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Parton distributions in the proton and pion

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Abstract

We use detailed balance for a hadron composed of quark and gluon Fock states to obtain parton distributions in the proton and pion on the basis of a simple statistical model.

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1. Introduction: parton distributions in the proton

There has been considerable interest in the flavor dependence of the proton's quark and antiquark distributions. The first measurement of the $\bar{d} - \bar{u}$ asymmetry was made by the NMC group [1]. The integral of this distribution showed a violation of the Gottfried sum rule. Later, Drell–Yan [2,3] and deep inelastic scattering [4] experiments determined the Bjorken- x dependence of the asymmetry. The meson cloud model and the Sullivan process were used to explain the momentum fraction distribution of $\bar{d} - \bar{u}$; many other models have been proposed [5].

Most recently, Zhang and collaborators [6–8] have used a simple statistical model to calculate the $\bar{d} - \bar{u}$ distribution in the proton. They consider the proton to

be an ensemble of quark–gluon Fock states, and use detailed balance [6,7] or “the principle of balance” [8] to determine the distribution functions for all partons of the proton. Despite its simplicity, the model does reasonably well in predicting the distributions of partons, as well as for that of $\bar{d} - \bar{u}$. The excess of \bar{d} over \bar{u} comes about from the 2:1 ratio of $u:d$, which provides an excess of u quarks for the annihilation of \bar{u} 's.

Zhang, Zou, and Yang (ZZY) [7] write a general Fock state expansion for the proton as

$$|p\rangle = \sum_{i,j,k} c_{ijk} |\{uud\}\{ijk\}\rangle, \quad (1)$$

with i the number of $\bar{u}u$ pairs, j the number of $\bar{d}d$ pairs and k the number of gluons. The states are normalized such that the sum of the probabilities $\rho_{ijk} = |c_{ijk}|^2$ of finding a proton in the state $|\{uud\}\{ijk\}\rangle$,

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summed over all i, j , and k , is unity [6],

$$\sum_{i,j,k} \rho_{ijk} = 1. \quad (2)$$

In ZZY statistical model, detailed balance between any two Fock states requires that

$$\begin{aligned} \rho_{ijk} N(|\{uud\}\{ijk\}\rangle \rightarrow |\{uud\}\{i'j'k'\}\rangle) \\ \equiv \rho_{i'j'k'} N(|\{uud\}\{i'j'k'\}\rangle \rightarrow |\{uud\}\{ijk\}\rangle), \end{aligned} \quad (3)$$

in which $N(A \rightarrow B)$ is the transfer rate of state A into state B . Transfer rates between states are assumed to be proportional to the number of partons that can split or recombine. Taking into account two processes, $q \leftrightarrow qg$ and $g \leftrightarrow q\bar{q}$, ZZY find that

$$\frac{\rho_{ijk}}{\rho_{000}} = \frac{1}{i!(i+2)!j!(j+1)!k!}. \quad (4)$$

This equation, together with the normalization condition (2), determines all the ρ_{ijk} . It is clear from this equation that $u\bar{u}$ states, labelled by i , are suppressed relative to $d\bar{d}$ states, labelled by j . Summing over all states, Zhang et al. [6] find $\bar{d} - \bar{u} \approx 0.124$, remarkably close to the experimental value of 0.118 ± 0.012 [3].

ZZY determined parton distribution functions for the proton by using a Monte Carlo simulation of the distribution of momenta among the n partons in each Fock state. The phase space volume f_n^F for n free partons is determined by

$$df_n^F = \delta^4\left(P - \sum_{i=1}^n p_i\right) \prod_{i=1}^n \frac{d^3 p_i}{(2\pi)^3 2E_i}, \quad (5)$$

with P and p_i the 4-momenta of the proton and the i th parton, respectively. The masses of the partons are neglected so that $E_i = |\vec{p}_i|$, and

$$df_n^F = \delta^4\left(P - \sum_{i=1}^n p_i\right) \prod_{i=1}^n \frac{E_i dE_i d\Omega_i}{2(2\pi)^3}. \quad (6)$$

ZZY argue that this free parton phase space distribution should be multiplied by $\prod E_i^{-1}$ because partons with smaller momenta spend more time at the center of the proton where they are almost free; these partons are thus weighted with a higher probability. Then df_n , the distribution for confined partons, is:

$$df_n = \delta^4\left(P - \sum_{i=1}^n p_i\right) \prod_{i=1}^n \frac{dE_i d\Omega_i}{2(2\pi)^3}. \quad (7)$$

We have found that the effect of the weighting factor is quite small, except for the very lowest and highest parton momenta.

