Tax evasion under behavioral structures

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Abstract

We study the strategic interactions between the fiscal authority and the taxpayer regarding tax evasion and auditing. We fit this interaction into a Bayesian game and introduce the concept of behavioral consistency, which helps reducing the number of available strategies and models the stylized fact according to which the choice to evade is subject to behavioral patterns.

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1. Introduction

The evolution of the government’s role in modern societies and the strengthening of its institutions has brought to theoretical attention one of the most serious problems for the functioning of the government: the tax evasion. Indeed, since the government’s role in education, health and infrastructure are costly, the need for financing has increased, for which reason audit has become an important mechanism in the hands of fiscal authorities. There is, however, a clear trade-off between the cost to audit and the benefit from recovering tax revenues. In addition, the taxpayer’s income

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is, to some extent, private information, hence the fiscal authority faces an asymmetric information problem, which amounts to an extra cost.

In this paper we study the problem of tax evasion by taking into account incentive issues. On the one hand, the taxpayer faces a trade-off between reporting and not reporting his true income. On the other hand, the government faces a trade-off between auditing and not auditing. The interaction between the government and the taxpayer leads to a variety of equilibria dependent on parameters such as costs, tax rates and so on. Yitzhaki (1974) and Andreoni et al. (1998) and Andvig and Moene (1990) show stylized facts that should be taken into consideration regarding this matter. The taxpayer behaves according to moral principles that are external to the game. For instance, if he does not evade, it is because it is morally incorrect to do so. In addition, his action is often influenced by the actions of other taxpayers. For example, a poor taxpayer evades because the rich one evades. The seminal article in the literature on tax evasion is Allingham and Sandmo (1972)'s, henceforth A–S. In that article, they built a model of tax evasion in which labor supply and return on capital are given. The agent decides how much of his income to report and there is an endogenous probability that his non reported income be detected by the government, in which case the taxpayer is forced to pay a fine higher than the initial tax share. The reported income is chosen so as to maximize his expected utility. This choice depends on the probability of detection, on his risk aversion and on the penalty fine. Yitzhaki (1974) pointed out that in their model an increase in the tax share yields to an ambiguous effect on tax evasion. There is a negative income-effect in the sense that an increase on the tax rate makes the taxpayer poorer and hence less risk averse, so that the reported value of income increases. There is also a substitution-effect. Since the fine levied on the taxpayer for the same non-reported amount of income does not vary when there is an increase of the tax share, there is a smaller difference between the tax share and the penalty fine, which then creates an incentive for his to increase the non-reported income. Yitzhaki then suggested a new approach, according to which the penalty fine for not reporting the true income is not proportional to the reported income, but to the nonpaid portion of the tax rate. With this, the ambiguity would be prone to disappear. However, as Sandmo (2005) observed, the disappearance of this ambiguity does not match empirical evidence and intuition, since the agent has an incentive to reduce his reported income should the difference between the tax rate and the penalty rate decrease.

Sousa et al. (2008), based on a model by Fisman and Wei (2004), use commerce data between Brazil and the United States and data on import tariffs in order to measure the impact of tariffs on tax evasion. They showed that higher tariffs imply higher degree of evasion. In addition, this relation is not linear, so the impact is meaningful only after some level of import aliquot. Siqueira and Ramos (2006) extends the A–S model and find a result that points to the opposite direction. They showed that an increase of the marginal aliquot reduces tax evasion and, in addition, that an increase of the probability of detection and of the penalty fine also leads to a reduction of tax evasion. The differences between these results may reflect the income and substitution-effect pointed out by Yitzhaki (1974). Richter and Boadway (2005) use the A–S model as well as Yitzhaki’s in order to study the interaction between tax evasion and tax structure. Under Yitzhaki’s framework, the optimal tax design remained invariant with respect to the introduction of risks inherent to tax evasion. Under A–S framework, on the other hand, it showed a trade-off between tax distortion and the magnitude of tax evasion. Goerke (2003) studies what happens with the amount of labor in the market as the tax structure becomes more progressive. When opportunities to evade are introduced into the model, employment increases as taxes become more progressive. In particular, this result holds only when part of the penalty fine is dependent on the non-reported income, as in the A–S model. From these two papers, it is possible to conclude that tax evasion influences the tax design and its impact on taxpayers.

Most of the papers focus on the individual decision-making. Schneider and Klinglmair (2004) estimate the size of the informal labor market in 110 countries and show that the size varies with the country. Sandmo (2005) shows that these variations cannot be explained by the magnitudes of the tax rates and fines alone. Cowell (1990) emphasizes that tax evasion requires a theory of social interaction, since it is a social phenomenon. Therefore, part of the evasion could be explained by factors related to the social interaction between agents. In the A–S model, the taxpayer gets to an opinion about the probability of detection also by observing the other agents and their probabilities of being audited. Then the taxpayer’s subjective belief of being detected depends on his own evasion and the evasion of others. If he perceives that the non-reported income by others increases, his subjective belief of being detected is reduced and his non-reported income increases. On the top of that, there is a disutility from not reporting the true income, though this could be lower in case he perceives that many other do not report truthfully. In their study on corruption, Andvig and Moene (1990) also find the same pattern: the more corrupt the environment the individual is in, the harder it is for the individual to be honest.
A dilemma extensively studied in the literature of tax evasion is the existence of people who declare fully their income if the expected value of the utility when the taxpayer do not report part of its income, is positive. According to Andreoni et al. (1998), there are moral and social factors that influence the decision to evade. Among these factors are the feelings of guilt and shame that agents feel by not declaring all their income. There is a disutility when the agent feels he did something wrong. Furthermore, the perception of fairness in the tax burden from the taxpayer also influences their decisions. If he perceives that its tax burden is unfair compared with the tax burden of others, or if it perceives that others do not fully declare their income and therefore is at a disadvantage, there is an incentive to evade his income. Another factor that influences the amount of reported income mentioned by Andreoni et al. (1998) is the satisfaction of the taxpayer with respect to government policies. The misuse of taxes by the government is another incentive to circumvent the system of tax payments. Another study that explains evasion as a social phenomenon is that of Barth et al. (2005) who consider the case of two people who receive the same income, the one working longer and having a lower remuneration and other working less but getting more for time worked, and both paying the same amount of taxes. The first group feels wronged and has an incentive to lie about its income. All these analyzes consider the interactions among taxpayers, not just individual motivations to explain tax evasion.

