Selective maintenance problem for series–parallel system under economic dependence

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Received 20 January 2016; revised 26 April 2016; accepted 27 April 2016
Available online 19 May 2016

Abstract

In view of the high complexity of the objective world, an economic dependence between subsystems (paired and unpaired) is proposed, and then the maintenance cost and time under different economic dependences are formulated in a simple and consistent manner. Selective maintenance problem under economic dependence (EDSMP) is presented based on a series–parallel system in this paper. A case study shows that the system reliability is promoted to a certain extent, which can validate the validity of the EDSMP model. The influence of the ratio of set-up cost on system performance is mainly discussed under different economic dependences. Several existing improvements of classical exhaust algorithm are further modified to solve a large sized EDSMP rapidly. Experimental results illustrate that these improvements can reduce CPU time significantly. Furthermore the contribution of each improvement is defined here, and then their contributions are compared thoroughly.

Keywords: Selective maintenance; Economic dependence; Series–parallel system; Exhaust algorithm

1. Introduction

In the field of system reliability and maintenance engineering, a typical series–parallel system consists of multiple subsystems in series, which is composed of several components in parallel. As an important kind of multi-component system, such model can describe distinctly the basic features of a complex system, such as aircraft engine [1], naval vessel [2], machining line in automobile engine shop [3] and coal transportation system in power station [4].

In many applications, including manufacturing equipment, nuclear power plants, electrical transmission networks, rail industry, aircrafts, and military weapons, a system has to perform some consecutive missions. However, the components in the system cannot suffer any maintenance activities during these missions. During the interval between any adjacent missions, it is a wonderful opportunity for maintenance staff to make a good maintenance on the failed components in system in order to maintain the system in normal and safety state, and decrease the probability of failure during the next mission.

These maintenance activities should include at least inspection, lubrication, adjustment, repair, replacement, and so on. However, it is impossible to perform all necessary maintenance activities before the next mission for the limited maintenance resources, such as time, budget, staff, spare parts and workshop. Therefore, decision-makers have to determine a subset from the complete set of all available maintenance activities. Only the selected maintenance activities are to be completed by a maintenance team during a mission interval, while the rest of the activities will be carried out for the next chance. The main purpose of such tough choice for maintenance activities is that the overall performance and reliability of the system should be restored as much as possible on the premise of meeting the maintenance resource requirements. Such problem, called selective maintenance problem (SMP), falls into maintenance resources allocation and mission planning. It was first proposed by Rice et al. in 1998 [5].

The original selective maintenance model for series–parallel system was extended following two ways in Refs. [6,7]. First, each independent subsystem in a complex system was composed of inhomogeneous components, which could be connected in any way. Second, it was also extended to a general case where both maintenance time and budget were constrained. Three alternative selective maintenance models were defined and the
most appropriate one could be determined by decision-makers based on the interested performance index. One possible objective was to maximize system reliability, the second possible objective was to minimize total repair cost, and the last possible objective was to minimize total repair time. After that, in order to make the maintenance model more feasible and exercisable in engineering applications, the component life was assumed to follow Weibull distribution in series–parallel system, and the decision-makers could also perform different maintenance policies [8].

After maintenance activities, the component state is not the exact combination of as good as new and as bad as old, and it may be somewhere between two extreme states, i.e., the so-called imperfect maintenance. An imperfect maintenance model was built ingeniously to reveal the relationship among maintenance cost, age reduction factor and effective age, and then a novel selective maintenance model was proposed by Pandey et al. [9]. Following this initial idea, a selective maintenance model with two failure modes, maintainable failure mode and non-maintainable failure mode, was considered in Ref. [10]. Notice that the probability of the former relies on the cumulated probability of the latter, which is different from that in Ref. [9]. This selective maintenance model was further extended to multi-state system with multi-state component [11].

Optimization of selective maintenance for multi-mission series–parallel system was addressed by Knatab et al. [12]. The optimal order of preventive maintenances was found for the purpose of minimizing total maintenance cost subject to necessary system reliability constraint for each mission. Maaroufi et al. studied carefully a specific complicated system with global failure propagation and isolation effects under limited maintenance resources, such as maintenance time and cost before the next mission [13].

The optimization methods applied to selective maintenance models are either exact method or heuristic method. Exact methods always find the global optimum solution. For improving the efficiency of total enumeration, Rajagopalan and Cassady explored several avenues, described in detail in Section 3.2 [14]. Along with the similar idea, several rules that potentially narrow the solution space and consequently accelerate the optimization procedure were also proposed in Ref. [13]. Given a system reliability requirement, an exact algorithm based on the original Kettele’s algorithm was proposed to minimize the total maintenance cost for series–parallel system in Ref. [2].

If the complexity of a selective maintenance problem is high and the computing time of the exact method increases exponentially with the size of the optimization problem, the heuristic method can be used to find a near-optimal solution in reasonable time. Three new methods (construction heuristic method, exact method based on branch and bound procedure, and heuristic approach based on Tabu search) were developed by Lust and then applied to various system configurations [15]. The modified great deluge algorithm, a local search meta-heuristic method, incorporated both the worse solution acceptance and the well-known hill climbing rule [16]. This method requires only one parameter, while it provides the competitive results within a good reasonable computational time. Other heuristic approaches often mentioned in related literatures are genetic algorithm (GA) [4], differential evolution (DE) [11], simulated annealing algorithm (SAA) [17], and so on.

