Measurement of clamping forces in a 3 jaw chuck through an instrumented Aluminium ring

M. Estremsa,*, M. Arizmendi a, W. E. Cumbicus a, A. López a

a Dept. of Mechanical Eng. TECNUN. Univ of Navarra, P de Manuel Lardizabal 13, Donostia-San Sebastián (Gipuzkoa) 20018, España

Abstract
Measuring the clamping forces on cylindrical workpieces is a key factor in the geometrical tolerances of such components, especially if they are slender as the case of thin rings. The lower the clamping force, better tolerances will be achieved, but with the disadvantage of reducing friction and, therefore, increasing the risk of slipping. Therefore, achieving a minimum but safe clamping force is a key factor to control the process. Usually, these parts are made in lathes that have concentric plate clutches and these are fixed mechanically by wrenches or hydraulically through the control of pressure by valves. A simple and economic method is proposed to measure the clamping forces in lathes, although it is necessary the use of a model for the ring deformation. This method allows knowing the clamping force from the torque applied by a dynamometric wrench, or from the hydraulic pressure controlled by valves.

Keywords: Chuck Forces; Clamping Forces; Strain Gages; Ring Deformation; Torque Wrench.

1. Introduction
Measuring the clamping forces on cylindrical workpieces is a key factor in the geometrical tolerances of such components, especially if they are slender as the case of thin rings [1, 2]. The lower the clamping force, better tolerances will be achieved, but with the disadvantage of reducing friction and, therefore, increasing the risk of slipping. Therefore, achieving a minimum but safe clamping force is a key factor to control the process. There is an influence also in the dynamic response of the system to cutting [3, 4]. As the spindle speed increases, the camping...
force decreases as a consequence of centrifugal forces. Many times the clamping force must compensate the strong bending moments [5], vibrations [6], etc. Anyway, the control of the camping force is important in any operations of cutting.

Usually, these parts are made in lathes that have concentric plate clutches and these are fixed mechanically by wrenches or hydraulically through the control of pressure by valves. There exist load cells to check the clamping forces, but these sensors are expensive and difficult to assembly. Due to the disposition of jaws, and the concentric mechanism, three loads would be necessary to have and acceptable date. Several solutions have been proposed in literature and in patents, such of Nyamekye et al. [7] that is depicted in Fig 1. This load cell presents lacks of the triple symmetry of the grip, as one of the jaws is more flexible than the other two jaws. The advantage of this solution is that there is not a complicated formulation to relate the signal with the force of the jaw.

The method proposed in this paper consist in putting strain gages in a ring and relate mathematically the values of the circumferential tension strains with the normal loads of the chuck. This relation is determined through a formulation developed in this work similar to that of Malluck et al. [8]. This method deals well with the ring geometry, is ease to implement, and can evaluate the mechanical amplification of the complex transmissions and wedges that are common in current chucks.

As a solution, a simple and economic method is proposed, although it is necessary the use of a model for the ring deformation. This method allows knowing the clamping force from the torque applied by a dynamometric wrench, or from the hydraulic pressure controlled by valves.

2. Experimental setup

The instrument designed to measure the clamping force in a 3-jaw chuck consist of an aluminum ring instrumented with 2 strain gages placed in positions where maximum tensile strain occurs.

The Aluminium ring has 80 mm in diameter, 4.5 mm thickness and 10 mm width. The two gages are placed in symmetrical positions so that they have the same deformation and can be mounted in parallel in a Wheatstone bridge. Measuring the voltage variation, the deformation of the two gages is averaged giving a robust datum for the clamping force acting on the ring.

Fig. 2 (left) shows the Aluminum ring with the two strain gages and the marks necessary to get position of the ring in the chuck.

The method is calibrated using uniaxial known loads. The equations of deformations in this case are simpler and it gives a magnitude of the elastic constants of the material, as well as a relation between load and deformation in the points in which the strain gages are placed.

The lathe used is a ZMM-SLIVEN CU-400M, with a chuck Mk-G. It is run with the following cutting conditions: feed \( f = 0.1 \text{ mm} \), depth of cut \( a_p = 0.5 \text{ mm} \), and speed \( v = 4.2 \text{ m/s} \).

A dynamometric wrench BACHO 7455-20 has been used to apply a torque to the chuck. It has been calibrated by a piezoelectric torque measurement device Kistler 9275 bought with its calibration certificate. A torque of 10 N·m has been applied for the experimental test. For each torque, the dynamometric wrench has been applied to each three entrance in the chuck, pursuing the triple symmetry.
After machining, the part has been unclamped and its roundness was measured with a profilometer (RA-400 Mitutoyo) at 3 different depths. The number of samples per revolution in the measure of roundness has been 120, with a division of scale of 1 µm.