From the Monte Carlo distribution of parton momenta \vec{p}_i , the parton distributions can be found in terms of the light cone variable Bjorken- x ,

$$x_i = \frac{E_i - p_{zi}}{M}, \quad (8)$$

in which M is the proton mass. We used RAMBO [9] for our Monte Carlo event generator. Then for an n -parton state, for which $n = 3 + 2(i + j) + k$, the x -distributions for \bar{u} and \bar{d} are

$$\bar{u}_{ijk}(x) = f_n(x)i, \quad \bar{d}_{ijk}(x) = f_n(x)j, \quad (9)$$

for u and d are

$$\begin{aligned} u_{ijk}(x) &= f_n(x)(2+i), \\ d_{ijk}(x) &= f_n(x)(1+j), \end{aligned} \quad (10)$$

and for the gluons is

$$g_{ijk}(x) = f_n(x)k. \quad (11)$$

Thus, we find, in accord with ZZY,

$$\bar{u}(x) = \sum_{i,j,k} \rho_{ijk} \bar{u}_{ijk}(x), \quad (12)$$

and corresponding equations for $\bar{d}(x)$, $u(x)$, $d(x)$ and $g(x)$, normalized so that

$$\int_0^1 x [u(x) + d(x) + \bar{u}(x) + \bar{d}(x) + g(x)] dx = 1. \quad (13)$$

The average number of partons in the proton, \bar{n} , is given by

$$\begin{aligned} \bar{n} &= \int_0^1 [u(x) + d(x) + \bar{u}(x) + \bar{d}(x) + g(x)] dx \\ &\approx 5.6. \end{aligned} \quad (14)$$

ZZY use the zeroth moment of their distributions to set the starting scale $\mu_0 \approx \bar{E} = M/\bar{n} \approx 0.17$ GeV for $Q_0^2 = \mu_0^2$. One might argue that it would be better to use the first and/or second moments, because of the sea distributions' divergence as $x \rightarrow 0$, but their choice of scale is irrelevant, because they did not evolve their distributions. They note that the experimental data for

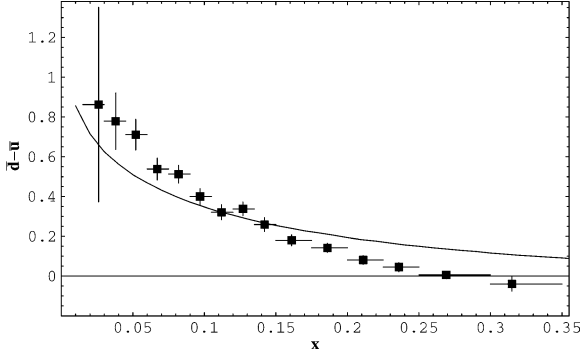


Fig. 1. Comparison of statistical model calculation with E866 experimental results [3] for $\bar{d} - \bar{u}$.

flavor asymmetry shows little Q^2 -dependence. The Hermes results [4] at $Q^2 = 2.3 \text{ GeV}^2$ are consistent with the more precise E866 data [3] at $Q^2 = 54 \text{ GeV}^2$. ZZY point out that this can be understood because flavor asymmetry is due primarily to the non-perturbative connection of “intrinsic” partons in the higher-order terms in the Fock state expansion to the leading valence quark term. This non-perturbative connection is Q^2 -independent, in contrast with the Q^2 -dependent flavor symmetric generation of “extrinsic” partons in the lepton scattering process. The plot in Fig. 1, which reproduces ZZY’s results, shows that the experimentally deduced $\bar{d} - \bar{u}$ is fit qualitatively in this model, but is low at small x and high at large x . The discrepancy with experiment shows up more starkly in Fig. 2, our plot of $\bar{d}(x)/\bar{u}(x)$. We find that these results are changed very little if the phase space weighting factors $\prod E_i^{-m}$ are varied from $m = 1$ to $m = 3$. Nevertheless, we believe that it is remarkable that such a simple model does so well.