For a detailed overview on the state-of-the-art selective maintenance modeling and optimization, we strongly recommend a recent review article for interested readers, entitled “Recent advances in selective maintenance from 1998 to 2014” [18].

Thomas classified the multi-component maintenance models on the basis of the interaction between components and defined three distinct types of interactions: economic dependence, structural dependence, and stochastic dependence [19]. Dekker and Wildeman dealt exclusively with maintenance models under economic dependence, which were classified based on the planning aspect: stationary model and dynamic model [20]. Some latest advances in maintenance decision modeling on multi-component system were summarized by Nicolai and Dekker [21] and Nowakowski and Werbinka [22], respectively.

In a word, economic dependence implies that the cost can be saved when several components are jointly maintained instead of separately; structural dependence is applied if several components structurally form a part, so the repair or replacement of a failed component implies maintenance or dismantling of other working components as well; and stochastic dependence occurs if the state of a component (valid age, lifetime distribution, failure probability, failure state, etc.) can influence those of other components in a given probability. Comparatively speaking, structural dependence and stochastic dependence mainly focus on the lifetime distribution or degradation process of components in a repairable system. In contrast, the economic dependence cannot modify the failure model of each component. Its core issue is that how to assemble these maintenance activities in all components in order to save more maintenance costs. The economic dependence can be formulated in a simple manner and still widely applied in many real-world situations. Until now, its research are relatively abundant among all dependences among the components [23–26]. So the maintenance policies under economic dependence only are discussed in the present paper.

A selective maintenance model incorporating economic dependence was recently presented by Dao et al. [27,28]. Time and cost savings are achieved when several components are simultaneously repaired in a selective maintenance strategy. The authors considered two types of time and cost savings in multi-state contexts as follows: (1) the fixed savings due to common set-up activities on all components and (2) the additional advantage due to specific maintenance activity on multiple identical components. However, these two interactions between components considered in Refs. [27,28] are not comprehensive enough and should entail more thorough discussion. Furthermore, GA was used to solve the proposed selective maintenance model for multi-state series–parallel system. In order to keep down the scope of solution space, they dealt with a series–parallel system with only nine components, which application is limited.

On the basis of the previous works, the selective maintenance problem for the two-state series–parallel system under economic dependence was studied from the perspective of mathematic modeling and algorithm design in the present paper. First, the economic dependence was investigated seriously and distinguished
in three major categories (economic dependence in system, economic dependence in subsystem, and economic dependence between subsystems), and then the corresponding computation equations of total time and cost savings were given in a simple and consistent manner for the first time. Second, some existing improvements in Ref. [14] were analyzed and revised in order to fit the novel selective maintenance model, and then the contributions of each improvement on algorithm performance were also discussed. Note that these improvements are based on the classical exhaust algorithm and were proposed for the original selective maintenance model.

## 2. Selective maintenance model under economic dependence

### 2.1. System description

We consider a system composed of \(m\) independent subsystems, referred to as subsystem 1, subsystem 2, \ldots, subsystem \(m\), connected in series. Each subsystem \(i\) contains a set of \(n_i\) independent and identical components connected in parallel. This type of system configuration, indeed, represents a wide variety of equipment utilized in both industrial and military environments. Fig. 1 shows a reliability block diagram of a system. Note that each component in the series–parallel system is denoted by \((i, j)\), where \(i\) is the subsystem number and \(j\) is the component number.

In addition, each component, subsystem and system are assumed to have two possible states denoted by 0 (complete failure state) and 1 (perfect functioning state). As the system is a series arrangement of subsystems, all the subsystems must function for a subsystem to function because of the parallel arrangement of components.

Within a given time period, the system is required to perform a series of consecutive identical missions, and the maintenance activities are executed only during the breaks between adjacent missions. For the sake of later discussion, it is assumed that the amount of maintenance resources (budget, time, etc.) required for a single maintenance activity is deterministic and known by the maintenance staff in advance. The reliability of a component, subsystem or system, which indicates the probability of successfully accomplishing the given mission, is also given in advance.

### 2.2. Selective maintenance modeling

Let \(r_i\) denote the probability that a component of subsystem \(i\) successfully completes a mission given that the component is functioning at the start of the mission, \(a_i\) denote the number of failed components in subsystem \(i\), \(d_i\) denote the number of failed components in subsystem \(i\) to be repaired before the next mission, \(t_i\) and \(c_i\) denote the amount of time and cost required to repair a component of subsystem \(i\), respectively, and \(T_0\) and \(C_0\) denote the total amount of time and cost allotted to perform maintenance on failed components between missions, respectively.

Under these definitions and assumptions, an initial selective maintenance model was introduced for assisting in the selective maintenance decision [5]. The final form of mathematical programming model is given below.

