Historia Mathematica 9 (1982) 54-58

NOTE

MINOAN AND MYCENAEAN NUMERALS

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In Memory of Ken May

Ever since the discovery by Heinrich Schliemann in 1876 of the shaft graves in Mycenae with the so-called treasury of Agamemnon, more and more light has been thrown upon pre-Homeric Greece and Crete. High points have been the Knossos excavations by Sir Arthur Evans, beginning in 1900 with his discoveries of clay tablets with the script called Linear A and Linear B, and the decoding of Linear B by Michael Ventris in 1952. This Linear B turned out to be written in archaic Greek. Linear A is still somewhat of an enigma. Roughly speaking, Linear A belongs to the first, Linear B to the second half of the second millennium B.C.

The excavations of "Nestor's palace" near Pylos on the Peloponnese by Carl Blegen in 1929, followed by later excavations, have added a large number of clay tablets in Linear B. They have given us much information on the daily life at the court of a Mycenaen feudal lord during a period from circa 1400 to circa 1200 B.C., the period ending in the days of the Trojan war.

These tablets, both in A and B, have taught us that the Mycenaeans, and the Minoans before them, used a decimal system of counting integers, based on (and perhaps inspired by) the same additive principle as the system used in Egypt. Integers are expressed by adding symbols standing for the principal decimal units (see Fig. 1). (The sign for 10000 as 10 times 1000 reminds us of the *deka-chilioi* of Homer. In accordance with the Egyptian custom there is no special sign for 5; yet there must have been counting with 5's when we think of Homer's

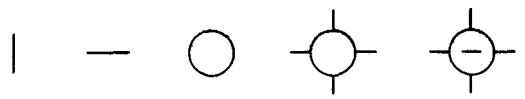

Figure 1

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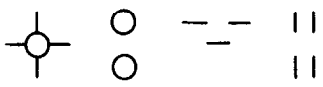
pempazein in *Odyssey* 4, 412.) The number 1234, for example, is expressed by



This had already been discovered by Evans, except for the sign for 10000 [1].

There also exist signs expressing fractions, used in expressing weights and measures. Not all of them have values beyond doubt; for instance, the sign for 1/2, as Evans saw, is the fractional symbol with the greatest frequency. To discover the values of the tokens used for fractions, statistical methods have been used, based on frequency, as well as trial solutions, based on common sense, of systems of indeterminate linear equations. The latter method is possible because the tablets not infrequently register the sum of a set of numbers. Evans was able to verify his interpretation of integers by using a tablet in which 57 was added to 23 to give 80. The result of a computation due to D. A. Was [2] is shown in Fig. 2.

Numerical fractions in Linear A

		1/4	𐄼	1/2	𐄷	3/4	𐄾	
1/24	𐄁	7/24	[𐄷+]	13/24	[𐄷+]	19/24	[𐄾+]	
1/12	𐄂	1/3	𐄼	7/12	𐄻	5/6	𐄼𐄻	
1/8	𐄃	3/8	𐄷	5/8	𐄾	7/8	𐄷𐄷	
1/6	𐄀	5/12	𐄼	2/3	[𐄷+]	11/12	𐄾	
5/24	𐄄	11/24	𐄾	17/24	𐄷𐄄	23/24	𐄷𐄻	

Table VII — Possible system for the Minoan fractions

FIG. 2. Reproduced, with permission, from "Numerical Fractions in Minoan Linear Script," by D. A. Was (Kadmos 10 (1971), 35-51; see also 13 (1974), 82).

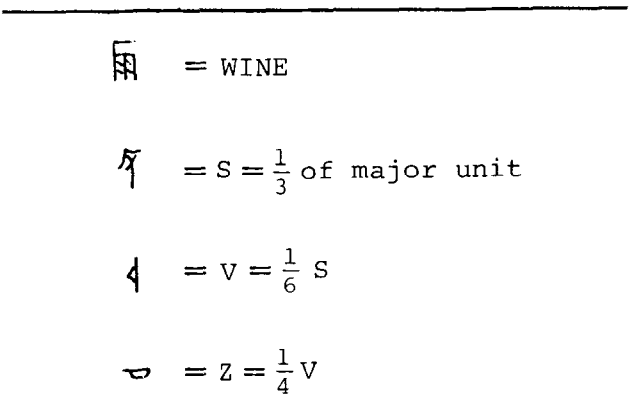
It is seen that the symbol for 3/4 indicates the sum of 1/4 and 1/2. Similarly,

$$\frac{7}{24} = \frac{1}{8} + \frac{1}{6}\,, \quad \frac{19}{24} = \frac{1}{4} + \frac{3}{8} + \frac{1}{6}\,, \quad \frac{11}{12} = \frac{1}{4} + \frac{1}{2} + \frac{1}{6}\,.$$

Here the influence of the Egyptians' use of unit fractions is seen, but not consistently, if the interpretation given in Fig. 2 is correct.

