Footprint Problem with Angle of Attack Optimization for High Lifting Reentry Vehicle

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Abstract
A formal analysis to footprint problem with effects of angle of attack (AOA) is presented. First a flexible and rapid standardized method for footprint generation is developed. Zero bank angle control strategy and the maximum crossrange method are used to obtain virtual target set; afterward, closed-loop bank angle guidance law is used to find footprint by solving closest approach problem for each element in virtual target set. Then based on quasi-equilibrium glide condition, the typical inequality reentry trajectory constraints are converted to angle of attack lower boundary constraint. Constrained by the lower boundary, an original and practical angle of attack parametric method is proposed. By using parametric angle of attack profile, optimization algorithm for angle of attack is designed and the impact of angle of attack to footprint is discussed. Simulations with different angle of attack profiles are presented to demonstrate the performance of the proposed footprint solution method and validity of optimal algorithm.

Keywords: reentry; angle of attack optimization; footprint; trajectory constraints; bank angle control

1. Introduction

High lifting reentry vehicles (such as space shuttle, X37B and HTV) have received extensive attention in recent years because of their substantial downrange and crossrange maneuverability. The long time endo-atmospheric reentry flight presents material and thermal insulation challenges due to the increased integrated heat loads and active aerodynamic control. The safe and reliable reentry has become a very important issue. Footprint is a performance to evaluate the ranging capability of vehicle in reentry flight [1]. It is defined as the boundary of all reachable landing locations on the surface of the Earth from a certain reentry interface. Several factors influence footprint, including initial condition, terminal condition, aerodynamic heating and load constraints, glide capability, etc. For mission planning, the information of footprint is needed as a reference to determine landing site. While trajectory reconfiguration is needed to accommodate vehicle state variation (such as engine failure during ascent, unforeseen control failure and damage), the landing site must be selected based on the footprint [2]. In addition, footprint can be used to evaluate the reentry guidance algorithms that combine a trajectory planner and a trajectory tracker [3].

The footprint problem is traditionally formulated as an optimal control problem [4]. In recent years, there are four typical methods to solve footprint problem. Through coordinate rotation, the footprint problem was converted into parameter searching problem for the maximum crossrange with free downrange [5]. It could also be converted into trajectory optimization using direct method. The well-known direct methods such as pseudospectral were used [6-9]. The highest drag profile and minimum drag profile were used to approximately determine the far side and near side of footprint [10-11]. References [12]-[14] discussed the closest-approach...
problem and the maximum crossrange at prescribed downrange problem. And then the equivalence of these two problems was proved. After that the footprint was taken as a single parameter searching problem with closest approach.

For this class of vehicles, bank angle is chosen as the primary trajectory control parameter because angle of attack (AOA) can then be selected to minimize aerodynamic heating environment while achieving the required range [15]. Usually the design of trajectory uses nominal AOA profile which is planned ahead. In most literature of footprint problem, the rapid generation of footprint is also based on the nominal AOA. Though these methods perform with good versatility, the effects of nominal AOA are not taken into account. In fact, AOA is important for the footprint problem and the effects should be investigated profoundly.

This paper investigates footprint problem with effects of AOA. We first develop a flexible and rapid method for footprint generation. It is very convenient to use it to analyze the ranging capability for high lift/drag ratio (L/D) vehicles and its calculation efficiency has advantage over traditional methods. With parametric AOA profile analysis and optimization results for the maximum downrange and minimum integrated heat load, a conclusion is derived: the high lifting reentry vehicle’s downrange maneuverability comes at the cost of the damage of crossrange.

2. Footprint Analysis

2.1. Entry dynamics

The point-mass dynamics of reentry vehicle over a sphere rotating Earth are described by the following dimensionless equations of 3D motion [16]:

\[
\begin{align*}
\dot{r} &= V \sin \gamma \\
\dot{\theta} &= (V \cos \gamma \sin \psi) / (r \cos \phi) \\
\dot{\phi} &= (V \cos \gamma \cos \psi) / r \\
V &= -D - \frac{\sin \gamma}{r} + \Omega^2 r (\sin \gamma \cos \phi - \cos \gamma \sin \phi \cos \psi) \cos \phi \\
\dot{\gamma} &= \left[ L \cos \sigma + [(V^2 - 1/r) \cos \gamma] / r + 2 \Omega V \sin \psi \cos \phi + \right.
\left. \Omega^2 \left( \cos \gamma \cos \phi + \sin \gamma \cos \psi \sin \phi \right) \cos \phi / r \right] / V \\
\dot{\psi} &= \left[ L \sin \sigma / \cos \gamma + (V^2 \cos \gamma \sin \psi \tan \phi) / r - 2 \Omega V \cdot 
\left. (\tan \gamma \cos \psi \cos \phi - \sin \phi) + \right. \right.
\left. \Omega^2 r \sin \psi \sin \phi \cos \phi / \cos \gamma \right] / V \\
\dot{Q} &= k_Q \sqrt{r^3 \cdot 15}
\end{align*}
\]

