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CDM approach applied to fatigue crack propagation on airframe structural alloys

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Abstract

Characterization of materials requires a large number of experimental tests with various loading modes in order to ensure that it meets the design requirements. The aim of his work is to develop a numerical model to predict the behaviour of cracked panels, in place of expensive and complex experiments to generate data for designers. In particular, the model has to simulate toughness test and fatigue crack propagation at high level of ΔK for a wide variety of metallic alloys including aluminium alloys, titanium alloys and steels.
The proposed model is based on a thermodynamic approach within the framework of Continuum Damage Mechanics which provides numerical robustness as well as a precise description of the physical micromechanism of fracture. An extension of a classical CDM approach, named Lemaitre model [1], is presented in this paper. Lemaitre model [1], which has been shown to be particularly well suited to describe ductile damage under monotonic loading, has been only applied separately to low cycle fatigue and high cycle fatigue. The extension has to simulate fatigue crack propagation. Experimental results have evidenced that the role of two distinct mechanisms in the high fatigue crack growth rate regime. In order to account for the different mechanisms involved in fatigue crack propagation test two damage variables are introduced in the Lemaitre model [1]. The first variable, called static damage, represents the damage variable associated with ductile fracture modes. While the static damage evolution law is consistent with the classical Lemaitre model [1], the second variable is assumed as a consequence of cumulated plastic strain and does not influence plasticity. It is named cyclic damage.
Then, to correctly predict fatigue crack propagation rates, an internal length has to be defined. An energetic equivalence criterion has been inserted to model crack propagation and to reduce mesh dependence. This method, originally developed by J.Mazars and G. Pijaudier-Cabot [2], fills the gap between damage and fracture and avoids the introduction of additional boundary conditions or complex code developments associated with non-locality. The damage law coefficients have been calculated by taking into account the size of the elements. This model has been implemented in the commercial finite element code ABAQUS/CAE. Explicit computations have been performed to highlight drawbacks and advantages of the method.

Keywords : CDM approach, Energetic equivalent criteria, Internal lenght, Crack propagation, Finite element simulations, Aeronautical metallic alloys.

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1. Introduction

The qualification of a material for aircraft structures requires many tests in order to determine mechanical properties. These tests are sometimes long and quite expensive. The aim of this study is to develop a robust numerical model to predict the fracture behaviour of cracked panels and by this means reduce the number and the cost of experiments. This study focuses on three tests: facture toughness, R-curve and fatigue crack propagation tests and four aeronautical metallic alloys : two aluminium alloys 7175 T7351 and 2024 T351, a Ti-6Al-4V titanium alloy and a precipitation hardened martensitic stainless steel 15-5PH. For the sake of simplicity, this paper will mainly focus on the fatigue crack propagation behaviour at high level of ΔK of the Ti-6Al-4V alloy.

The model used to simulate these tests had to meet several requirements imposed by the industrial partner. It has to be simple, robust and to take into account micromechanical mechanisms. The initial model has been chosen using a literature survey. Many approaches address the difficult issue of crack propagation simulation under monotonic loading. However, models that can also account for the crack growth under cyclic loading are much less numerous. The Continuum Damage Mechanics (CDM) approach, originally proposed by Lemaitre [1], was one of them. Applied on monotonic and cyclic loading, it takes into account the effects associated to a given damage state through the definition of an internal variable, *D*. The set of constitutive equations for the damaged material is then derived from a thermodynamic potential. The evolution of the internal variable is assumed to follow the normality principle with respect to a convex dissipation potential. It has been used under many loading cases and on different classes of material ([3], [4]). Simple and robust, this model appears as the best option to achieve the aforementioned objectives.

Damage can be defined as the progressive loss of material load carrying capability as a result of some irreversible processes that occur in the material microstructure during the deformation history. A CDM framework for ductile damage in metals, in which damage is one of the thermodynamic variables. With the additional assumption of isotropic damage, Lemaitre [1] has characterized damage as a single scalar *D* given by:

$$D = 1 - \frac{E}{E_0} \qquad (1)$$

where E and E_0 are the Young's moduli of the damaged and undamaged material, respectively.