There are, of course, other explanations of the excess of \bar{d} over \bar{u} , which have been considered previously, particularly the pion cloud [10–14] and ω mesons [15]. The leading term in the Fock state expansion of the pion cloud model is a “bare” proton which consists of valence quarks plus $\bar{q}q$ pairs due to gluon splitting. It can be argued that this gluon splitting is what is being considered by ZZY and thus the perturbative sea in the “bare” proton should not be symmetric, i.e., $\bar{d} \neq \bar{u}$. The pion cloud, represented by the higher-order terms in the expansion, would then be an additional effect. If this is the case, the pions must have a larger role at high x where the fall-off of the ra-

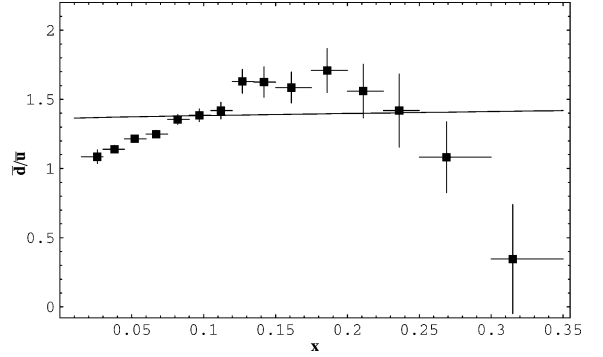


Fig. 2. Comparison of statistical model calculation with E866 experimental results [3] for \bar{d}/\bar{u} .

tio \bar{d}/\bar{u} is not reproduced by the statistical model. We will not pursue this argument further here.

2. Parton distributions in the pion

If the statistical model has some validity, then it should not only work for the proton, but also for the pion. This distribution function is, in fact, required in the pion cloud model. We have therefore investigated the valence and sea quark distributions for the π^+ in the statistical model. The formulas are similar to those for the proton, but because there is only one valence quark of each flavor in the π^+ , the sea is flavor-symmetric.

We write the Fock state expansion for the pion as

$$|\pi^+\rangle = \sum_{i,j,k} c_{ijk} |u\bar{d}\rangle \{ijk\}. \quad (15)$$

The analysis of Section 1 is unchanged, except that $n = 2 + 2(i + j) + k$, and the ratio of probabilities for different Fock states (4) is now

$$\frac{\rho_{ijk}}{\rho_{000}} = \frac{1}{i!(i+1)!j!(j+1)!k!}, \quad (16)$$

so that the π^+ sea is symmetric, i.e., $\bar{u}(x) = d(x)$. We find $\bar{n} = 4.7$ in the pion.

The phase space distributions $f_n(x)$ for the pion are identical to those of the proton for $n \geq 3$. For the 2-parton state, $f_2(x)$ is a constant. We used the same weighting factor as for the proton, $\prod E_i^{-1}$, and again found that our results were insensitive to the use of this weighting factor.

The quark distributions for a specific n -parton state are

$$\begin{aligned}\bar{d}_{ijk}(x) &= f_n(x)(1+j), \\ u_{ijk}(x) &= f_n(x)(1+i),\end{aligned}\quad (17)$$

$$\bar{u}_{ijk}(x) = f_n(x)i, \quad d_{ijk}(x) = f_n(x)j, \quad (18)$$

and for the gluons is

$$g_{ijk}(x) = f_n(x)k. \quad (19)$$

The parton distributions are found by summing these distribution functions over all values of $\{ijk\}$

$$u(x) = \sum_{i,j,k} \rho_{ijk} u_{ijk}(x) = \bar{d}(x), \quad (20)$$

$$d(x) = \sum_{i,j,k} \rho_{ijk} d_{ijk}(x) = \bar{u}(x), \quad (21)$$

$$g(x) = \sum_{i,j,k} \rho_{ijk} g_{ijk}(x). \quad (22)$$

The valence quark distribution function is

$$v(x) = u(x) - \bar{u}(x) = \bar{d}(x) - d(x). \quad (23)$$

Our results are shown in Fig. 3. The valence quark distributions are too high for large x because of the dominant contribution of the $n=2$, $\{ijk\} = \{000\}$ state, the leading term in the Fock expansion, for which $f_2(x)$ is a uniform distribution in x . The sea quark distribution is flavor symmetric, as noted above. Unlike the

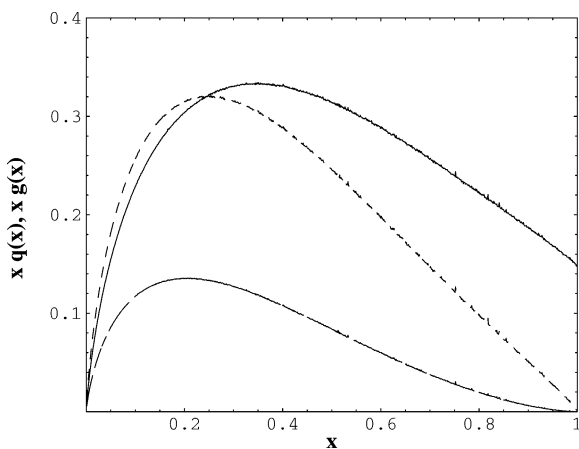


Fig. 3. Our results for parton density distributions $xq(x)$ and $xg(x)$ for the pion. Solid curve: valence quark distribution; long-dashed curve: sea quark distribution; short-dashed curve: gluon distribution.