Some authors use game theory to analyze tax evasion. Pruzhansky (2004) views the honesty of taxpayers differently. The model arises from the idea that there are no completely honest taxpayers, that is, under certain conditions everyone can escape. The model is a Bayesian game between the taxpayer and the government and this concept of honesty is included in the equilibrium they find. This work is the closest to ours. However, in the model of Pruzhansky (2004), given two income levels, low and high, the actions of each taxpayer are to declare their own income or the income of the other. This implies that the low-income taxpayer may decide to declare that his income is high. In our model, as we shall see later, the decision is between evading and not evading, and income level is simply private information, i.e., the type of taxpayer.

Another subject of great importance in the literature of tax evasion is the relation of the probability of detecting evasion with the level of income reported to the government. Reinganum and Wilde (1985) were one of the first to study this relationship and used for analysis an audit system different from the usual random audit. They considered that the government has some information about the income of the population so that it can establish a threshold level of income reported. Given a reported income, if it is below the threshold, it is considered to be a very low income, then the taxpayer will be audited with a 100% probability. In contrast, if above, it will not be considered low and will not be audited. According to their findings, the cost of auditing to the government is higher when the audits are random. In addition, the audit with a established minimum level of income weakly dominates the random audit in cases of a lump sum tax and a tax proportional to income.

This paper addresses the issue of tax evasion from the perspective of the government and the taxpayer. It aims to analyze the relations between government and taxpayer upon the incentives that the government has to audit and incentives that the taxpayer has to evade. For this, we build a Bayesian game in which the taxpayer may be either of two types, a taxpayer with high income or low income.

Our contribution is the adoption of what we call behavioral consistency. This concept facilitates the computation of equilibria, since it reduces the set of strategies available, and is able to model the widely recognized phenomenon whereby the evasion or the non-evasion by a taxpayer is a result of the evasion or non-evasion that he observes in the other taxpayers or, alternatively, the idea that the act of evading or not evading is subject to behavioral principles external to the game. Basically, a strategy is behaviorally consistent if the action taken is invariant with respect to the outcome of the random variable that determines the type of taxpayer. Thus, if a taxpayer can be of two types, rich or poor, and can take one of two possible actions, evade and not to evade, and given that a pure strategy in a Bayesian game could be, for example, evade if rich and not evade if poor, and another pure strategy could be not evade if rich and not evade if poor, then the latter is a behaviorally consistent pure strategy and the former is therefore behaviorally inconsistent. Thus, we have two distinct behavioral structures (consistency and inconsistency of behavior) which, as already mentioned, in addition to describe stylized facts also serve as a criterion for elimination of pure strategies, a property useful for our model, as the criterion for selection of rationalizable strategies seems unable to reduce the size of the normal form game.

We determine, moreover, the Bayesian equilibria in mixed strategies under the condition of behavioral consistency and also behavioral inconsistency, due to the fact that there are no equilibria in pure strategies. In each case, whether in behavioral consistency or inconsistency, we interpret the mixed strategies in terms of the tax parameters of our model, particularly the cost of tax audit and fines in case of evasion and also in terms of the distribution of types.
In Section 2 we construct the Bayesian game, which we call the tax evasion game, and introduce the concept of behavioral consistency. We calculate the Bayesian equilibria in mixed strategies both in the case of behavioral consistency and inconsistency. Then we analyze the equilibria in terms of the model parameters. Section 3 concludes the paper.

2. Tax evasion game under behavioral structures

In this section, we build a Bayesian game between the fiscal authority (government) and the taxpayer. Since the taxpayer’s wealth is his private information, the fiscal authority faces asymmetric information. In order to take into account the existence of external moral rules that induce the taxpayer to behave consistently whatever his wealth, we introduce the concept of taxpayer’s behavioral consistency into the game, which is the novelty of our paper. In addition, we also show what happens with the equilibrium of the game should the taxpayer behave inconsistently.

In Section 2.1 we present the primitives of our tax evasion Bayesian game. In Section 2.2 we find the mixed strategy Bayesian equilibria under behavioral consistency and in Section 2.3 the equilibrium under behavioral inconsistency. In Section 2.4 we analyze the equilibria in terms of the relevant parameters of the game: the tax rate, the fee on evasion, average income and auditing costs.

2.1. Primitives of the tax evasion Bayesian game

There are two players: the government (player 1) and the taxpayer (player 2). We introduce asymmetric information by assuming that player 2 can be either of two types. Let $T_2 = \{Y, y\}$ be the set of types:

$$T_2 = \begin{cases} y, & \text{with probability } p \\ Y, & \text{with probability } 1 - p \end{cases}$$

where $0 < y < Y$ are the levels of income, $y$ is low income and $Y$ is high income. The taxpayer’s type is private information, everything else is common knowledge. Denote by $\pi = \{p, 1 - p\}$ the probability distribution of the taxpayer’s types $y$ and $Y$. Hence $y$ occurs with probability $p$, $Y$ with probability $1 - p$. Let $T_1 = \{\theta\}$ be the government’s set of types, which is a singleton, hence there is no asymmetric information with respect to its type, that is, it has only one type, which occur with probability 1.