**Maximize**

\[
R = \prod_{i=1}^{m} \left[ 1 - (1 - r_i)^{(n_i - a_i + d_i)} \right] 
\]  

subject to

\[
\sum_{i=1}^{m} t_i d_i \leq T_0 \tag{2}
\]

\[
\sum_{i=1}^{m} c_i d_i \leq C_0 \tag{3}
\]

\[0 \leq d_i \leq a_i \tag{4}\]

\[d_i \text{ is integer } i = 1, 2, \ldots, m \tag{5}\]

The objective of this maintenance model, as shown in Eq. (1), is to obtain the optimal maintenance action subset to maximize the system reliability for the next single mission. The constraints given by Eqs. (2) and (3) imply that all maintenance activities should be completed within the allotted time and cost. The constraint given by Eq. (4) imposes that the number of components in a subsystem that are repaired cannot exceed the number of components that are failed. The constraint in Eq. (5) means that the number of components in subsystem that are repaired must be a non-negative, integer value.

### 2.3. Repair time and cost under economic dependence

In a formal way, the selective maintenance model listed above can be applicable for both the cases without and with considering the economic dependence. The apparent difference between them is the calculating process and results of total actual time and cost consumed by all maintenance activities. For the former case (such as in Ref. [5]), total actual cost can be simply formulated as the left side of Eq. (3). However, for the latter case, total actual cost is equal to the difference between total theoretical cost and cost savings due to economic dependence. Thus we should discuss the accumulative impact on maintenance time and cost, and also attempt to provide the corresponding equations of time and cost savings due to different economic dependence categories in this subsection.

In most industrial and military systems, the total maintenance cost of multi-component system usually consists of variable costs and a fixed cost. The first part is directly related to the components to be repaired; however, the other part has nothing to do with the category and amount of all failed components.
Economic dependence between subsystems typically requires the occurrence of the economic dependence in subsystem results from repairing these failed components is \((N_r - 1) \times \text{cost}_{\text{set}}\) for unpaired and \((2 \times \text{min}(N_1, N_2) - 1) \times (\text{cost}_{\text{set}} + \text{cost}_{\text{add}})\) for paired, respectively. It is obvious that the same maintenance cost savings can be attained when \(N_1 = N_2 = N/2\) for two cases.

If \(N_r\) components in series–parallel system are maintained, \(N_r \times (\text{cost}_{\text{set}} + \text{cost}_{\text{add}})\) is supposed to be the original total maintenance cost. According to the above analysis, the saved set-up cost and actual total maintenance cost of these components can be determined for different circumstances, as shown in Table 1, when economic dependence is considered seriously into maintenance activities for repairable systems.

It is noted that, although the discussion and results are based on maintenance cost, the analysis process and main conclusion are also suitable for maintenance time. Hence, the total maintenance time of repairable systems can also be calculated in a similar way, and we will not go further on this topic here. As a final note on the selective maintenance model under economic dependence, when the total actual maintenance cost (modification of left side of Eq. (3)) is calculated, the decision-maker can adopt the results from Table 1 directly as cost savings due to economic dependence.

3. Solution methodology

3.1. Exhaust algorithm

For all optimal solutions, the classical exhaust algorithm, called exhaust algorithm I in the present paper, was utilized for solving the basic selective maintenance model [5]. Its pseudocode is presented as follows.

As seen from exhaust algorithm I, the decision variable \(d_i\) \((i = 1, 2, \ldots, m)\) is restricted (see lines 2–6 in Algorithm 1) to
satisfy the constraint condition (Eq. (4)), and the maintenance cost and time are also restricted (see lines 7 and 8 in Algorithm 1) to satisfy the other two maintenance resource constraints (Eqs. (2) and (3)). For the relaxed ranges of variable \(d_i\), some candidate solutions do not satisfy the maintenance cost constraint and/or the maintenance time constraint obviously, which may waste massive computation and memory resources inevitably. Because of this, the ranges of all variables are further compressed when three constraint conditions are considered simultaneously [14]. As a result, it is unnecessary to calculate the actual total maintenance time and cost (line 7 in Algorithm 1) and decide whether or not candidate solutions satisfy all constraints (line 8 in Algorithm 1). Apparently it can speed up the search rate and improve the algorithm efficiency. The pseudo-code of the revised exhaust algorithm, called exhaust algorithm II, is also presented as follows.

For the nonlinear and discrete selective maintenance model, the classical exhaust algorithm takes a longer time to cover the entire solution space, thus it cannot meet the active demands in large scale engineering problems.

3.2. Existing improvements

Four improvements of exhaust algorithm were proposed by Rajagopalan and Cassady [14]. On the premise of ensuring the global optimal solution, the improved exhaust algorithm can shorten running time and improve efficiency evidently. For the purpose of convenient analysis, four existing improvements are explained briefly as follows.

1) Since the amounts \((d_1, d_2, \ldots, d_{m-1})\) of repaired components in the former \(m-1\) subsystems are already determined, the variable \((d_m)\) of the last subsystem has to provide the maximum value to obtain the optimal system reliability. Therefore, the first improvement can be summarized as

\[
\begin{align*}
\text{dd d} & = \max_{\max} (6) \\

2) & \text{Because the object investigated in the present paper is a series–parallel system, at least one component in each subsystem must be in working state. Otherwise, the entire system will fail in performing its function. If all components in a subsystem failed, at least one of the components in such failed subsystem must be repaired prior to the next mission. Hence,}

\[
\begin{align*}
d_m &= d_m^{\max} \\

3) & \text{At the same time, the upper bounds of the decision variables are revised as follows:}

\[
\begin{align*}
d_i^{\max} &= \min \left\{ a_i, \left[ \frac{C_i - \sum_{j<i} c_j d_j - \sum_{k>j} c_k d_k^{\min}}{c_i} \right], \frac{T_j - \sum_{j<i} t_j d_j - \sum_{k>j} t_k d_k^{\min}}{t_j} \right\} (8) \\