Numbers are always used to count people, animals, and objects and to express weights and measures; no interest is shown in the study of numbers for their own sake, as we know it from Mesopotamia and Egypt. But we must be careful with generalizations, since only documents of one particular kind have been found, and they deal with the economy of the royal household.

As for weights and measures, quantities of such goods as grain or wine were counted in units of decreasing size. Here we find both decimal and sexagesimal division, as well as other types. For instance, the basic unit of dry measure is divided into tenths, and these again into sixths, and these again into quarts. The major weight is divided into sixty units (or thirty double units according to the symbol) and this unit again into quarts (comp. Latin *talentum* divided into 60 *minae*), and this quart, it seems, is again divided into twelve lower units (see Fig. 3).



45 The Mycenaean signs for liquid measure

FIG. 3. Reproduced, with permission, from *The Mycenaean World*, by J. Chadwick (Cambridge Univ. Press, 1976).

Unlike those in Mesopotamia, Minoan and Mycenaean tablets were never deliberately baked. Made of ordinary clay, they seem to have been used for temporary purposes only, as scrap material. The results of the registration would have been transferred eventually to a (for that time) more permanent record, perhaps to papyrus scrolls. But such scrolls, although preserved in part in the Egyptian climate, have not been preserved, it seems, in Crete and Greece. By sheer "accident" the supposedly impermanent clay tablets have survived because, in the great fires which reduced the palaces to ashes, the clay tablets were baked. Had it not been for this destruction of the buildings in which the tablets were kept, they would long since have crumbled away.

Our records are thus only from the last days of these palaces. We do not know what precious written materials (arithmetic problems or tables, perhaps) may have been lost forever.

Chadwick believes that the scribes were not professional writers, as in Egypt and Mesopotamia; instead, they were literate officials able to register data when required, but they had other duties as well [3]. Most may have been rather like bailiffs or majordomos, or their clerks. There were no schools of scribes, it seems, in pre-Homeric Greece or Crete, and yet it was in such schools elsewhere that an independent arithmetical-geometrical art was later developed with excursions into pure mathematics, such as the expression of fractions by unit fractions or Pythagorean triples. So far as we know, however, this did not happen in pre-Homeric Greece or Crete. But who knows? Some romantic archaeologists may still be dreaming of finding documents analogous to the Dead Sea Scrolls containing an early account of the wrath of Achilles.

The destruction of the pre-Homeric civilization was followed by what is often called the Dark Ages, connected with Sea-Raiders and the Dorian invasion. Several centuries later, when a new Greek civilization developed in the city states of the Aegean, only legends remained. The Bronze Age had passed into the Iron Age. With the invention of the alphabet, the ancient script was forgotten and with it the ancient system of writing numbers. The new system used the letters of the newly adopted alphabet. Only in Cyprus did a script resembling the ancient Linear A and B remain until nearly the end of classical times.

NOTES

1. Ventris, M., & Chadwick, J. 1956. *Documents in Mycenaean Greek*, p. 36. New York/London: Cambridge Univ. Press; Evans, A. J. 1935. *The palace of Minos*, p. 691. New York/

London: Oxford Univ. Press. See also Gordon, C. H. 1976. Further notices of the Hagia Triada tablet No. 11. *Kadmos* 15, 28-30.

2. Was, D. A. 1971. Numerical fractions in Minoan linear script A. *Kadmos* 10, 35-51. I thank Professor Mabel Lang of Bryn Mawr College for this reference, as well as the references: Bennett, E. L. 1950. Fractional quantities in Minoan book-keeping. *American Journal of Archeology* 54, 201-222; and Lang, M. 1964. Es proportions in *Mycenaean studies*, pp. 37-51. Madison: Univ. of Wisconsin Press.

3. Chadwick, J. 1976. *The Mycenaean world*, p. 24. New York/London: Cambridge Univ. Press. See also pp. 32, 102-108.