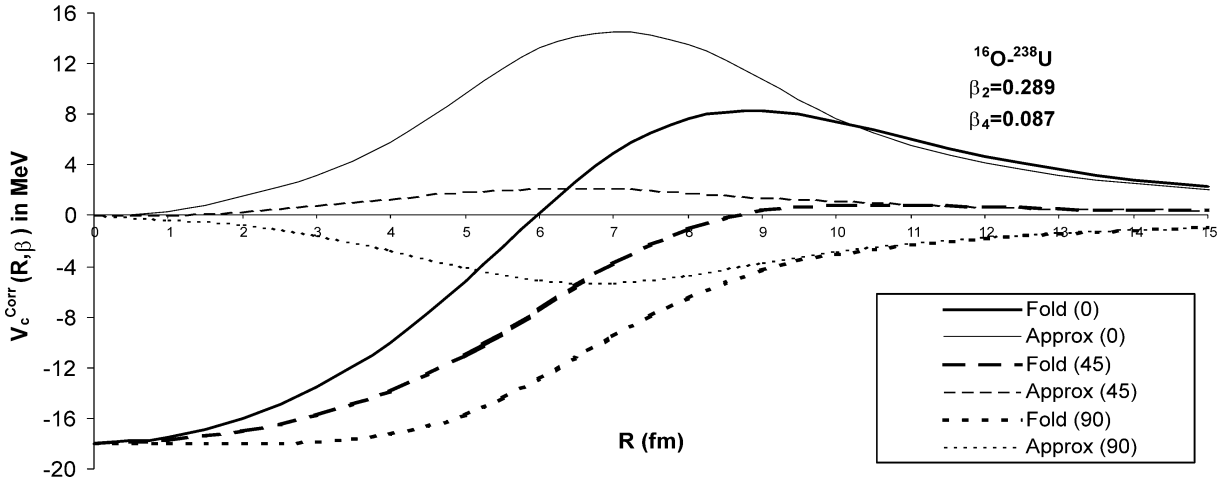


Fig. 3. Comparison between $V_C^{\text{corr}}(R,\beta)$ calculated in the present Letter, bold lines, and that calculated in Ref. [4], regular lines, for three values of the orientation angle, $\beta = 0°$, 45° and 90°, of the deformed target with $\beta_2 = 0.289$ and $\beta_4 = 0.087$.

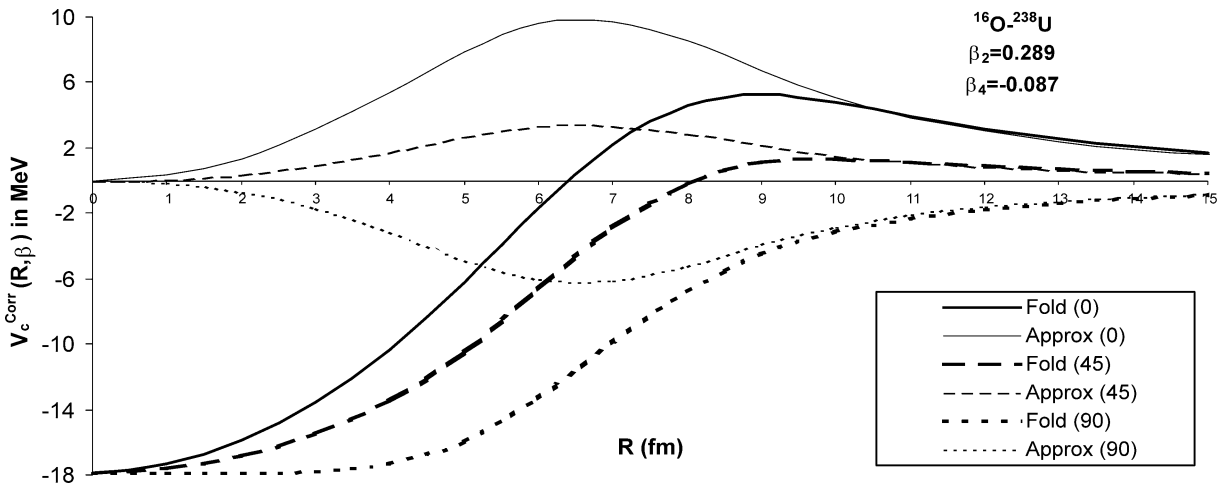


Fig. 4. Comparison between $V_C^{\text{corr}}(R,\beta)$ calculated in the present Letter, bold lines, and that calculated in Ref. [4], regular lines, for three values of the orientation angle, $\beta = 0°$, 45° and 90°, of the deformed target with $\beta_2 = 0.289$ and $\beta_4 = -0.087$.