Let $C_1 = \{A, \tilde{A}\}$ be the pair of possible actions available to player 1, where $A$ is to audit and $\tilde{A}$ is not to audit. Let $C_2 = \{E, \tilde{E}\}$ be the pair of possible actions available to player 2, where $E$ is to evade and $\tilde{E}$ is not to evade. The decision to evade is the decision not to fulfill the individual income tax form, that is, the taxpayer does not disclose his income. The income tax share is denoted by $t$ and the penalty fee for evasion is denoted by $\varphi$, where $\varphi > t$. The assumption $\varphi > t$ is pretty natural, since it punishes evasion. When the government audits, it incurs into a cost $c > 0$ and identifies the taxpayer who did not report his true type.

The payoff matrix conditional on player 2 being of type $y$ is:

<table>
<thead>
<tr>
<th>Player 2</th>
<th>Player 1</th>
<th>Player 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>$E$</td>
<td>$y(1 - \varphi), \varphi y - c$</td>
</tr>
<tr>
<td>$\tilde{E}$</td>
<td>$y(1 - t), yt - c$</td>
<td>$y(1 - t), yt$</td>
</tr>
</tbody>
</table>
Matrix 2.

<table>
<thead>
<tr>
<th>Player 2</th>
<th>Player 1</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$A$</td>
</tr>
<tr>
<td>$E$</td>
<td>$Y(1-\varphi), \varphi Y - c$</td>
</tr>
<tr>
<td>$\tilde{E}$</td>
<td>$Y(1-t), Yt - c$</td>
</tr>
</tbody>
</table>

from tax minus the cost of auditing, $yt - c$. Finally, if the taxpayer does not evade and the government does not audit, $(\tilde{E}, \tilde{A})$, then the taxpayer’s payoff is also $Y(1-t)$, and the government’s equals the revenue from tax, $yt$. An identical reasoning applies to the payoff matrix conditional on player 2 being of type $Y$:

Both conditional matrices can be synthesized in the normal form of the Bayesian game, whose matrix is given below:

Matrix 3.

<table>
<thead>
<tr>
<th>Player 2</th>
<th>Player 1</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$A$</td>
</tr>
<tr>
<td>$EE$</td>
<td>$\bar{\gamma} - \varphi \bar{\gamma}, \varphi \bar{\gamma} - c$</td>
</tr>
<tr>
<td>$E\tilde{E}$</td>
<td>$\bar{\gamma} - \varphi \bar{\gamma} - Yt(1-p), \varphi \bar{\gamma} + Yt(1-p) - c$</td>
</tr>
<tr>
<td>$\tilde{E}E$</td>
<td>$\bar{\gamma} - t\gamma - \varphi Y(1-p), t\gamma + \varphi Y(1-p) - c$</td>
</tr>
<tr>
<td>$\tilde{E}\tilde{E}$</td>
<td>$\bar{\gamma} - t\gamma, t\gamma - c$</td>
</tr>
</tbody>
</table>

The pure strategies available to player 2 are $EE, E\tilde{E}, \tilde{E}E, \tilde{E}\tilde{E}$, and for player 1 $A, \tilde{A}$. The entries in matrix 3 are the expected payoffs for each player, where $\bar{\gamma} = yp + Y(1-p)$ is the average income.

Note that the government audit is random, so that the decision to audit or not to audit is taken before observing the realization of its effective payoff. Indeed, in the Bayesian game, players choose simultaneously the random variables (mixed strategies) that they will announce to each other. Thus, the fact that the government knows that it gets zero payoff when it does not audit and taxpayers evade, as we said this does not mean that it observed this payoff and, therefore, that it knows that taxpayers evaded. The government knows only that the zero payoff is a possible realization of a random variable, since in the Bayesian game, the mixed strategies are decided ex ante.

In the first entry of matrix 3, in which the actions are $EE$ (taxpayer) and $A$ (government), the taxpayer’s payoff is his average income minus the average penalty paid to the government. The government’s payoff is the tax collected from the taxpayer minus the cost to audit. We observe a pattern in all entries. The payoffs to the taxpayer (first and third column) are the average income minus the amount paid to the government. The government’s payoff (second and fourth column) will always be fine collected minus the cost to audit. To simplify matters, define the following: $\Gamma_1 = \varphi \bar{\gamma}$, $\Gamma_2 = \varphi \bar{\gamma} + t(1-p)Y$, $\Gamma_3 = t\gamma + \varphi(1-p)Y$, $\Gamma_4 = t\gamma$, $\Gamma_5 = 0$, $\Gamma_6 = t(1-p)Y$, $\Gamma_7 = t\gamma$ and $\Gamma_8 = t\gamma$.

Matrix 3 can then be rewritten as:

Matrix 4.

<table>
<thead>
<tr>
<th>Player 2</th>
<th>Player 1</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$A$</td>
</tr>
<tr>
<td>$EE$</td>
<td>$\bar{\gamma} - \Gamma_1, \Gamma_1 - c$</td>
</tr>
<tr>
<td>$E\tilde{E}$</td>
<td>$\bar{\gamma} - \Gamma_2, \Gamma_2 - c$</td>
</tr>
<tr>
<td>$\tilde{E}E$</td>
<td>$\bar{\gamma} - \Gamma_3, \Gamma_3 - c$</td>
</tr>
<tr>
<td>$\tilde{E}\tilde{E}$</td>
<td>$\bar{\gamma} - \Gamma_4, \Gamma_4 - c$</td>
</tr>
</tbody>
</table>

Matrix 4 describes the normal form of the Bayesian game. The payoffs $U_1$ and $U_2$ of players 1 and 2 are given by the elements of the matrix above. In each cell, the payoff $U_1$ is to the right and the payoff $U_2$ is to the left. For example, given the profile of strategies $((E, E), A)$, we have $U_2((E, E), A) = \bar{\gamma} - \Gamma_1$ and $U_1((E, E), A) = \Gamma_1 - c$. 