The effective stress concept is then used. In multiaxial case of isotropic damage, damage acts in the same way on each stress component and the effective stress tensor is given by:

$$\underline{\underline{\bar{\sigma}}} = \frac{\underline{\underline{\sigma}}}{1-D} \qquad (2)$$

Since damage is the result of progressive plastic deformation, the damage variable is dependent on plastic strain. Meanwhile, since the presence of damage affects elastic strain, the damage and plastic dissipation potentials can be considered as uncoupled. According to this, the entire set of constitutive equations for the damaged material can be written as follows. The decomposition of total strain, $\underline{\underline{\varepsilon}}$, into elastic $\underline{\underline{\varepsilon}}^e$ and plastic $\underline{\underline{\varepsilon}}^p$ contributions is assumed:

$$\underline{\underline{\varepsilon}}^p = \underline{\underline{\varepsilon}} - \underline{\underline{\varepsilon}}^e \qquad (3)$$

The effective stress tensor is given as a function of elastic strain:

$$\underline{\underline{\sigma}} = (1-D)\underline{\underline{\underline{E}}} : \underline{\underline{\varepsilon}}^e \qquad (4)$$

where $\underline{\underline{\underline{E}}}$ is the stiffness matrix. The plastic and damage loading function *f* corresponds to the von Mises criterion with isotropic and kinematic hardening:

$$f = \sigma_{eq} - R - \sigma_y = 0 \tag{5}$$

where R is the isotropic hardening force, σ_y the yield stress. The equivalent stress, σ_{eq}, is classically defined as:

$$\sigma_{eq} = \left[\frac{3}{2}\left(\frac{\underline{\underline{s}}}{1-D} - \underline{\underline{x}}\right) : \left(\frac{\underline{\underline{s}}}{1-D} - \underline{\underline{x}}\right)\right]^{1/2} \tag{6}$$

where $\underline{\underline{s}}$ and $\underline{\underline{x}}$ represent the deviatoric part of the tensor stress $\underline{\underline{\sigma}}$ and kinematic hardening tensor $\underline{\underline{X}}$ respectively.

Standard isotropic plasticity, associated with a von Mises yield criterion, leads to the following expression for the plastic potential F_p:

$$F_p(R;D) = \frac{\sigma_{eq}}{(1-D)} - R - \sigma_y \tag{7}$$

The plastic strain components and the internal variables associated to R and $\underline{\underline{X}}$, can be derived from Fp from the normality rule.

$$\dot{\varepsilon}_{ij}^{p} = \dot{\lambda}\frac{\partial F_p}{\partial \sigma_{ij}} = \dot{\lambda}\frac{3}{2}\frac{s_{ij}}{(1-D)\sigma_{eq}} \tag{8}$$

$$\dot{\alpha}_{ij} = -\dot{\lambda}\frac{\partial F_p}{\partial X_{ij}} = \dot{\lambda}\frac{3}{2}\frac{s_{ij}}{\sigma_{eq}} \tag{9}$$

$$\dot{r} = -\dot{\lambda}\frac{\partial F_p}{\partial R} = \dot{\lambda} = \dot{p}(1-D) \tag{10}$$

where $\dot{\lambda}$ is the plastic multiplier and p the accumulated plastic strain. The evolution law of damage is also assumed to be normal with respect to a distinct damage dissipation potential F_D to be determined:

$$\dot{D} = \dot{\lambda}\frac{\partial F_D}{\partial Y} \tag{11}$$

where Y is the thermodynamic force associated to D, and given by:

$$Y = \underline{\underline{\varepsilon}}^{e} : \underline{\underline{\underline{E}}} : \underline{\underline{\varepsilon}}^{e} \tag{12}$$

The choice of the damage dissipation potential is a key point of every CDM model. It is primarily a function of Y. The following expression was proposed by Lemaitre [5]:

$$F_D(Y;D) = \frac{S}{(1+s)(1-D)}\left(\frac{Y}{S}\right)^{s+1} \tag{13}$$

In this expression, S and s are material parameters. The substitution of (13) in (11) leads to:

$$\dot{D} = \dot{\lambda}\frac{\partial F_D}{\partial Y} = \left(\frac{Y}{S}\right)^s \dot{p} \qquad (14)$$

Equation (14) illustrates the coupling between plasticity and damage: $\dot{D}$ is explicitly dependent on the accumulated plastic strain rate $\dot{p}$.