4) & \text{where} \quad a_i = \begin{cases} 0 & \text{if} \quad a_i < n_i \\ 1 & \text{if} \quad a_i = n_i \end{cases} (7)
\end{align*}
\]
Thus the scopes of the decision variables are further tightened as follows

\[ d_i = d_i^{\text{min}} : d_i^{\text{max}} \quad (9) \]

3) The concept of branch-and-bound method is utilized in the third improvement effort. In each iteration process in the exhaust strategy, an upper bound on mission reliability is estimated at the current state of system and then determined if the iteration is worthwhile. If it is possible to improve the system performance, the iteration is executed continuously. Otherwise, this iteration process is stopped in advance. Similarly, the revised exhaust algorithm is summarized using the following pseudo-code. Note that the basic ideas of the former two improvements are also incorporated in Algorithm 3. Simply, we directly utilize Eq. (9) (lines 2, 6 and 10 in Algorithm 3), in which the lower bounds \( d_i^{\text{min}} \) and upper bounds \( d_i^{\text{max}} \) of the decision variables can be expressed as Eqs. (7) and (8), respectively.

4) As we all know, the goal of selective maintenance problem is to maximize system reliability. The optimal values of the decision variable \( d_i \) are typically closer to their upper bounds than their lower bounds. Hence, changing their search method from ascending order to descending order should also speed up the search rate. So,

\[ d_i = d_i^{\text{max}} : d_i^{\text{min}} \quad (10) \]

3.3. Application in EDSMP of the revised exhaust algorithm

For the classical SMP, the main purpose of four improvements proposed by Rajagopalan is to shorten CPU time of exhaust algorithm massively without decreasing its accuracy. Then can these improvements continue to function properly when solving EDSMP proposed in Section 2? This issue needs to be analyzed carefully, and then some corresponding revision suggestion should be presented. It is also one of the primary contributions of this paper.

It is obvious that the exhaust algorithm II is the common ground for four improvements. However, the interactions among components may occur indefinitely, leading to the complicated calculation of maintenance cost and time in EDSMP model. We have to collect all maintenance resource information first and then compute the total actual maintenance time and cost consumed only once after determining the last decision variable, instead of calculating it step by step. Therefore, it is useless to determine the maximum number \( d_m^{\text{max}} \) of repaired components in each subsystem independently. In view of this, four improvements explained in Section 3.1 should be further modified to suit EDSMP model accordingly.

First, the maximum number \( d_m^{\text{max}} \) of repaired components in the last subsystem cannot be defined even when the maintenance strategies of the former \( m-1 \) subsystems have been determined. In fact, it is possible that the value of \( d_m^{\text{max}} \) is less than \( a_m \), which means that it does not satisfy the constraint in

\[ d_m = d_m^{\text{max}} : d_m^{\text{min}} \quad (10) \]
Eqs. (2) or/and (3). On the other hand, it may also be greater than \( \min_{i=1}^{m} \left[ \left( 1 - r_j \right)^{n_j} \right] \). The reason is that, due to economic dependence in repairable systems, the saved maintenance time and cost can support us to repair more failed components than the case not considering economic dependence. Therefore, the first improvement may be no longer applied in the EDSMP model.

For the same reason, Eq. (8) for computing the upper bounds \( d_{i}^{\text{max}} \) of decision variables \( d_i \) is not suitable for new EDSMP model, while Eq. (7) for computing the lower bounds \( d_{i}^{\text{min}} \) is not affected in the second improvement.

We have not modified the value of \( d_{i}^{\text{max}} \) in the third and fourth improvements. Hence they should be used well in the revised exhaust algorithm. It should be noted that, in fact, \( d_{i}^{\text{max}} \) in our revised algorithm is calculated as classical exhaustive algorithm I in Ref. [5], not as exhaustive algorithm II in Ref. [14].

What we have to indicate more especially is that, for all algorithms in this paper, the total actual maintenance time and cost should be calculated based on the analysis results in Table 1.

4. Experimental results and discussion

In the present paper, all exhaust algorithms were programmed by MATLAB 2013. All simulation experiments were run on Windows XP, with Inter Core 2.3 GHz and 2.5 GB memory capacity. The data were analyzed with software IBM SPSS Statistics 19.0 version, and significance is assumed at 0.05.

4.1. Effectivity of selective maintenance model under economic dependence

A simple series–parallel system is considered, as shown in Fig. 2. Let \( m = 3 \), \( T_0 = 23 \), \( C_0 = 80 \), while other specific parameters are listed in Table 2. Total maintenance time \( T = 3 \times 2 + 2 \times 3 + 4 \times 4 = 28 \) and maintenance cost \( C = 3 \times 9 + 2 \times 15 + 4 \times 13 = 109 \) when all failed components are repaired at one time. It is clear that \( T > T_0 \) and \( C > C_0 \).