The variable $\Gamma_5$ does not appear in the above matrix because it is null, as the government do not have any gain when all taxpayers do not pay their taxes and are not audited. A relevant fact is that there is no strictly dominated strategies in the tax evasion game. Indeed, given the condition $p > t$, one can easily show that there are only two possible orderings for the variables $\Gamma$ above. To auditing, we have $\Gamma_1 > \Gamma_3 > \Gamma_2 > \Gamma_4$ and $\Gamma_1 > \Gamma_2 > \Gamma_3 > \Gamma_4$. The first ordering occurs when $pY < (1 - p)Y$, the second one occurs when $pY > (1 - p)Y$. To non-auditing, we have $\Gamma_3 < \Gamma_7 < \Gamma_6 < \Gamma_8$ and $\Gamma_3 < \Gamma_6 < \Gamma_7 < \Gamma_8$. With these orderings, in any case, there is no strictly dominated strategies, that is, all strategies are rationalizable. Therefore, we cannot delete any lines and there is no Bayesian Nash equilibrium in pure strategies.

A Bayesian game is the collection $J = \{N, T_1, T_2, \pi, C_1, C_2, U_1, U_2\}$, in which $N = \{1, 2\}$ is the set of players, $T_i$ is the set of types of player $i \in N$, $\pi_i$ is the probability distribution over the types of player $i \in N$, $C_i$ is the set of actions of player $i \in N$ and $U_i$ is the payoff of player $i \in N$. Our game will be called the tax evasion game and its normal form is given by matrix 4 above.

A pure strategy for player $i \in N$ is a prescription of action for each type, i.e., it is a function $s_j : T_i \rightarrow C_i$. A mixed (or random) strategy for player $i \in N$ is a prescription of probability distribution for each type, i.e., it is a function $\mu_j : T_i \rightarrow \Delta(C_i)$, in which $\Delta(C_i)$ is the set of probability distributions over $C_i$. Define $\mu_i(t_i) = \sigma_i$. A profile of pure strategies for the game $J$ is a pair $s = (s_1, s_2)$. A profile of mixed strategies for $J$ is a pair $\mu = (\mu_1, \mu_2)$. Without loss of generality, a mixed strategy will be identified with its image $\sigma = (\sigma_1, \sigma_2) \in \Delta(C_1) \times \Delta(C_2)$. Let $|T_i|$ be the (finite) cardinality of $T_i$. Define:

$$C_i^{[T_i]} = C_i \times \cdots \times C_j$$

and define the $a$-trace of $C_i^{[T_i]}$ as the vector $(a, \ldots, a) \in C_i^{[T_i]}$, where $a \in C_i$. Denote by $tr(C_i)$ the set of all $a$-traces of $C_i^{[T_i]}$.

**Definition.** A Bayesian Nash equilibrium in pure strategies (mixed) of the game $J = \{N, T_1, T_2, \pi, C_1, C_2, U_1, U_2\}$ is a Nash equilibrium in pure strategies (mixed) of the game described by normal form.

**Definition.** Let $(\sigma_1^*, \sigma_2^*)$ be a Bayesian Nash equilibrium in mixed strategies of the tax evasion game $J$. We say that $(\sigma_1^*, \sigma_2^*)$ is:

(a) **behaviorally consistent** (or simply consistent) if the action taken by each player is invariant with respect to its type, that is, if $\text{supp}(\sigma_i^*) \subset tr(C_i)$, $\forall i \in N$, in which $\text{supp}(\sigma_i^*)$ is the support of $\sigma_i^*$.

(b) **behaviorally inconsistent** (or simply inconsistent) if it is not consistent, that is, if $\text{supp}(\sigma_i^*) \cap tr(C_i) = \emptyset$.

In the tax evasion game, the behaviorally consistent actions available to the taxpayer are $(E, E)$ and $(\tilde{E}, \tilde{E})$. The behaviorally inconsistent actions to the taxpayer are $(E, \tilde{E})$ and $(\tilde{E}, E)$. Note that our concept of behavioral consistency is not exactly the usual concept of consistency of beliefs (Myerson, 1997, pp. 168–177). The concept of consistency of beliefs has to do with the behavior in states that have zero probability in equilibrium. That is because these probabilities are endogenous. However, by exogenously placing zero probability on strategies that characterize behavioral inconsistency, it is possible to find equilibria that match the stylized facts that we mentioned. Therefore, if on the one hand, the concept of consistency belief solves problems of rationality in states of zero probability, in other hand, what we state is that there are exogenous constraints (moral, cultural, whatever) which alone play this role. Therefore, the presupposed environment for the game is fundamentally different, for instance, from Kreps and Wilson (1982), who deal with consistent belief systems.

2.2. **Bayesian equilibrium under behavioral consistency**

We say that the Bayesian equilibrium is consistent if the action chosen by the player is the same regardless of its type, i.e., if the taxpayer behavior is consistent with some behavioral or moral principle external to the game. Thus, a consistent equilibrium indicates that the taxpayer either always evades or never evades. A strategy in which the taxpayer evades if he is of a type and does not evade if he is of another type, is not part of a consistent equilibrium. If $s_2 : T_2 \rightarrow C_2$ denotes a pure strategy consistent to the taxpayer then there exists an action $a \in C_2$ such that $s_2(t) = a$, $\forall t \in T_2$. With
respect to the government (player 1), the restriction to consistent strategies is irrelevant, since the government has only one type.