2. Experimental investigation

2.1. Experimental device

Fatigue crack propagation tests are performed on compact tension specimens, CT W = 75 mm, B = 20 mm. A COD clip gauge is used to monitor crack length. Tests are carried out according to the standard ASTM E-647. The load, *F*, follows a sinusoidal signal and two values of the load ratio are considered R = 0.1 and R = 0.7.

2.2. Experimental results

The $\frac{da}{dN}$ curves for the two values of the load ratio are plotted as a function of ΔK and of K_{max} in Fig 1a and Fig 1b, respectively (a denotes the crack length, N the number of cycle and $\frac{da}{dN}$ the fatigue crack growth rate. K is the stress intensity factor). The slight shift noticed In Fig 1a) in the low ΔK region can be attributed to crack closure effect. However one can consider that in this regime crack propagation is mainly controlled by ΔK. For R = 0.7 failure occurs for relatively low ΔK value due to the high K_{max} value while at R = 0.1 a stable crack growth is observed over a wide ΔK range. This difference might therefore be attributed to a K_{max} effect. This K_{max} effect is less marked in the Ti-6Al-4V alloy and the 15-5 PH steel than in aluminium alloys [ref Hamon, to be published].

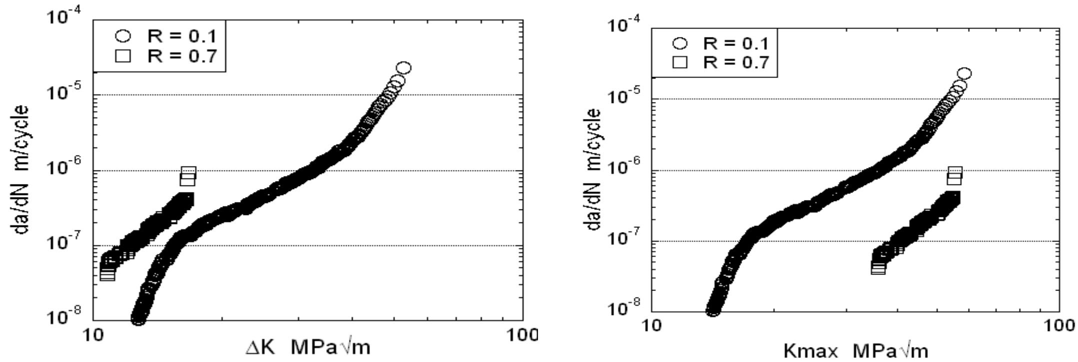


Fig 1 : $\frac{da}{dN}$ curves for the selected Ti-6Al-4V alloy. a) da/dN versus ΔK ; b) da/dN versus K_{max}

However Fig 1b) demonstrates that a single parameter such as K_{max} cannot account for the observed behaviour since the two curves do not merge in a single curve. This observation suggests in this high crack growth rates regime a cyclic damage mechanism, mainly related to ΔK, and a static damage mechanism controlled by K_{max} have to be considered.

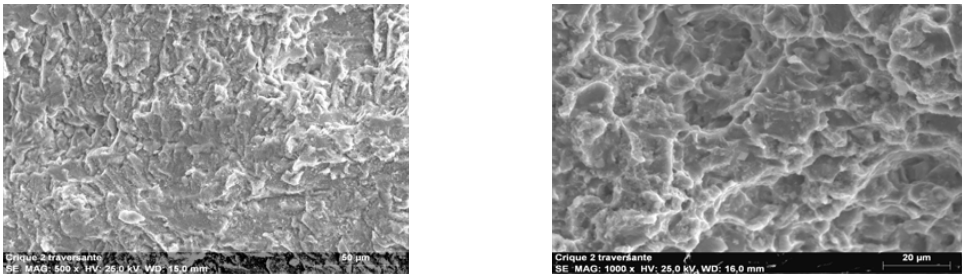


Fig 2 : Fracture surfaces for two values of ΔK. a) ΔK =10MPa √ m, da/dN=5.46x 10-9 m/cycle, R=0.1 ; b) ΔK =26.6MPa √ m1/2 , da/dN=2.9 x 10-5 m/cycle, R=0.7