2.1. Strain gages on the ring

The strain gages used in the work are specific for aluminum material. They are Vishay CEA-13-250UW-120, of 120 Ω. They have the same thermal expansion coefficient as the material base. The gain factor of the gage is k=2.005, and it has been mounted by an expertise technician. They have been placed in two of the three points where a maximum moment is expected to occur, as it is shown in Fig. 2 (right). The number of 2 is due to the compensation of values in the Wheatstone bridge as will be seen in the following subsection.

The gage factor k relates the variation of electrical resistance R with the elastic strain ε.

\[
k = \frac{1}{\varepsilon} \frac{dR}{R}
\]  

(1)

2.2. Wheatstone bridge

The method to measure the variation of resistance of the two strain gages is a Wheatstone bridge as it is shown in Fig.3, where the strain gages are disposed in a half opposite way, i.e. R1=R3=Rg and R2=R4=Ri, where Rg is the resistance of the strain gage, while Ri is the internal resistance of the device used. In a relaxed state both resistances are the same i.e. 120 Ω. Both strain gages are placed in symmetrical position so they will have identical variation due to the strain of the point. Meanwhile, the internal resistance Ri will be constant in all the process. The equipment used to make the measurement is a SanEi AS1203 with connections Bridge Box 5370 which add the Ri of 120 Ω.
The non-equilibrium of the bridge gives a relation between the voltage of the source \( V \) and the voltage measured \( \Delta \) which is multiply with the gain factor of the amplifier to read the \( \varepsilon \) value at the display. In equilibrium conditions \( \Delta = 0 \) because all the resistances have the same value.

\[
\Delta = \frac{R_1 R_3 - R_2 R_4}{(R_1 + R_2)(R_3 + R_4)} V = 0
\]  

(2)

Supposing that all resistances have small variation, the increment of the voltage measured will be, and furthermore as initially the bridge is in equilibrium \((\Delta = 0)\) the increment of the voltage measured could be calculated form the variation of the resistance according the following expression:

\[
\Delta = \frac{R_2 R_4}{(R_1 + R_2)^2} \left( \frac{dR_1}{R_1} - \frac{dR_2}{R_2} - \frac{dR_3}{R_3} + \frac{dR_4}{R_4} \right) V
\]  

(3)

Using the expression of the strain gage factor (1), the variation of the voltage is related with the strain of each point where the strain gage is placed.

\[
\Delta = \frac{R_2 R_4}{(R_1 + R_2)^2} k (\varepsilon_1 - \varepsilon_2 + \varepsilon_3 - \varepsilon_4) V
\]  

(4)

In this case, \( \varepsilon_2 = \varepsilon_4 = 0 \) due to the resistance \( R_i \) are constantan while \( \varepsilon_1 = \varepsilon_3 = \varepsilon \) and initially the 4 resistances are equal:

\[
\Delta = \frac{k V \varepsilon}{2}
\]  

(5)

The device SanEi has a system of self-calibration with an internal resistance that simulate a strain gage with a factor \( k = 2 \) that occupies \( 1/4 \) of the bridge. In this case we have put a simulated strain \( \varepsilon_c \) of 500 microdeformations so the voltage measured \( \varepsilon_c \) shows the gain and voltage of the power source that can be variable.

\[
\Delta_c = \frac{2V \varepsilon_c}{4} = \frac{V \varepsilon_c}{2}
\]  

(6)

Dividing equations (5) and (6) \( V \) is eliminated and also the gain factor \( G \) of the amplifier. As \( \varepsilon = G \Delta \) being \( \varepsilon \) the value that is read in the amplifier device, thus the strain of each point where the strain gage is put from the voltage read in the amplifier \( \varepsilon_c \):

\[
\varepsilon_c = \frac{e \cdot \varepsilon_c}{e_c \cdot k} = \frac{500 \cdot 10^{-6}}{1.34 \cdot 2.005} e
\]  

(7)

3. Static model

In order to calculate the strain in the points where the gages are located, we use a model that is developed in [11], but as in our problem there is no cutting force, and therefore, there is not any tangential reaction in the jaws, a simplify model is presented here.
3.1. Castigliano’s Theorem to calculate the moments in the ring.