In order to analyze the Bayesian equilibrium in mixed strategies, assume the belief of player 2 with respect to the probabilities associated with the actions of player 1 is given by the distribution $\alpha$ and $1 - \alpha$ to audit and not to audit, respectively. Similarly the belief of player 1 with respect to the probabilities associated with the actions of player 2 is given by the probabilities $\beta, \gamma, \delta$ and $\epsilon$ for the actions $EE, E\bar{E}, \bar{E}E$ and $\bar{E}\bar{E}$, respectively.

Matrix 5.

<table>
<thead>
<tr>
<th>Action</th>
<th>Subjective probabilities</th>
<th>$A$</th>
<th>$\bar{A}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$EE$</td>
<td>$\beta$</td>
<td>$\gamma - \Gamma_1, \Gamma_1 - c$</td>
<td>$\gamma, 0$</td>
</tr>
<tr>
<td>$E\bar{E}$</td>
<td>$\gamma$</td>
<td>$\gamma - \Gamma_2, \Gamma_2 - c$</td>
<td>$\gamma - \Gamma_6, \Gamma_6$</td>
</tr>
<tr>
<td>$\bar{E}E$</td>
<td>$\delta$</td>
<td>$\gamma - \Gamma_3, \Gamma_3 - c$</td>
<td>$\gamma - \Gamma_7, \Gamma_7$</td>
</tr>
<tr>
<td>$\bar{E}\bar{E}$</td>
<td>$\epsilon$</td>
<td>$\gamma - \Gamma_4, \Gamma_4 - c$</td>
<td>$\gamma - \Gamma_8, \Gamma_8$</td>
</tr>
</tbody>
</table>

**Proposition 1.** Suppose $0 < c < \varphi \bar{\gamma}$. Then the behaviorally consistent Bayesian equilibrium in mixed strategies is given by:

$$B_{cons} = \left\{ \left[ \frac{t}{\varphi} \right] \circ A \oplus \left[ 1 - \frac{t}{\varphi} \right] \circ \tilde{A}, \left[ \frac{c}{\varphi \bar{\gamma}} \right] \circ EE \oplus \left[ \frac{\varphi \bar{\gamma} - c}{\varphi \bar{\gamma}} \right] \circ \tilde{E}\bar{E} \right\}$$

**Proof.** First, we show that $\alpha = t/\varphi$, i.e., the value of $\alpha$ is equal to the tax rate divided by the punishment suffered by the taxpayer when he does not pay the tax and is audited. Making $U_2(EE) | \alpha \circ A \oplus (1 - \alpha) \circ \bar{A}) = U_2(\bar{E}\bar{E}) | \alpha \circ A \oplus (1 - \alpha) \circ \bar{A})$, we have $(\gamma - \Gamma_1)\alpha + \bar{\gamma}(1 - \alpha) = (\gamma - \Gamma_6)\alpha + (\gamma - \Gamma_8)(1 - \alpha)$. Solving for $\alpha$, we find $\alpha_1 = ((\Gamma_8)/(\Gamma_1 + \Gamma_8 - \Gamma_4))$. Making $U_2(EE) | \alpha \circ A \oplus (1 - \alpha) \circ \bar{A}) = U_2(\bar{E}\bar{E}) | \alpha \circ A \oplus (1 - \alpha) \circ \bar{A})$, we have $(\gamma - \Gamma_1)\alpha + \bar{\gamma}(1 - \alpha) = (\gamma - \Gamma_7)\alpha + (\gamma - \Gamma_8)(1 - \alpha)$. Solving for $\alpha$, we find $\alpha_2 = ((\Gamma_7)/(\Gamma_1 + \Gamma_7 - \Gamma_3))$. Making $U_2(EE) | \alpha \circ A \oplus (1 - \alpha) \circ \bar{A}) = U_2(\bar{E}\bar{E}) | \alpha \circ A \oplus (1 - \alpha) \circ \bar{A})$, we have $(\gamma - \Gamma_1)\alpha + \bar{\gamma}(1 - \alpha) = (\gamma - \Gamma_6)\alpha + (\gamma - \Gamma_8)(1 - \alpha)$. Solving for $\alpha$, we find $\alpha_3 = ((\Gamma_6)/(\Gamma_1 + \Gamma_6 - \Gamma_2))$. Substituting the variables $\Gamma$ in terms of $\gamma, \varphi, t$ and $\varphi$, we have $\alpha_1 = \alpha_2 = \alpha_3$, a common value that we denote by $\alpha$. To calculate $\alpha$, is sufficient to use $\alpha_3$. Thus, $\alpha = ((\gamma)/(\varphi \bar{\gamma}))$. Then, $\alpha = ((\gamma)/(\varphi \bar{\gamma}))$ and $\epsilon = ((\varphi \bar{\gamma} - c)/(\varphi \bar{\gamma}))$. Indeed, from $U_1(A | \beta \circ EE \oplus \gamma \circ \bar{E}E \oplus \delta \circ \bar{E}\bar{E}) = U_1(A | \beta \circ EE \oplus \gamma \circ \bar{E}E \oplus \delta \circ \bar{E}\bar{E})$, we get $\beta(\Gamma_1 - c) + \gamma(\Gamma_2 - c) + \delta(\Gamma_3 - c) + \epsilon(\Gamma_4 - c) = \beta \Gamma_5 + \gamma \Gamma_6 + \delta \Gamma_7 + \epsilon \Gamma_8$. Since $\gamma = \delta = 0$ and $\Gamma_5 = 0$, we have $\beta(\Gamma_1 - c) + \epsilon(\Gamma_4 - c) = \epsilon \Gamma_8$. Therefore $\beta(\Gamma_1 - c) = \epsilon(\Gamma_8 - \Gamma_4 + c)$. Substituting $\Gamma_1, \Gamma_4$ and $\Gamma_8$ in terms of $\gamma, t$ and $\varphi$, we get $\beta(\varphi \bar{\gamma} - c) = \epsilon(\gamma - \bar{\gamma} + c)$. Then $\epsilon = (\beta(\varphi \bar{\gamma} - c))/(c)$. We also know that $\beta + \gamma + \delta + \epsilon = 1$. By substituting $\epsilon = (\beta(\varphi \bar{\gamma} - c))/(c)$ into $\beta + \gamma + \delta + \epsilon = 1$, we get $(\beta + \beta(\varphi \bar{\gamma} - c))/(c) = 1$, in which $(\beta(c + \varphi \bar{\gamma} - c))/(c) = 1$. Thus, $\beta = (c)/(\varphi \bar{\gamma})$. Isolating the term $\epsilon$ we have $\epsilon = (1 - c)/(\varphi \bar{\gamma})$. Therefore, $\epsilon = (\varphi \bar{\gamma} - c)/(\varphi \bar{\gamma})$. It remains to show the necessity of the condition $0 < c < \varphi \bar{\gamma}$. The probabilities $\beta$ are $\epsilon$ are positive. In addition, if $\beta + \epsilon = 1$, we have that $0 < \beta < 1$ and $0 < \epsilon < 1$. Substituting the value of $\beta$ into the previous inequality we get $0 < (c)/(\varphi \bar{\gamma}) < 1$. Therefore, $0 < c < \varphi \bar{\gamma}$, as claimed. $\square$