This latter idea is confirmed by observations of the fracture surfaces presented in Fig 2. For lowest ΔK values, the fracture surface is mainly occupied by quasi cleavage, especially at R = 0.1 (Fig 2 a). As the value K_{max} is increased by increasing ΔK and/or considering higher values of the load ratio, a second mechanism characterized by the presence of dimples appears (Fig 2 b). These dimples are furthermore similar to those observed after fracture toughness tests [ref Hamon to be published]. These observations globally hold for the other alloys selected in this study despite of some quantitative differences.

Consequently these observations suggest that in the high crack growth rate regime two damage modes influence fatigue crack propagation depending on the respective values of ΔK and K_{max}:

- a cyclic damage, D_{cyc}, which is active at low and high values of ΔK.
- a static damage, D_{sta}, which is prevailing at high values of K_{max}. It corresponds with the classical definition of damage for ductile fracture in CDM models.

3. Damage variables and internal length definition

3.1. Static and cyclic damage formulations

Experimental results have evidenced that the role of two distinct mechanisms in the high fatigue crack growth rate regime. In order to account for the different mechanisms involved in fatigue crack propagation test two damage variables are introduced in the Lemaitre model.

The first variable Dsta represents the damage variable associated with ductile fracture modes and characterized by the presence of dimples on fracture surfaces at high Kmax values, Figure 2. Its associated force is denoted Ysta. The variable Dsta and the force Ysta correspond to D and Y in the previously described Lemaitre model. However, the evolution law of static damage is deduced from another dissipative damage potential F'D in order to distinguish the two types of damage. The decomposition of this potential into a dissipative static and a dissipative cyclic damage potential is assumed:

$$F_D^{'} = F_{D\,sta} + F_{Dcyc} \tag{15}$$

F_{Dsta} correspond to F_D, in (13), with the introduction of a function Q to separate static and cyclic loading case. S and s become the static damage parameters S_t and s_t.

$$F_{Dsta}\left(Y; D_{sta}\right) = \frac{S_t}{\left(1+s_t\right)\left(1-D_{sta}\right)}\left(\frac{Y_{sta}}{S_t}\right)^{s_t+1} Q\left(\varepsilon_{\max}^{p}, p\right) \tag{16}$$

where:

$$\begin{cases} Q(\varepsilon^p_{\max}, p) = 1 & \text{If } p = \varepsilon^p_{\max} \\ Q(\varepsilon^p_{\max}, p) = 0 & \text{If } p > \varepsilon^p_{\max} \end{cases} \quad (17)$$

and $\varepsilon^p_{\max}$ the maximal equivalent plastic strain. The function *Q*, whose value is zero under cyclic loading, permits to limits the variation of D_{sta}. Consequently, the evolution law of static damage is then given by:

$$\dot{D}_{sta} = \left(\frac{Y_{sta}}{S_t}\right)^{s_t} \dot{p} Q(\varepsilon^p_{eq}, p) \quad (18)$$

The expressions of the evolution law of D_{cyc} and its associated thermodynamic force Y_{cyc} are established consistently with the thermodynamic approach. The thermodynamic potential of Lemaitre model [9] is modified by adding an extra term involving D_{cyc} only.

$$w(\underline{\underline{\varepsilon}}^e, \underline{\underline{\alpha}}, r, D_{sta}, D_{cyc}) = w_e(\underline{\underline{\varepsilon}}^e, D_{sta}) + w_p(\underline{\underline{\alpha}}, r,) + w_d(D_{cyc}) \quad (19)$$

The term w_d is chosen so that Y_{cyc} writtes:

$$Y_{cyc} = -\frac{\partial w}{\partial D_{cyc}} = -\frac{\partial w_d}{\partial D_{cyc}} = \frac{(p - \varepsilon^p_{\max})^{s_{cyc}+1}}{s_{cyc} + 1} \quad (20)$$