where the differentiation is with respect to the dimensionless time \( \tau = t / \sqrt{R_0 / g_0} \). \( r \) is radial distance from center of the Earth to vehicle and normalized by the radius of the Earth \( R_0 \). \( \theta \) and \( \phi \) are the longitude and latitude. \( V \) is the Earth-relative velocity and normalized by \( \sqrt{g_0 R_0} \) with \( g_0 \) being the gravitational acceleration magnitude on the surface of the Earth. \( \rho \) is atmosphere density. \( L = \rho g_0 R_0^2 S_{\text{ref}} C_L \) and \( D = \rho g_0 R_0^2 S_{\text{ref}} C_D \) are the aerodynamic lift and drag acceleration in \( g \), where \( S_{\text{ref}} \) is reference area, \( m \) is the mass and normalized by initial mass \( m_0 \). \( g \) gravitational acceleration, \( C_L \) lift coefficient and \( C_D \) drag coefficient. \( \gamma \) is flight path angle and \( \psi \) velocity azimuth angle. \( \Omega \) is the Earth’s rotation rate normalized by \( \sqrt{R_0 / g_0} \). The heat load \( Q \) is taken as a state variable and brings an additional equation about heating rate. \( k_Q \) is a constant determined by the radius of the reference sphere. It is introduced to convert the integral type performance index \( \int \dot{Q} dt \) into terminal type performance index and normalized by \( m g_0 R_0 \). Bank angle \( \sigma \) is usually taken as control input together with AOA \( \alpha \).

2.2. Quasi-equilibrium gliding condition (QEGC)

QEGC is a well-known flight mechanics for reentry guidance [17]. Setting \( \gamma = 0 \) and \( \psi = 0 \) in Eq. (1) and ignoring Earth rotation will give the following formula:

\[
L \cos \sigma + (V^2 - 1/r) / r = 0
\]

Because the altitude during entry flight is much smaller than the radius of the Earth, then

\[
r \approx 1
\]

Substituting Eq. (3) into Eq. (2), we can get

\[
L \cos \sigma + (V^2 - 1) = 0
\]

Solving \( L \) from the QEGC in Eq. (4) and expressing \( D = L / (C_L / C_D) \), we have

\[
L = (1 - V^2) / \cos \sigma
\]

\[
D = C_D (1 - V^2) / (C_L \cos \sigma)
\]

The downrange \( S \) can be expressed as

\[
S = - \int_{v_t}^{v_f} \frac{1}{D} de
\]

where \( e = 1/r - V^2/2 \), and the subscript “0” means value at the initial time while the subscript “f” means value at the final time. This notation holds ture in the whole paper. Since \( D = L / (C_L / C_D) \) and considering QEGC and \( r \approx 1 \), then Eq. (7) turns to

\[
S = - \int_{v_t}^{v_f} \frac{C_L \cos \sigma}{C_D (1 - V^2)} de
\]

Equation (8) will be used in AOA optimization (see Section 3) and the maximum downrange appears when \( \cos \sigma \) and \( L/D \) are selected as the maximum values. That means \( \sigma = 0 \) and AOA should make \( L/D \) the maximum value \( (L/D)_{\text{max}} \). However, the \( (L/D)_{\text{max}} \) will be against path constraint, and the optimization of AOA in this paper will be discussed with path constraints.
2.3. Closed-loop guidance law

Based on QEGC and Eq. (3) and ignoring Earth rotation, the reduced dynamic equations are

$$\begin{align*}
\dot{\theta} &= (V \sin \psi) / \cos \phi, \\
\dot{\phi} &= V \cos \psi, \\
V &= -C_D (1 - V^2) / (C_L \cos \sigma), \\
\dot{\psi} &= [(1 - V^2) \tan \sigma + V^2 \sin \psi \tan \phi] / V.
\end{align*}$$  

(9)

According to the maximal principle, the Hamiltonian function $H$ is defined as

$$H = \lambda_\psi V \sin \psi / \cos \phi + \lambda_\phi V \cos \psi + \lambda_3 \frac{C_D}{C_L} V^2 \frac{1}{\cos \sigma} + \lambda_4 \frac{1}{V} [(1 - V^2) \tan \sigma + V^2 \sin \psi \tan \phi] = \frac{A}{\cos \sigma} + B \tan \sigma + C$$  