With the help of selective maintenance model, the subset of failed components should be determined to be repaired before the next mission, within the given maintenance time and cost limitation.
Exhaust algorithm I or II mentioned above can find the optimal solution $d^* = [2, 1, 3]$ easily in this example. The system reliability was increased from 0.5400 to 0.9782 (approximately 81%) after maintenance activities. The maintenance time $T$ consumed is 19, and the total maintenance cost $C$ is 72, which meet all constraint conditions of maintenance resources (Eqs. (2) and (3)).

The influence of economic dependence on the system performance is discussed later in this section. The simulated results are summarized in Table 3.

The ratio of set-up cost to maintenance cost is assumed as 10% when the economic dependence in system is considered. Exhaust algorithm I can find the new optimal solution $d^* = [2, 2, 3]$ in this example. In addition, the system reliability was increased further to 0.9871, the maintenance time $T$ consumed is 20.1143, and the maintenance cost $C$ is 79.5429. Compared with the selective maintenance model without considering economic dependence, the series–parallel system can repair an additional component in Subsystem 2 due to the saved maintenance resources in this case, thus resulting in improving the overall system reliability slightly by 0.91%.

Similarly, the ratio of set-up cost to maintenance cost is assumed as 13% when the economic dependence in subsystem is considered. The optimal maintenance strategy (optimal solution $d^* = [3, 1, 3]$ in this example) will repair one more component in Subsystem 1 than the case without considering the economic dependence. Under such environment, the system reliability is 0.9846, the total maintenance time is 19.4400, and the total maintenance cost is 75.2800. Compared with the selective maintenance model under economic dependence in system, the system reliability decreases slightly, although the ratio of set-up cost increases. The reason is that the second type of economic dependence exists only when two failed identical components in the same subsystem are repaired simultaneously. Hence the maintenance resource will be saved under tougher conditions in such type of economic dependence. In this example, only one component in Subsystem 2 is repaired, and no more maintenance resources are saved. Although several failed components in Subsystems 1 and 3 are repaired, the maintenance resources in two subsystems are calculated independently. Hence the saved maintenance time is equal to $2 \times 13\% \times (3 - 1) + 4 \times 13\% \times (3 - 1) = 1.56$, and the saved maintenance cost is equal to $9 \times 13\% \times (3 - 1) + 13 \times 13\% \times (3 - 1) = 5.72$.

For economic dependence between subsystems, the ratio of set-up cost to maintenance cost is also assumed as 13%, and Subsystems 1 and 3 are two given ones. The optimal solution and the system reliability found by exhaust algorithm I in this example are the same as the case under economic dependence in subsystem. The difference between two cases is the total maintenance resources consumed by the series–parallel system. For this case, the total maintenance time is 19.0500, and the total maintenance cost is 73.8500. Obviously, compared with the case under economic dependence in subsystem, the series–parallel system can retain the same system reliability with less resource. It is because that the maintenance resources consumed in Subsystems 1 and 3 are calculated together in this example. Hence the saved maintenance time is equal to $(3 + 3 - 1) \times (2 \times 13\% \times 3 + 4 \times 13\% \times 3) / (3 + 3) = 1.95$, and the saved maintenance cost is equal to $(3 + 3 - 1) \times (9 \times 13\% \times 3 + 13 \times 13\% \times 3) / (3 + 3) = 7.15$. It can also be observed that, by coincidence, Subsystems 1 and 3 have a uniform amount of failed components to be repaired in the optimal solution. As a result, for the selective maintenance models under two subtypes of the third economic dependence, we can obtain identical optimal solution and consumed maintenance resources in this example.

### 4.2. Influence of the ratio of set-up cost to maintenance cost

It appears that the ratio of set-up cost to maintenance cost has clear implications for the optimal maintenance strategy. Now it will be analyzed using the quantitative approach.

Generally speaking, it is impossible that the set-up cost is more than half of the total maintenance cost. Hence the ratio of set-up cost to maintenance cost increases from 1% to 50%, with 1% interval in the next experiment. The simulated results are illustrated in Fig. 3.

According to practical experiences, with the continuous increase in the ratio of set-up cost to maintenance cost, more maintenance resources can be saved and then utilized to repair more components. As seen from Fig. 3, the serial–parallel system suffers several optimal solutions, including $[2, 1, 3], [3, 1, 3], [2, 2, 3], [3, 1, 4], [3, 2, 3]$, and $[3, 2, 4]$. In this example, the amount of repaired components increases from 6 to 9

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**Table 2**

<table>
<thead>
<tr>
<th>Parameters of series–parallel system.</th>
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<tbody>
<tr>
<td>$i$</td>
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<tr>
<td>1</td>
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<td>2</td>
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<tr>
<td>3</td>
</tr>
</tbody>
</table>