An additional cyclic damage reversibility domain $f_{cyc} < 0$ is defined by the following expression:

$$f_{cyc} = Y_{cyc} - \frac{S_{cyc}}{\varepsilon^p_{\max}} D_{cyc} < 0 \quad (21)$$

S_{cyc} and s_{cyc} are material parameters related to cyclic damage. The evolution law of D_{cyc} can be derived from (21) by assuming normality with respect to f_{cyc} considered as dissipation potential:

$$\dot{D}_{cyc} = \frac{\dot{Y}_{cyc}}{S_{cyc}} \varepsilon^p_{\max} = \frac{(p - \varepsilon^p_{\max})^{s_{cyc}}}{S_{cyc}} \varepsilon^p_{\max} \dot{p} \quad (22)$$

It is noteworthy that, in the constitutive framework proposed in this paper, plasticity induces cyclic damage but cyclic damage does not affect elastic and plastic material properties. In fact, only static damage interferes with plasticity. Moreover, D_{cyc} and D_{sta} coexist but they don't evolve in the same time. For example, D_{cyc} does not vary during static loading because the accumulated plastic strain, p, and the maximal equivalent plastic strain, $\varepsilon^p_{\max}$, are equal. $(\dot{D}_{cyc} = 0)$. Even if the modified model approaches more the experimental observations, somes differences subsist.

3.2. Introduction of an internal length

In order to correctly predict fatigue crack propagation rates, the definition of an internal length is required. Indeed a finite element computation based on a classical local damage-plasticity theory may give rise to a physically unrealistic mesh dependence of results. It has been recognized that the most efficient method is the introduction of an internal length by the mean of an implicit formulation ([6], [7]). However the modification of the finite element formulation and the time required for such development are the major drawbacks of this technique. An alternative method is the delayed damage modelling for fracture prediction ([8], [9]) or the introduction of an internal length based on an energetic equivalence criterion developed by J.Mazars and G. Pijaudier-Cabot ([10], [2]). The latter solution is chosen in the present study because of its simplicity. The mesh dependence phenomenon has been mainly attributed to a consequence of the strain softening induced by the presence of damage [12]. The regularization method used in the present study consists in calculating the damage law coefficients by taking into account the size of the elements. It establishes a parallel between two main types of failure approaches, namely fracture mechanics and continuous damage mechanics. A principle of energy equivalence permits to relate both approaches. Considering the similarity of the two approaches, it seems natural to switch from one concept to the other. One possible solution is to transform a given damage zone into an equivalent crack or conversely. This equivalence must be thermodynamically acceptable, which means that during the evolution:

$$-\int_V Y_{sta}\dot{D}_{sta}dV - \int_V Y_{cyc}\dot{D}_{cyc}dV = -GA \tag{23}$$

where V is the overall volume of the structure and G is the fracture energy release rate. A is the crack area and is associated with the internal length l.

$$A = Bl \tag{24}$$

where B is the thickness. The unknown variable is Gc, the critical value of the fracture energy release rate which is a material parameter. For a given mesh size and on the basis of numerical simulation of a tensile test, it is possible to deduce the value Gc though the evaluation of the first term of (23) during computation. Therefore the mesh dependence phenomenon is solved by linking Gc to an internal length A.

4. Identification and simulations

4.1. Implementation

The simulations are run using ABAQUS v.6.7 software with user subroutines (Vumat) specifically developed for the model. It is a FORTRAN file where the constitutive equations have been introduced. Explicit resolution is used. This type of resolution is the simplest way to simulate crack propagation. To reduce time of computation, the option "Mass scaling" is used. It raises the density of the material to increase the increment size.

4.2. Identification of the model parameters

Table 1 : Model parameters.

Parameters	Name
E	Young modulus
σ_y	Yield stress
C	Kinematic hardening parameter
s_t & S_t	Static damage parameters
s_{cyc} & S_{cyc}	Cyclic damage parameters
Gc	critical value of the fracture energy release rate

This section briefly describes the various tests conducted in order to identify the values of the model parameters. These eight parameters are listed in Table 1.