(10)

where $\lambda_\psi, \lambda_\phi, \lambda_3$ and $\lambda_4$ are the costates and

$$A = C_D (V^2 - 1) \lambda_\phi / C_L, \\
B = (1 - V^2) \lambda_\psi / V, \\
C = V \left( \frac{\lambda_\psi \sin \psi}{\cos \phi} + \lambda_\phi \cos \psi + \lambda_3 \sin \psi \tan \phi \right)$$  

(13)

The optimality condition is

$$H(x^*(t), u^*(t), \lambda(t)) = \max H(x(t), u(t), \lambda(t))$$  

(14)

where $x^*$ is state vector, $u^*$ control input vector and $\lambda$ costate vector. The optimal solution necessitates

$$\partial H / \partial \sigma = 0$$  

(15)

Then we have

$$\sin \sigma = -B / A = (V^2 - 1) \lambda_\phi / (VA)$$  

(16)

Since the Hamiltonian function is not explicitly dependent on the variable $t$, along the optimal trajectory $H$ will be a constant which is set as $C_0$.

$$A / \cos \sigma + B \tan \sigma + C = C_0$$  

(17)

For free final time, $C_0 = 0$. The sub-optimal guidance laws for all kinds of terminal performance indexes can be easily obtained:

$$\tan \sigma = (c_1 \sin \phi + c_2 \cos \theta \cos \phi + c_3 \sin \theta \cos \phi) \cdot \frac{1 - V^2}{[c_2 \cos \phi \sin \psi + c_3 (\sin \theta \cos \psi - \cos \theta \sin \psi \sin \phi) - c_1 (\sin \theta \sin \psi \sin \phi + \cos \theta \cos \psi)]}$$  

(18)

where $c_1, c_2$ and $c_3$ are constants.

The different terminal performance indexes will result in different constants $(c_1, c_2, c_3)$. $c_2 = \sin \kappa$ and $c_3 = \cos \kappa$, where $\kappa$ is a parameter to be solved. For the first order and second order conditions, Ref. [13] provides detailed discussion.

The closed-loop sub-optimal guidance law without path constraint may be against the path constraints seriously. So the guidance law (Eq. (18)) should be improved based on QEGC [18]. The path constraints can be converted into the maximum bank angle boundary:

$$|\sigma| \leq \sigma_{\text{max}}$$  

(19)

The optimal control law of bank angle can be written as

$$\sigma_{\text{opt}} = \begin{cases} 
\sigma_n & |\sigma| \leq |\sigma_n| \\
\text{sgn}(\sigma) \sigma_{\text{max}} & |\sigma| \geq |\sigma_{\text{max}}|
\end{cases}$$  

(20)

where $|\sigma_n|$ is the unconstrained optimal bank angle solved by Eq. (18), and $|\sigma| \leq \pi / 2$.

2.4. Footprint

Footprint problems are usually solved as optimal control problems to find the maximum crossrange for a series of downranges. Lu and Xue [14] proposed a rapid generation method to obtain accurate footprints. They had proved that the original footprint problem can be converted to closest approach problem and solved by the closed form bank angle control law of Eq. (18). Actually, both the maximum crossrange method and closest approach method have good points and limits. The maximum crossrange method can provide accurate and detailed information of footprint. But it is a nonlinear equations root-finding problem and difficult to solve. If using this method to generate footprint, it will cost a great amount of time. The closest approach method is a univariate root-finding problem which can be numerically solved at a fast speed. However, it has to set a series of virtual targets which are unreachable before applying in footprint problem. These targets are chosen stochastically and usually need to be adjusted repeatedly to achieve a good result.

As Fig. 1 shows, the reentry point is denoted $E(\theta_0, \phi_0)$. $\theta_0$ and $\phi_0$ are longitude and latitude of entry point, respectively. $ED$ is the maximum downrange. $AD$ and $CD$ are sets of the maximum crossrange at prescribed downrange $EB_1, EB_2, \cdots, EB_N$. $AC$ is the set of minimum downrange. Footprint is the area covered by outer boundary $CD, AD$ and inner boundary $AC$.

The footprint is usually used for mission planning, trajectory reconfiguration onboard and evaluation of reentry guidance algorithms. These applications require a rapid and flexible footprint generation method. Here we propose a new solution to solve footprint problem with the benefits of both the maximum crossrange method and closest approach method.

1) Virtual target set

By solving the maximum crossrange problem, the virtual targets are obtained.

Firstly, the maximum downrange (the range represented by $ED$ in Fig. 1) should be calculated approximately. Integrate trajectory with $0^\circ$ bank angle control
law until the terminal states satisfy the setting condition.