**Table 3**

<table>
<thead>
<tr>
<th>Economic dependence</th>
<th>Ratio of set-up</th>
<th>Optimal solution</th>
<th>System reliability</th>
<th>Maintenance time</th>
<th>Saved time</th>
<th>Maintenance cost</th>
<th>Saved cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Without economic dependence</td>
<td>–</td>
<td>[2, 1, 3]</td>
<td>0.9782 (100%)</td>
<td>19</td>
<td>–</td>
<td>72</td>
<td>–</td>
</tr>
<tr>
<td>Economic dependence in system</td>
<td>10%</td>
<td>[2, 2, 3]</td>
<td>0.9871 (+0.91%)</td>
<td>20.1143</td>
<td>1.8857</td>
<td>79.5429</td>
<td>7.4571</td>
</tr>
<tr>
<td>Economic dependence in subsystem</td>
<td>13%</td>
<td>[3, 1, 3]</td>
<td>0.9846 (+0.65%)</td>
<td>19.4400</td>
<td>1.5600</td>
<td>75.2800</td>
<td>5.7200</td>
</tr>
<tr>
<td>Economic dependence between subsystems (unpaired)</td>
<td>13%</td>
<td>[3, 1, 3]</td>
<td>0.9846 (+0.65%)</td>
<td>19.0500</td>
<td>1.9500</td>
<td>73.8500</td>
<td>7.1500</td>
</tr>
<tr>
<td>Economic dependence between subsystems (paired)</td>
<td>13%</td>
<td>[3, 1, 3]</td>
<td>0.9846 (+0.65%)</td>
<td>19.0500</td>
<td>1.9500</td>
<td>73.8500</td>
<td>7.1500</td>
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gradually, and the system reliability shows an increased tendency from 0.978244 to 0.996428.

As a whole, the selective maintenance model under economic dependence in system can obtain the optimal system reliability and thus show the best performance among all. The second best strategy is economic dependence in the subsystem, and the third and fourth best strategies are the economic dependence between subsystems (unpaired and paired). The experimental results are identical to our expectation.

The variation of maintenance cost savings and time savings is demonstrated with different ratios of set-up cost to maintenance cost in Figs. 4 and 5, respectively. It is clear that the saved maintenance cost and time are increased for all EDSMP models when the ratio of set-up cost to maintenance cost has risen steadily. The fitting results of the experimental data are as follows in our example: $C_1 = 102.4380x - 2.8860$ and $T_1 = 26.3265x - 0.7722$ for economic dependence in a system; $C_2 = 73.8532x - 3.3072$ and $T_2 = 19.2330x - 0.8420$ for economic dependence in a subsystem; $C_3 = 65.1486x - 1.2409$ and $T_3 = 18.0356x - 0.3536$ for economic dependence between subsystems (unpaired); $C_4 = 55.4384x - 0.5298$ and $T_4 = 15.1196x - 0.1445$ for economic dependence between subsystems (paired). By a comparison of the slopes of the fit lines, we can also deduce the difference of four economic dependence models and then determine the precedence.

In order to measure the difference degree among all types of economic dependence, the coincidence between two models is first introduced in this paper and defined as follows. If the system performance of maintenance model A is better than that of maintenance model B, the coincidence between models A and B, $Coin_{AB}$, is defined as the ratio of the number of finding the identical system reliability (or optimal solution) by two models in all experiments against the total number. Obviously, a lower value of $Coin_{AB}$ indicates more clear advantage of maintenance model A. As shown in Table 4, the model coincidences among different economic dependences are calculated based on the results in Fig. 3.

It can be found from Table 4 that the coincidence between two subtypes of economic dependence between subsystems (unpaired and paired) is greater than 50%. In a sense, the difference between them is not obvious. Moreover, the coincidences between economic dependence in subsystem and two economic dependences between subsystems are also greater than 50%, showing very high similarity. In fact, the series–parallel system is composed of three subsystems in series in this example. In the present paper, the interaction relationships of

![Fig. 3. Influence of the ratio of set-up cost to maintenance cost on the system reliability.](image)

![Fig. 4. Maintenance cost savings vs. ratio of set-up cost to maintenance cost.](image)

![Fig. 5. Maintenance time savings vs. ratio of set-up cost to maintenance cost.](image)

<table>
<thead>
<tr>
<th>Economic dependence</th>
<th>Economic dependence in system</th>
<th>Economic dependence in subsystem</th>
<th>Economic dependence between subsystems (unpaired)</th>
<th>Economic dependence between subsystems (paired)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Economic dependence in system</td>
<td>–</td>
<td>46%</td>
<td>38%</td>
<td>18%</td>
</tr>
<tr>
<td>Economic dependence in subsystem</td>
<td>Undefined</td>
<td>–</td>
<td>80%</td>
<td>58%</td>
</tr>
<tr>
<td>Economic dependence between systems (unpaired)</td>
<td>Undefined</td>
<td>Undefined</td>
<td>–</td>
<td>64%</td>
</tr>
<tr>
<td>Economic dependence between systems (paired)</td>
<td>Undefined</td>
<td>Undefined</td>
<td>Undefined</td>
<td>–</td>
</tr>
</tbody>
</table>
two given subsystems are considered for the third type of economic dependence, while the interaction relationship of three subsystems are considered independently for the second type of economic dependence. Intuitively, the similar quantity of subsystems is considered for two types of economic dependence in our experiments. Of cause, when the series–parallel system consists of more subsystems, more subsystems can be considered for the second type of economic dependence, which results in more clear advantage.

4.3. Application and contribution of algorithm improvements

The optimal maintenance strategy and the corresponding system reliability are found using exhaust algorithm, and then the effectiveness of three EDSMP models is validated under the fixed system configuration in two sets of experiments above. However the classical exhaust algorithm is run in low efficiency, and with the increase in size and complexity of repairable system, some improvements must be integrated to speed up the computation process. The influence of improvements on algorithm efficiency was fully discussed in this subsection. In our sample, we take selective maintenance problem under the second type of economic dependence as an illustration. For other types of economic dependence, the research idea and main conclusion are similar with this case, and it is unnecessary to go into details here.