$\bar{d} - \bar{u}$ or \bar{d}/\bar{u} distributions of the proton, we expect our valence quark distributions to be Q^2 -dependent. To compare to experiment, we carried out an evolution in Q^2 . We determined the starting scale of our distributions by requiring that the first and second moments of our valence quark distribution at $Q^2 = 4 \text{ GeV}^2$ be equal to those found by Sutton et al. [17]. This gave us a starting scale of $Q_0^2 = 1.96 \text{ GeV}^2$. We used Miyama and Kumano's code BF1 [18] for the DGLAP [19] evolution. We compare our pion valence quark distribution with that obtained by E615 from pion scattering on tungsten [16] in Fig. 4. The dashed curve shows our results for the valence quark distributions, without any evolution, as in Fig. 3. The solid curve shows our results evolved to $Q^2 = 16 \text{ GeV}^2$ of the E615 experiment. The agreement between theory and experiment is good. Other theoretical calculations of pion parton distribution functions have used constituent quark models, the Nambu–Jona-Lasinio model, instantons, or the Dyson–Schwinger equations. Moments of the distributions can be calculated in lattice QCD, from which particular forms of the distributions can be reconstructed. For references see the recent papers of Hecht, Roberts and Schmidt [20] and Detmold, Melnitchouk and Thomas [21]. In Fig. 5 we compare our valence quark distribution to the Dyson–Schwinger calculation of Hecht et al. and to experiment. Both

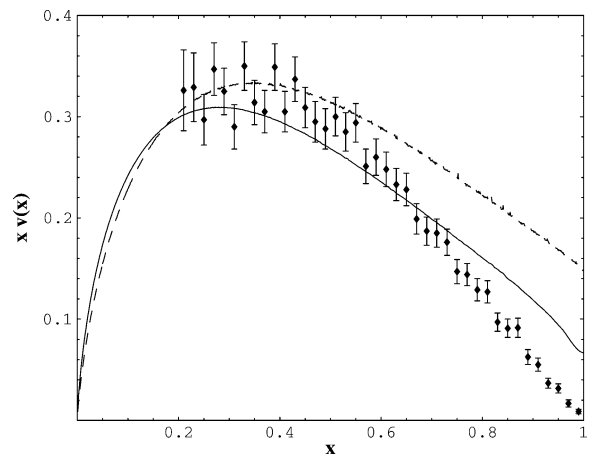


Fig. 4. Our calculation of the valence quark distribution $xv(x)$ in the pion, compared to the experimental results of Conway et al. [16]. The dashed curve shows our results without any evolution, as in Fig. 3, which correspond to a scale of $Q_0^2 = 1.96 \text{ GeV}^2$. The solid curve shows our results evolved to $Q^2 = 16 \text{ GeV}^2$ of the E615 experiment.

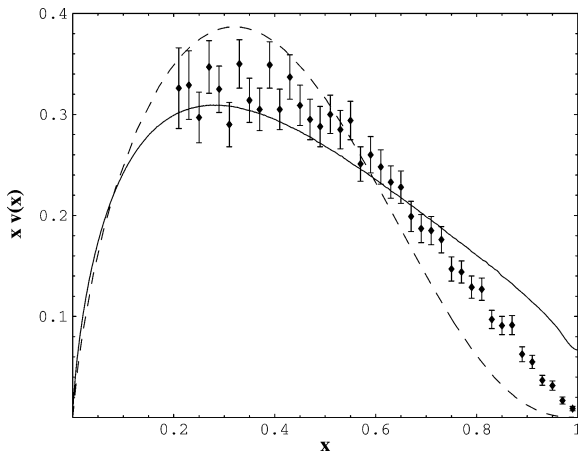


Fig. 5. Our results (solid curve) for the valence quark distribution $xv(x)$ in the pion, compared to the calculation of Hecht, Roberts and Schmidt [20] (dashed curve) and the experimental results of Conway et al. [16]. Both calculations were evolved to $Q^2 = 16 \text{ GeV}^2$ of the E615 experiment.

distributions were evolved to $Q^2 = 16 \text{ GeV}^2$. We find it remarkable that our simple statistical model agrees with experiment as well as the covariant, QCD-based model.

3. Conclusions

The calculation of parton distribution functions is an important goal of non-perturbative QCD. We have used the statistical model of Zhang et al., developed for the calculation of parton distribution functions in the proton, to calculate the parton distribution functions of the pion. We find that this simple model, with no free parameters, is in good agreement with experiment and other calculations.

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