The model presented uses the Castigliano’s Theorem for calculating the bending moments along the ring perimeter. From the sector of the ring, according to Fig. 5, the balance equations are expressed in (10).

The clamping forces $P_i$ are introduced only if acting on the sector of the ring studied. The equilibrium conditions, the symmetry of the problem, the uniformity of the ring and the absence of friction, result in that all three $P_i$’s are equal in magnitude to $P$.

\[
M_\phi = M_0 + Q_0 r \sin(\phi) + N_0 r (1 - \cos(\phi)) - \sum_i P_i \cos(2\phi_i) - \cos(\phi + \phi_i))
\]
\[
Q_\phi = N_0 \sin(\phi) + Q_0 \cos(\phi) - \sum_i P_i \cos(\phi - \phi_i)
\]
\[
N_\phi = N_0 \cos(\phi) + Q_0 \sin(\phi) - \sum_i P_i \sin(\phi + \phi_i)
\]

\[ \text{(8)} \]

$M_0$, $Q_0$ and $N_0$, are calculated through the three conditions given by the continuity in point 0 in Fig. 4. They are: that for $\phi=2\pi$, then $\delta_x=0$, $\delta_y=0$ and $\delta_\theta=0$. To calculate $\delta_x$, $\delta_y$ and $\delta_\theta$, the Castigliano’s Theorem is used. First, as if $M_0=0$, $Q_0=0$ and $N_0=0$, and then obtaining an analytical relation between the constant of integration and the $M_0$, $Q_0$ and $N_0$ values.

\[ \text{Fig. 4. Forces acting on the ring, and a sector of this one.} \]

As deformations, both previous and at machining are small, the radius and the ring radial area can be considered as constant. From the value of the strain energy, $U_f$, of the ring in sector $[0, \phi]$ of Fig. 5:

\[
2U_f = \int_0^\phi \left( \frac{M_\phi^2}{EI_z} + \frac{kQ_\phi^2}{GA} + \frac{N_\phi^2}{EA} \right) r d\phi
\]

\[ \text{(9)} \]

Where $E$ is the Young modulus, $G$ is the Shear modulus, $I_z$ is the moment of inertia of the section, and $A$ is the area of the ring. $M_\phi$, $Q_\phi$ and $N_\phi$ can be obtained from the equations (10). According to the Castigliano’s Theorem the displacements $\delta_x$ and $\delta_y$, and the torsion $\delta_\theta$ are the derivatives of the function $U_f$ with the forces and torques acting in such a point. In point 0, when $\phi=2\pi$, then:

\[
\delta_{y0} = \frac{\partial U_f}{\partial Q_0}; \delta_{x0} = \frac{\partial U_f}{\partial N_0}; \delta_{\theta0} = \frac{\partial U_f}{\partial M_0}
\]

\[ \text{(10)} \]

$\delta_{y0} = \delta_{x0} = \delta_{\theta0} = 0$
Solving the three integrals gives constants of integration $C_1$, $C_2$, and $C_3$ that can be obtained through the three conditions, giving a set of equations from which we get the values of $M_0$, $N_0$, and $H_0$, and then, the values of $M_\phi$, $Q_\phi$, and $N_\phi$ in equations (10).

Applying equation (10), the bending moment $M_\phi$ and the compression $N_\phi$ at any ring section can be calculated and used to calculate the stress at the point where the strain gages are placed.

Bending moment $M_\phi$ is also used as input in the differential equation of the deformation of the ring \([9]\). This equation is solved using Chebyshev polynomials with the method described in \([10]\) with some adaptations to the current problem that are described in \([11]\). This solution is used to check the methodology.

\[
y'' + y = \frac{-M_\phi r(\phi)^2}{EI_z}
\]

As a first approximation, the moment in the instrumented ring is calculated if the clamping force is 1 N. The constants of material used in this model are: Young modulus $E=7\cdot10^{-10}$ MPa, Poisson modulus $\nu=0.33$. In Fig. 5 the bending moment is represented along the ring perimeter, showing singularities $C_1$ in the contact points between the jaws and the ring. The compressive force of the section is $N_0=-0.577$ N, the maximum moment is $M_0=0.0038$ N·m, and it occurs at $0, 2\pi/3, 4\pi/3$ radians, and the minimum moment is $M_i=-0.007$ N·m, which occurs at the jaw position. Shear force is $V_0=0$ in all sections.