In order to eliminate the effect of system configuration on algorithm performance, some series–parallel systems are generated automatically as follows. The number of subsystem, \( m \), is set as 3, 4, 5, 6, 9, 12 and 15, and other parameters are listed as Eqs. (11)–(17).

\[
\begin{align*}
\tau_i & = 0.65 + 0.05 \times DU (1, 6) \\
n_i & = DU (a_n, b_n) \\
a_i & = DU (0, n_i) \\
t_i & = DU (1, 5) \\
c_i & = DU (1, 5) \\
T_0 & = DU \left( \sum_{i=0}^{n-1} t_i, \sum_{i=1}^{m} t_i a_i \right) \\
C_0 & = DU \left( \sum_{i=0}^{n-1} c_i, \sum_{i=1}^{m} c_i a_i \right)
\end{align*}
\]

where \( a_n = \begin{cases} 1 & \text{if } \eta_i \geq 0.85 \\ 2 & \text{if } \eta_i \leq 0.80 \end{cases}, \quad b_n = \begin{cases} 2 & \text{if } \eta_i = 0.95 \\ 3 & \text{if } 0.75 \leq \eta_i \leq 0.90, \\ 4 & \text{if } \eta_i = 0.70 \end{cases} \)

\( DU(a, b) \) refers to a discrete uniform random variable over the integers \( \{a, a+1, \ldots, b\} \). In all potential instances, the largest size of this optimization problem is the series–parallel system composed of 15 subsystems, each containing 4 components. It distinctly surpasses the sizes of series–parallel systems in Refs. [27,28].

<table>
<thead>
<tr>
<th>( m )</th>
<th>( \overline{X}_0 )</th>
<th>( \overline{X}_1 )</th>
<th>( t_0 )</th>
<th>Accept</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>0.1825</td>
<td>0.2086</td>
<td>14.0986</td>
<td>( H_1 )</td>
</tr>
<tr>
<td>4</td>
<td>0.2912</td>
<td>0.3325</td>
<td>12.3903</td>
<td>( H_1 )</td>
</tr>
<tr>
<td>5</td>
<td>0.4806</td>
<td>0.5176</td>
<td>15.0897</td>
<td>( H_1 )</td>
</tr>
<tr>
<td>6</td>
<td>0.8681</td>
<td>0.8313</td>
<td>−5.8083</td>
<td>( H_2 )</td>
</tr>
<tr>
<td>9</td>
<td>6.7058</td>
<td>4.0412</td>
<td>−53.7025</td>
<td>( H_2 )</td>
</tr>
<tr>
<td>12</td>
<td>65.3133</td>
<td>23.7733</td>
<td>−71.2480</td>
<td>( H_2 )</td>
</tr>
<tr>
<td>15</td>
<td>638.1045</td>
<td>144.5653</td>
<td>−60.1926</td>
<td>( H_2 )</td>
</tr>
</tbody>
</table>

The efficiencies of two exhaust algorithms were discussed by a comparison of their CPU time. In this example, \( \mu_0 \) and \( \mu_i \) are the average CPU time required using the original exhaust algorithm I and the exhaust algorithm I with improvements, respectively. The null and two alternative hypotheses for the tests are as follows

\[
H_0: \mu_i - \mu_0 = 0, \\
H_1: \mu_i - \mu_0 > 0, \\
H_2: \mu_i - \mu_0 < 0.
\]

For each value of \( m \), a paired \( t \)-test with a level of significance of \( \alpha = 0.05 \) is used. For each value of \( m \), each system configuration, one for the original exhaust algorithm I and one for the exhaust algorithm I with improvements, contains 50 observations on the CPU time required to solve 1000 selective maintenance instances, and \( \overline{X}_0 \) and \( \overline{X}_1 \) are the average values for two algorithms, respectively. According to the theory of mathematical statistics, the corresponding critical value for the hypothesis test is \( t_{a,m-1} = t_{0.05,50} = 1.676 = t_{0.05,49} = 1.676 \). The experimental results are listed in Table 5.

It can be seen from Table 5 that CPU time required by two algorithms increases steadily with the number of subsystems. It is a well-known fact that additional storage and computation resources must be called for any algorithm improvements, and they also consume more CPU time for their extra process. In our example, when the value of \( m \) is small, the better algorithm efficiency cannot offset the extra CPU time generated by the improvements, which may result in a longer CPU time of the revised exhaust algorithm. The algorithm efficiency is promoted significantly as more subsystems are integrated, while the computation cost invited by the improvements increases indistinctively. For example, if the improvements are combined into exhaust algorithm I, its CPU time reduces to 22.66% of the original one for \( m = 15 \).

These results fully demonstrate that these improvements explained in Section 3 are very useful to reduce CPU time, and the impact on CPU time is more distinct when the size of optimization problem is large. Then the next question to be answered is how to ascertain the contribution of each improvement. Let us take \( m = 9 \) for instance in this section. As shown in Fig. 6, the amount of candidate solutions, available solutions and updated solutions are recorded for different algorithms. Here the candidate solutions (recorded after line 6 in exhaust algorithm I, the same below) mean all solutions tried by the exhaust algorithm, the available solutions (recorded after line 8) imply all candidate solutions that satisfy the given constraint conditions of maintenance resources, and the updated solutions
It appears that the number of available solutions and updated solutions may decrease significantly for all exhaust algorithms. For example, the ratio of the number of available solutions to that of all candidate solutions and the ratio of the number of updated solutions to that of all candidate solutions are only 49.17% and 0.99% for exhaust algorithm I, respectively.