Tensile tests have been performed on flat rectangular hourglass specimens to evaluate the Young modulus and the yield stress. The values obtained here are consistent with the literature

Cyclic deformation tests are conducted on cylindrical specimens to identify strain hardening parameters. The specimen dimensions were taken from the ASTM E-606 standard test method for strain-controlled fatigue testing. The experiment is carried out under total strain-range control using a 7.5 mm gauge length extensometer. One specimen for each total strain amplitude level was tested using a sinusoidal symmetrical wave shape cycle. ($R\varepsilon$ = -1)

The results indicate that isotropic hardening is negligible compared to kinematic hardening. A simple linear relationship has been selected to take into account for kinematic hardening:

$$\underline{\underline{X}} = C\underline{\underline{\alpha}} \qquad (25)$$

where C is a parameter which has to be experimentally determined.

An identification based on a tensile test is carried out to identify the values of S_t, s_t and G_c, which appear in (18) and (23) respectively. The comparison presented in Fig 3 indicates a good agreement between the experimental tensile curve and the numerical simulation. Furthermore the values of static damage parameters agreed with the values determined by Lemaitre and Desmorat [1]. For Ti-6Al-4V, $S_t = 3$ MPa and $s_t = 0.08$.

Finally, a comparison between a fatigue crack propagation test at low value of ΔK, at R=0.7, and simulations provides the values of cyclic damage parameters (S_{cyc}, s_{cyc}).

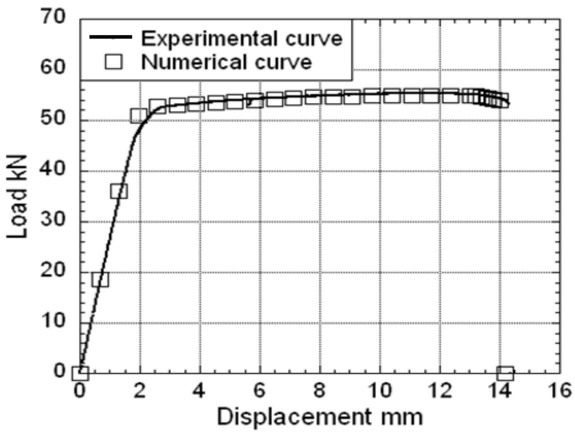


Fig 3 : Comparison between experimental and simulated curve for a tensile test.

Therefore the complete identification requires three types of tests, tensile test, cyclic deformation test and a $\frac{da}{dN}$ -ΔK value. The identification is based on a comparison between experiments and simulations.

4.3. Determination of the $\frac{da}{dN}$ -ΔK curve

This paper focus on the simulation of fatigue crack propagation test because the differences between the Lemaitre model and the proposed model appear only under cyclic loading. The simulations of toughness and R curve tests give quite good results and will be presented in other paper [ref Hamon, to be published].

A 2D planar finite element model of the CT specimen described in the previous part is developed in using 4-nodes, fully integrated elements. Linear elements were chosen to capture details of the stress distribution and reduce computational burden at the same time. Symmetry conditions are imposed in order to model only one half of the geometry.

Boundary conditions are summarized in Fig 4.