Table 1
Values for percentage error in the analytical formula, X(%), defined by Eq. (21)

	$\beta_2 = 0.289\ \beta_4 = 0.0$			$\beta_2 = 0.289\ \beta_4 = 0.087$			$\beta_2 = 0.289\ \beta_4 = -0.087$		
R	$\beta = 0°$	45°	90°	$\beta = 0°$	45°	90°	$\beta = 0°$	45°	90°
3	123.8	105.4	90.7	123.7	104.8	91.3	123.1	105.8	90.4
5	258.0	121.5	71.4	285.1	116.9	77.0	226.2	125.7	69.3
7	−234.9	184.3	41.2	−197.4	155.9	44.6	−339.9	217.1	38.8
9	−29.0	−134.1	13.1	−28.8	−259.3	13.5	−26.7	−97.3	14.1
10	−4.9	−24.4	9.0	−2.4	−39.4	8.7	−5.0	−16.4	10.8
11	3.7	−5.1	7.7	7.6	−13.2	7.2	2.2	0.5	9.7
12	6.4	−0.1	7.5	10.7	−6.0	7.1	4.6	4.8	9.4

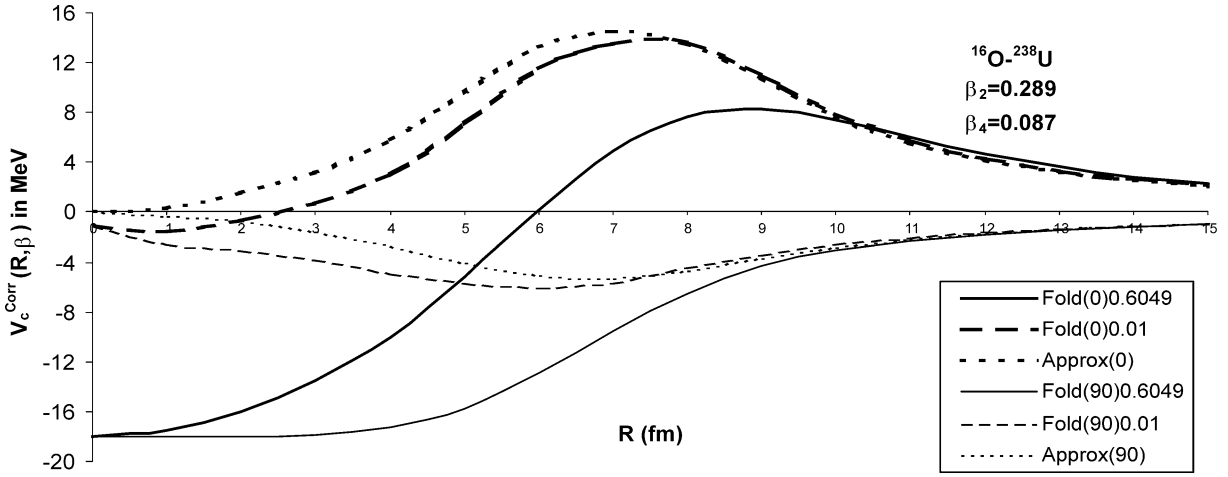


Fig. 5. Comparison between $V_C^{\text{corr}}(R,\beta)$ calculated in the present Letter (for two values of diffuseness parameter $\boldsymbol{a} = 0.6049$ (solid lines) and $\boldsymbol{a} = 0.01$(dashed lines)) and that calculated in Ref. [4] (dotted lines). The calculations are made at two orientation angle, $\beta = 0°$ and $90°$, of the deformed target with $\beta_2 = 0.289$ and $\beta_4 = 0.087$.

between the folding model potential and the results of Ref. [4] became small in both the surface and internal regions (as shown in Fig. 5). This is because if one allows for a surface diffuseness of the nuclei, some of the charges before the surface immigrate to larger distances. Since the Coulomb force is proportional to the inverse of the distance between the interacting charges, the value of Coulomb interaction between uniformly charged objects is larger in the internal region than its value for objects with diffused surfaces. Although the sharp surface assumption affects strongly the internal region, it has less effect on both the surface and tail regions. For orientation angles $\beta = 0°$ and $\beta = 90°$, Fig. 5 shows that the sharp surface assumptions reduce the error in the Coulomb form factors in the surface region with small amount. In this case the rest of the produced error is most probably due to neglecting the non-linear higher order terms of deformation parameters.

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