### 2.3. Bayesian equilibrium under behavioral inconsistency

The behaviorally inconsistent Bayesian equilibrium occurs when the two types of taxpayers take contrary actions. We then say that the equilibrium is inconsistent. In this case the subjective probability of player 2 associated with the

\[^2\] [p] \odot x \oplus [1 - p] \odot y denotes the random variable $V$ defined by $V = x$, with probability $p$, and $V = y$, with probability $1 - p$.\]
action of player 1 remains $\alpha$ and $(1-\alpha)$ to audit and non-audit respectively. Thus, the equality $\alpha = (t/\varphi)$ still applies. On the other hand, subjective probabilities of player 1 with respect to the actions of player 2 are different. We consider $\beta$ and $\varepsilon$ null which are the odds related to actions $(EE)$ and $(\tilde{E}E)$.

**Proposition 2.** Suppose $\varphi Y(1-p) < c < \varphi y_p$ and $c < ((\varphi y)/(2))$. Then the behaviorally inconsistent Bayesian equilibrium in mixed strategies is given by:

$$
B_{incons} = \left\{ t \varepsilon \right\} \circ A \oplus \left\{ 1 - t \varepsilon \right\} \circ \tilde{A}, \left\{ c - \varphi Y(1-p) \right\} \circ \tilde{E}E \oplus \left\{ \frac{\varphi y_p - c}{\varphi y_p - \varphi Y(1-p)} \right\} \circ \tilde{E}E
$$

**Proof.** Consider the subjective probabilities over the actions of player 2 (line player, taxpayer) as described in matrix 5. By definition, $\beta = \varepsilon = 0$. The expected utility of player 1 is denoted by:

$$
U_1(A | \beta \circ EE \oplus \gamma \circ \tilde{E}E \oplus \delta \circ EE \oplus \varepsilon \circ \tilde{E}E) = U_1(\tilde{A} | \beta \circ EE \oplus \gamma \circ \tilde{E}E \oplus \delta \circ EE \oplus \varepsilon \circ \tilde{E}E).
$$

From this it follows that $\beta(\Gamma_1 - c) + \gamma(\Gamma_2 - c) + \delta(\Gamma_3 - c) + \varepsilon(\Gamma_4 - c) = \beta \Gamma_5 + \gamma \Gamma_6 + \delta \Gamma_7 + \varepsilon \Gamma_8$. Since $\beta = \varepsilon = 0$, we have $\gamma(\Gamma_2 - \Gamma_6 - c) = \delta(\Gamma_7 - \Gamma_3 + c)$. Substituting $\Gamma_2, \Gamma_3, \Gamma_6$ and $\Gamma_7$ in terms of $\gamma, Y, t$ and $\varphi$, we get $\gamma[\varphi y + t\gamma Y(1-p) - t\gamma Y(1-p) - c] = \delta[\gamma Y - \gamma Y(1-p) + c]$, in which $\gamma[\varphi y - c] = \delta[\gamma Y - \gamma Y(1-p)]$. Therefore, $\gamma = (\delta(c - \gamma Y(1-p)))/(\gamma[\varphi y - c])$ and $\delta = (\gamma(\varphi y - c))[\gamma Y(1-p)]$. Also, we know that $\beta + \gamma + \delta + \varepsilon = 1$. But $\beta = \varepsilon = 0$, so that $\gamma + \delta = 1$. Substituting $\delta = ((\gamma(\varphi y - c))[\gamma Y Y(1-p)])$ into $\gamma + \delta = 1$ we get $\gamma = \gamma Y(\varphi y - c)[(c - \varphi Y(1-p))]$, hence $\gamma = (\gamma Y(\varphi y - c))[\gamma Y Y(1-p)]$. Thus, $\gamma = (\delta(c - \gamma Y(1-p)))/(\gamma Y(1-p))$. Substituting $\gamma = (\delta(c - \gamma Y(1-p)))/(\gamma Y(1-p))$ into $\gamma + \delta = 1$, we get $\delta = ((\delta(c - \gamma Y(1-p)))/(\gamma Y(1-p)))$. Therefore, $\delta = ((\varphi y - c)[\gamma Y(1-p)])$. The distribution $(\alpha, 1-\alpha)$ of the subjective probability over the government’s action is obviously the same as in Proposition 1. It remains to show the necessity of the conditions $\varphi Y(1-p) < c < \varphi y_p$ and $c < ((\varphi y)/(2))$. Since, by necessity, $0 < \gamma < 1$ and $0 < \delta < 1$ and given that $\gamma = ((c - \varphi Y(1-p)))/(\varphi y_p - \varphi Y(1-p))$, then $0 < (c - \varphi Y(1-p))([\gamma y_p - \gamma Y(1-p)]) < 1$, i.e., $0 < \gamma Y - \gamma Y(1-p) < [\gamma y_p - \gamma Y(1-p)]$, so that $\gamma Y(1-p) < c < (\gamma y_p - \gamma Y(1-p)) + \gamma Y(1-p)$. Thus, $\Gamma(1-p) < c < \varphi y_p$. Since $\varphi y_p(1-p) < \gamma y_p$, we can compare the equations $\gamma Y(1-p) < c < \gamma y_p$. We know that $\delta > \gamma$. Substituting the values of $\delta$ and $\gamma$ into the previous inequality we get $(\gamma y_p - \gamma y_p Y(1-p)) > (\gamma y_p - \gamma Y(1-p))$, i.e., $\gamma y_p - \gamma y_p Y(1-p)$. Hence, $\gamma y_p Y(1-p) > 2c$, from which we get $\gamma Y > 2c$ and, therefore, $c < ((\varphi y)/(2))$, as claimed. \[ \Box \]