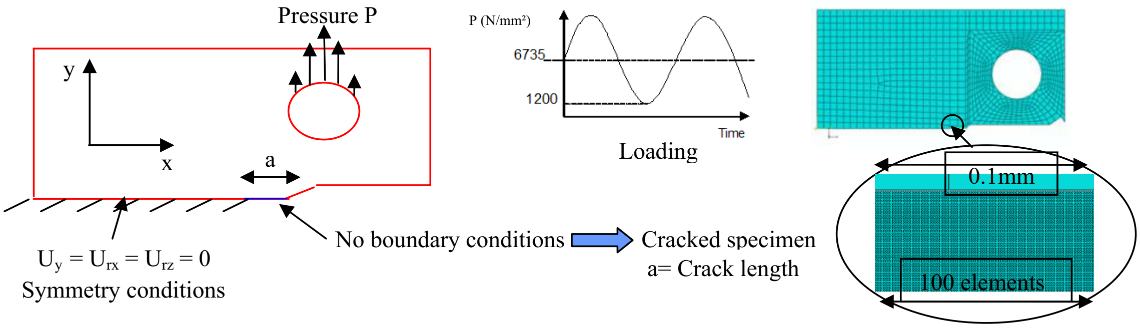
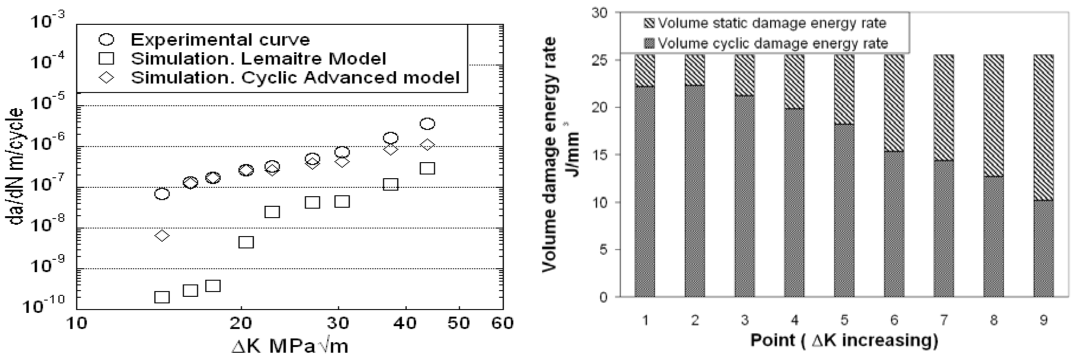


Fig 4 : ABAQUS model.

A smaller mesh size is used in the crack tip vicinity. Contrary to a critic damage criterion, an energetic equivalence criterion reduces the mesh dependency phenomenon. However, the mesh has to be less than to the size of the plastic zone to capture the evolution of state variables:

$$l \leq \frac{1}{2\pi}\left(\frac{K}{\sigma_y}\right)^2 \qquad (26)$$

At low ΔK value, the mesh is designed in order to have an element size of about 0.001 mm in the region where failure is expected to occur.



a) da/dN versus ΔK. R = 0.1 Ti-6Al-4V

b) Distribution of volume damage energy rate

Fig 5: Comparison between experimental and numerical curves and distribution of volume damage energy rate.

Then, the $\frac{da}{dN}$ -ΔK numerical curve is determined with the Lemaitre model and the cyclic advanced model, Fig 5 a). Contrary to the a priori with the classical CDM model using only one damage variable, results provided by the Lemaitre model are in the same order of magnitude than experimental results. However, the model quickly overvalues the crack propagation rate at higher value of R. In order to address this issue, the cyclic/advanced model limits the variation of static damage, which is similar to Lemaitre damage parameter. The critical value of *Gc* will be reached due to the cyclic damage increase. The proposed model improves results in the Paris regime, characterized by the equation:

$$\frac{da}{dN} = G * \Delta K^n \tag{27}$$

The parameters of the Paris law, identified with the curve Fig 5a), are listed in Table 2.

Table 2: Parameters of the Paris law.

	n	-Log (G)
Experimental	3	1.1×10^{-5}
Lemaitre model	7.4	4.9×10^{-9}
Cyclic / advanced model	3.6	4.0×10^{-6}

Fig 5 b) shows that the two damages coexist during the simulation of the fatigue crack propagation test as the observations stipulated it. When the value of ΔK increases, the volume static damage energy rate is more important and at high value of ΔK, the contribution of cyclic damage is negligible.

5. Conclusion

Experimental results have evidenced the role of two distinct mechanisms in the high fatigue crack growth rate regime. The Lemaitre model has thus been revised along these lines to account for propagation in this regime. More precisely, two damage parameters have been introduced: a static and a cyclic damage. In addition, an internal length is defined to solve mesh dependency. The proposed model provides more accurate predictions of fatigue crack propagation rates in Ti-6Al-4V alloy at R = 0.1. The predictions of $\frac{da}{dN}$ -ΔK curves for higher values of R with this model are quite good and better than the classical CDM model.

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