By observing Figs. 6(a) and (b), we can find that exhaust algorithm I and exhaust algorithm I with single improvement 4 can give the same numbers of candidate solutions and available solutions. The reason for this is that the search method of improvement 4 is only modified from ascending order to descending order. In contrast, improvements 2 and 3 can compress the population sizes of candidate solutions and available solutions obviously.

It can also be concluded from Fig. 6(c) that exhaust algorithm I and exhaust algorithm I with single improvement 2 can give the same number of updated solutions in this example. As explained above, for improvement 2, the lower bounds of decision variable $d_i$ are modified from all zeros to zeros in part (at least one component in subsystem $i$ is in working state) and ones in part (all components in subsystem $i$ are failed). This improvement is well designed for practice demands, and naturally the optimal solution must follow this constraint. It is impossible that the optimal solution of this example comes from the removed candidate solutions (or available solutions). Hence the updated solutions are unaffected by such single improvement.

Similarly, only a small portion of the removed candidate solutions due to single improvement 3 have the chance to be an optimal solution. Compared with exhaust algorithm I, the number of updated solutions decreases from 8752.5 to 8664.4, with the ratio of 1.01%. As a result, the numbers of updated solutions of improvements 2 and 3 are both similar to that of exhaust algorithm I.

It is well known that the local optimal solution of system reliability, which may not be updated by a new candidate solution (or variable solution), can be found from the larger values of decision variables $d_i$. Therefore, the number of updated solutions decreases from 8752.5 to 2716.76 by integrating improvement 4, with the ratio of 68.96% approximately.

It is assumed that the numbers of the removed solutions of each single improvement and all improvements are $dec_i$ ($i = 2, 3, 4$) and $dec_{all}$, respectively. Then the contribution of each improvement can be defined as follows. The number of removed solutions and the corresponding contributions of different improvements are calculated and listed in Table 6.

### Table 6
Contribution of different improvements.

<table>
<thead>
<tr>
<th>Different solutions</th>
<th>Evaluation criterion</th>
<th>All improvements</th>
<th>Single improvement</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Improvement 2</td>
<td>Improvement 3</td>
</tr>
<tr>
<td>Candidate solution</td>
<td>Number of removed solutions</td>
<td>713,490.62</td>
<td>600,793.32</td>
</tr>
<tr>
<td></td>
<td>Contribution of each improvement</td>
<td>100%</td>
<td>84.20%</td>
</tr>
<tr>
<td>Variable solution</td>
<td>Number of removed solutions</td>
<td>430,397.5</td>
<td>350,211.62</td>
</tr>
<tr>
<td></td>
<td>Contribution of each improvement</td>
<td>100%</td>
<td>81.37%</td>
</tr>
<tr>
<td>Updated solution</td>
<td>Number of removed solutions</td>
<td>6,053.18</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Contribution of each improvement</td>
<td>100%</td>
<td>0%</td>
</tr>
</tbody>
</table>

Fig. 6. Number of different solutions.
As seen from Table 6, improvements 2 and 3 have made great efforts to diminish the candidate solutions and available solutions, and improvement 4 contributes to a drastic reduction of updated solutions. Although improvements 2 and 3 produce similar effects, the experimental results by improvement 2 are much more significant than those by improvement 3. Therefore, it is deduced that the calculated results only by improvements 3 and 4 are almost equivalent to those by all three improvements.

In addition, for the three types of solutions, the sum of contributions of all single improvements is always larger than 100%. This paradoxical outcome means that there are some duplicate data produced by all single improvements. For example, for candidate solutions, the sum of contributions of improvements 2 and 3 is equal to 180.09% in this example. That is to say, the duplicate data produced by two improvements is 80.09%, and the additional removed candidate solutions by improvements 2 and 3 are about 4.11% and 15.8% of all candidate solutions, respectively.

5. Conclusions

Interactions among components offer the opportunity to group the maintenance activities, which may save resources. Economic dependence between subsystems (paired and unpaired) was first proposed in the present paper. The total time and cost savings under three types of economic dependence were formulated in a simple and consistent manner, and then the corresponding EDSMP models were established.

Experimental results illustrate that the reliabilities of systems under different economic dependences are both promoted to various certain extents, which can validate the validity of the EDSMP models. With the continuous increase in the ratio of set-up cost to maintenance cost, more maintenance resources can be saved and then utilized to repair more components.

According to the features of new EDSMP, several existing improvements of exhaust algorithm were modified to reduce CPU time significantly. In addition, the contribution of each improvement was first defined in the present paper, and then their contributions were compared in detail.

Obviously, the combination of different categories of interaction (economic, structural and stochastic dependences) complicates the modeling and optimization of selective maintenance. So, this seems to be a promising field for further research. We also observed that the optimization of maintenance policies via simulation is often done by using deterministic algorithms. Therefore, another upcoming field is efficient heuristic algorithms, such as evolutionary programming, biogeography-based optimization and fireworks algorithms.

References