2.4. Analysis of the equilibria

The equations found in the model allow us to analyze the incentives of both players in the tax evasion game. In Section 2.2, in which we found the results for the consistent Bayesian equilibrium, the probabilities $\alpha, \beta$ and $\varepsilon$ were related to the variables $t, \varphi, \gamma$ and $c$. Furthermore, we found the limit to the cost of auditing, $c$. The proposition of this section shows that the belief of player 2 with respect to the actions of player 1, $\{\alpha, 1-\alpha\}$, depends on the magnitude of the income tax rate, $t$, and on the punishment charged on the taxpayer when he does not report his income properly, $\varphi$. Specifically, $\alpha = (t/\varphi)$. Thus, the taxpayer believes more strongly that the government will audit when the income tax rate increases, and believes less strongly that the government will audit when the punishment imposed on the taxpayer increases. On the other hand, the belief of the taxpayer with respect to the government action not to audit is interpreted in the opposite way when the tax rate and punishment parameters vary.

**Proposition 1** shows that the degrees of belief prescribed by behaviorally consistent Bayesian equilibria, $\beta \in \varepsilon$, are given by $\beta = (c/(\varphi y))$ and $\varepsilon = (\varphi y - c)/(\varphi y P)$. Such equalities denote the subjective probabilities of player 1 with respect to $EE$ and $EE$ strategies of player 2. The probability $\beta$ is directly proportional to the cost of auditing and is inversely proportional to the punishment and to the average income. When the cost of auditing increases, the government tends to decrease the frequency of audits, the taxpayer in turn has more incentives to evade, which is consistent with the model, i.e., when the cost of auditing increases the government believes that the taxpayer will evade with a higher probability. The punishment is a variable that discourages the taxpayer to evade, because the greater the punishment, the greater his expenses if audited. Thus, a greater punishment decreases the belief of the government with regard to the evasion of the taxpayer. Another variable that behaves the same way as the punishment with respect to $\beta$ is the average income. The higher the average income, the lower is the belief that player 1 assigns
to the \( EE \) strategy of player 2. The average income depends on the different incomes of the taxpayer \((y\) and \(Y\)) and its probabilities \((p\) and \(1 - p\)). The higher the average income, the lower is the belief that player 1 assigns to the \( EE \) strategy of player 2. The average income depends on the different incomes of the taxpayer and its probabilities. If the taxpayer’s probability \( p \) of having low income falls, the average income increases, and therefore, the parameter \( \beta \) decreases. This means that the higher the proportion of low-income taxpayers, the greater is the belief of the government that taxpayers will not evade. This is a stylized fact in the literature, hence our model also captures this fact. See, for instance, the 2010 International Monetary Fund report on fiscal policy in developed and developing countries (IMF, 2010).3

The parameter \( \varepsilon \) denotes the belief of the government with regard to the \( E \bar{E} \) strategy of the taxpayer. This parameter depends on the same variables as the parameter \( \beta \). However, the variables influence the parameter differently: \( \varepsilon \) is directly proportional to the punishment and to the average income, and is inversely proportional to the cost of auditing. A very high cost is a disincentive to the frequency of government audits. Thus, the government believes that if the cost of auditing increases, the amount of taxpayers who declare their income correctly decreases. In the government’s view, the increase in average income leads to a decrease of taxpayers who do not declare their income correctly, and the same analysis can be made with respect to the increase of punishment.

Also according to Proposition 1, it is necessary that \( 0 < c < \varphi \bar{Y} \). Thus, the cost of auditing must necessarily be greater than zero and less than the amount of punishment multiplied by the average income. The value of \( \varphi \bar{Y} \) is the amount that the government collects from taxpayers if all of them evade and they are all audited, i.e., it is the highest amount that the government can raise. It is therefore evident that the cost of auditing should be less than the maximum government gain. Otherwise, this gain would be negative, i.e., the actual tax activity would be socially inefficient.

In Section 2.3 we got the degrees of belief prescribed by behaviorally inconsistent Bayesian equilibrium \( \gamma \) and \( \delta \) with respect to the variables \( t \), \( \varphi \), \( \bar{Y} \) and \( c \). If the penalty \( \varphi \) increases, then the government decreases the belief \( \gamma \) that the low-income will evade and that the high-income will not evade, since \( \gamma = ((c - \varphi Y(1 - p))(\varphi[\varphi - Y(1 - p)])\). So when the punishment is high, the government believes more strongly that the poor taxpayer will not evade and that the rich taxpayer will evade, which is corroborated by the value of \( \delta = ((\varphi \varphi p - c)(\varphi[\varphi - Y(1 - p)])\).