This moment diagram is coherent with the system as it has three identical intervals. The moment at the beginning of the ring is identical to that of the end point with continuity $C_1$. Using the Navier distribution of stress, in the jaw-workpiece contact points there are compressive stresses in the internal surface and compression in the external ones, but in the middle points (where maximum moments happens) the tensional stress are in the external surface. The stress in the point where the strain gages are located is $\sigma=0.0803$ MPa which correspond to a strain $\varepsilon=1.02\cdot10^{-6}$. Due to the linearity of the problem, the clamping force can be calculated dividing the strain measured by the value of $\varepsilon$.
4. Methodology to obtain the mechanical amplification of the chuck

The methodology to obtain the mechanical amplification in the chucks with 3 jaws consisted in the following steps:

- Place the instrumented ring in the chuck
- Apply a known torque in the entrance by a dynamometric wrench
- Measure the strain in the location of the strain gages with formula (9)
- Calculate the strain produced in the same location if a clamping force $P$ of 1 N is applied using the model developed in Section 3
- Divide the measured strain (step 3) and the calculated one (step 4) to get the real clamping force $P$
- The relation between the clamping force $P$ and the torque applied in the chuck is the mechanical amplification of the chuck. It is useful to know which torque is necessary to clamp a workpiece with a force $P$

The physical implementation is very simple consisting in a ring and 2 gages placed in a symmetrical way in the external part of the ring. When this ring is placed in the chuck and a known torque is applied in its three knots, the ring is deformed in a trilobular way, with maximum momentums in the places where the strain gages are located. With the adequate equipment the strain at these points is calculated and compared with a mechanical model of the behavior of the ring. In this way, the force of the jaws applied to produce the strain in location of the jaw is calculated. The relation of this force of the jaw and the torque applied by a dynamometric wrench is the amplificatory factor of the chuck that is useful to establish the requirements of clamping force in lathe operations.

5. Measurements

In order to check the method, a test has been developed. The test consisted in clamping the ring with a torque of 10 N·m measuring the clamping force according to step 5 in the methodology. As the voltage registered in the device is 4.96 V, this correspond to a strain of $\varepsilon=9.25\cdot10^{-4}$ applying equation (9), which corresponds to a stress $\sigma=64.8$ MPa well below the yield stress. As the model gives a strain of $1.02\cdot10^{-6}$ for a clamping force off 1 N, the clamping force required to get a deformation measured of $\varepsilon$ will be 930 N. This will give a mechanical amplification of $930/10=93$ m⁻¹, i.e. to get the clamping force in each jaw, it is enough to multiply the torque in N·m by 93.

After these measurements, a new ring is placed in the lathe applying the same torque and, as a consequence, the same clamping force of 930 N. This ring is slightly machined in its interior diameter. Logically the flute of the tool will cut with a major depth in the position of the jaws, in a way that when the part is released the form of the surface will be a copy of the deformed shape when it was clamped. The maximum radius will be in the position of the clamps where more material was removed.

Fig. 6. Deformation of the ring (m) theoretical and experimental with a clamping force of 930N.
To calculate the deformation of the ring while it is cutting, the method of polynomials of Chebyshev described in [11] has been applied. This form calculated is compared with the measured in the roundness machine.

Fig. 6 shows the comparison of both: the calculated and the measured. As it is observed, profiles generally agree well in form. The differences between theoretical and experimental deformations, shown in Fig. 6, can be due to several factors: a) the estimation of the material constants, b) the difficulty of obtaining precise gripping forces values, and c) the initial inaccuracy of the ring that amplifies the deformation when three equal loads are applied.

6. Conclusions

A new method to measure the mechanical amplification in the chucks with 3 jaws has been developed. The physical implementation is very simple consisting in a ring and 2 gages placed in a symmetrical way in the external part of the ring. When this ring is placed in the chuck and a known torque is applied in its three knots, the ring is deformed in a trilobular way, with maximum momentums in the places where the Strain Gages are located. With the adequate equipment the strain at these points is calculated and compared with a mechanical model of the behavior of the ring. In this way, the force of the jaws applied to produce the strain in location of the jaw is calculated. The relation of this force of the jaw and the torque applied by a dynamometric wrench is the amplification factor of the chuck that is useful to establish the requirements of clamping force in lathe operations.

The method has been tested comparing the form of a machined ring with the predicted with the model developed in [11].

This method has its limits due to the mechanical inequality between the different jaws of the chuck. Anyway, this factor could be estimated placing the ring in the three different positions and applying the same torque in every entrance with the dynamometric wrench.

This method can be extended to others kinds of mechanism of amplification of forces as the commonly used in CNC machines of hydraulic pressure to pistons and wedges.

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