The value of \( \delta \) relates to the strategy \( E \bar{E} \) and is positively related to the punishment. Therefore, the analysis of the impact on \( \delta \) due to an increase of penalty is the same as the one previously done, in which we studied the impact of a decline in the value of the punishment \( \gamma \).

The cost of auditing has a direct effect on the variable \( \gamma \). If auditing becomes more expensive, the government believes more strongly that the low-income taxpayer will evade and the high-income taxpayer will not evade. On the other hand, the cost of auditing has a reverse effect on the variable \( \delta \), i.e., if the cost increases, the government believes less strongly that the low-income taxpayer will not evade and that the high-income taxpayer will evade.

The probability that the taxpayer is a low-income individual also influences the government’s belief on the taxpayer’s strategies. When this probability increases, the variable \( \gamma \) increases, that is, the government believes more strongly that the low-income taxpayer will evade and that the high-income taxpayer will not evade. The variable \( \delta \), on its turn, decreases, which means that the government believes less intensely that the low-income taxpayer will not evade and the high-income taxpayer will evade.

Proposition 2 also gives the conditions for the equilibrium values of \( \gamma \) and \( \delta \) to exist, namely, \( \varphi Y(1 - p) < c < \varphi \varphi p \) and \( c < ((\varphi \bar{Y})/2) \). These conditions establish limits on the cost of auditing. The cost must be greater than the penalty amount paid by the high-income taxpayer and less than the penalty amount paid by the low-income taxpayer. Moreover, the second condition requires that the cost should be substantially less than the maximum that the government can collect from taxpayers, \( \varphi \bar{Y} \). On the other hand, in the consistent equilibrium, the cost should be lower than the maximum gain of the government. One possible interpretation for these different conditions in the equilibria is that, in the inconsistent equilibrium, the government is susceptible to variations in the behavior of the taxpayer. Thus, the condition that the cost must be substantially less than the maximum gain of the government would guarantee that it would not incur into a loss when auditing.

The inconsistent equilibrium equations allow us to identify the differences in the analysis of the government when it evaluates the impacts of the variables on the low-income taxpayer or on the high-income taxpayer. The punishment is seen by the government as something that encourages high-income taxpayers to evade. On the other hand, the cost of auditing is seen as a factor that discourages high-income taxpayers to evade.

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3 We thank Andrés Lemgruber Viol, from International Monetary Fund, Washington, DC, for this reference.
The analysis of the government with respect to the low-income taxpayer goes the other way round. The idea that with the increase of the fine, low-income taxpayers are more likely to evade is quite plausible. However, under the same conditions, the decision of the high-income taxpayers not to evade is counterintuitive. Similarly, given an increase of the cost, with the corresponding increase of \( \gamma \), it is also plausible that the low-income taxpayer will evade, but the decision of the high-income taxpayer not to evade is, once more, counterintuitive. Far from these counterintuitive results associated with inconsistent behavior being a burden on the model, we can conclude the following: if taxpayers’ reactions to changes in the cost of auditing and changes in the tax penalties are considered rationally plausible reactions, then it is justified to claim that taxpayers actually are behaviorally consistent. This thesis is supported by previous studies that shows that moral factors induce the agent to have a good behavior or bad behavior according to the behavior of society as a whole, as shown by Andvig and Moene (1990), Andreoni et al. (1998), among others. The behavioral consistency can easily be interpreted in accordance with this principle. The \( \tilde{EE} \) strategy, which means not to evade when the taxpayer’s type is low income and not to evade when it is high income, may refer the idea that the poor do not evade because he observes that the rich do not evade either and vice versa. Conversely, the \( EE \) strategy refers to the idea that the poor evades because he observes the rich evading and vice versa. Our contribution is the modeling of this principle through the concept of behavioral consistency.

3. Conclusion

The article analyzed the incentives that taxpayers and the government have in the tax evasion Bayesian game. In our model, taxpayers have two types, high income and low income. This is quite different from Pruzhanski (2004)’s model, in which the action available to the taxpayer is his reporting low income or high income. Why would a taxpayer report a higher income? In our model, on the contrary, the level of income is his private information and his decision is whether to evade or not, which, in our opinion, is much more reasonable. We studied the equilibria under consistency and inconsistency. In both instances, we also studied the equilibrium beliefs in terms of the parameters. Our concept of consistent Bayesian equilibrium fit quite well the stylized facts that agents follow moral principals external to the game.

An extension of the model is to differentiate the fine and the tax rate between rich and poor. Indeed, it is natural to assume that the fines and rates that would be levied on the low-income taxpayer would be lower than the ones levied on the high-income taxpayers. Suppose the rate on low-income is \( t_1 \) and the fine is \( \varphi_1 \). Similarly let \( t_2 \) and \( \varphi_2 \) be, respectively, the rate and the fine related to high-income taxpayers. One can easily show in this case, among other things, that the ratio \( (t_1/\varphi_1)=(t_2/\varphi_2) \) must hold for the equilibrium to exist. As pointed out in the literature review, tax evasion influences the tax design. An extension using different rates and fines to taxpayers with different incomes would bring quite plausible explanations for the effects of the interaction between tax evasion and tax design.

In sum, our model describes the strategic interaction between the tax authority and the taxpayer regarding tax evasion and tax audit, taking into account two distinct behavioral structures: behavioral consistency and behavioral inconsistency. The idea is to incorporate in the Bayesian game the notion that the taxpayer, whatever his type, may want to do so in accordance with the social behavior or according to moral principles external to the game. Our model, by describing stylized facts as a Bayesian game, has shown that behavioral consistency perfectly agrees with plausible and intuitive results, which means the relationship between the taxpayer and the tax authorities is indeed characterized by the submission to moral principles or imitation of social behavior.

References

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