Secondly, several points are selected along the maximum downrange trajectory and denoted as \(B_1, B_2, \cdots, B_N\). For each point, the corresponding maximum crossrange is calculated through solving the maximum crossrange problem. Notice that the \(N\) is not needed to be large (in the following simulation case, \(N=5\) is adequate enough).

Thirdly, the corresponding maximum crossrange is denoted by solid diamonds (the solid line in Fig. 2). The values of the maximum crossrange are added small positives to generate hollow diamonds (the dashed line in Fig. 2) which are taken as unreachable virtual targets. The other virtual targets can be determined by interpolate.

Since the virtual targets are obtained by solving the maximum crossrange problem, the choice of virtual targets is not arbitrary and will be much more rational.

2) Parameters searching algorithm

For each virtual target, we solve the closest approach problem and find the footprint. As Ref. [11], rather than taking square sum of terminal constraints for the maximum crossrange problem, which makes the searching results fall into local minima and sharp valleys very easily, we solve two nonlinear equations directly instead. Fortunately, the roots are easy to be found.

After determining virtual targets, then the footprint problem is solved by closed-loop bank angle guidance law (Eq. (20)).

Considering the continuity of the problem in nature [19], the virtual targets are chosen as close as possible. So we take the optimal parameter of the last target as initial guess of next one. It can improve the convergence and rapidity significantly of the searching algorithm.

To verify the proposed method, it is compared with the maximum crossrange method. The initial conditions are shown in Table 1, where \(h_0=r - R_0\) is the initial altitude.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(h_0/\text{km})</td>
<td>90</td>
</tr>
<tr>
<td>(\theta_0/(^\circ))</td>
<td>-167.007</td>
</tr>
<tr>
<td>(\phi/(^\circ))</td>
<td>-28.255</td>
</tr>
<tr>
<td>(V_0/(\text{m} \cdot \text{s}^{-1}))</td>
<td>7000</td>
</tr>
<tr>
<td>(\alpha/(^\circ))</td>
<td>0</td>
</tr>
<tr>
<td>(\omega/(^\circ))</td>
<td>38.329</td>
</tr>
</tbody>
</table>

The heating rate constraint is 5.2 \(\text{MW}/\text{s}^2\), dynamic pressure constraint 0.65 \(\text{MN}/\text{s}^2\), and loading constraint 5g. The terminal conditions are 2000 m/s and 35 km. The simulation results are shown in Figs. 3-4. Figure 3 shows the ground tracks of nominal trajectories. The

![Fig. 1 Illustration of footprint.](image1)

![Fig. 2 Virtual targets and the maximum crossrange.](image2)

![Fig. 3 Ground tracks of nominal trajectories.](image3)

![Fig. 4 AD and CD parts of footprint.](image4)
small circles are unreachable virtual targets generated by the above method. Figure 4 shows the AD and CD parts of footprint. It can be seen that the two footprints almost coincide with each other.

With the simulation comparison, the results show that both of the methods can find accurate footprint. But they have great difference in time cost. The proposed method takes 136 s while the maximum cross-range takes 3 186 s at the same calculation condition (Intel Core2 Duo CPU E8400 @3.00 Hz).

3. Optimization of AOA

3.1. Path constraints enforcement

Heating rate, dynamic pressure and aerodynamic load are the most critical hard path constraints during reentry. The optimal AOA has to satisfy these constraints to be flyable. Here the QEGC is used to convert these path constraints into AOA lower boundary, and the optimal trajectory will consequently satisfy constraints.

We divide the gliding reentry trajectory into initial descent part and quasi-equilibrium gliding part because the thin atmosphere density could not offer enough aerodynamic force to control the vehicle and quasi-equilibrium gliding cannot be achieved. The initial descent part is important since it determines whether the vehicle can turn into quasi-equilibrium gliding later. According to these two flight parts, the path constraints will be converted into AOA lower boundary constraint respectively.

1) Initial descent part

The path constraint during initial descent part is heating rate constraint. To avoid violating heating rate constraint, the initial AOA should not be too small. However, if AOA is too large, the trajectory will have a sharper skip after initial descent part (see Fig. 5).

![Fig. 5 Initial descent part with different AOs.](image)

From Fig. 5 we can find that while $\alpha=15^\circ$, the initial descent part violates heating rate constraint seriously. In order to satisfy heating rate constraint, the AOA at initial descent part should not be less than $\alpha_{\text{tan}}$. The $\alpha_{\text{tan}}$ can be found through the following two conditions at a certain velocity $V_{\text{tan}}$:

$$F(\alpha) = \dot{\theta} - \dot{\theta}_{\text{max}} = 0$$  \hspace{1cm} (21)

$$\left| \frac{dr}{dV} \right|_{\alpha=\text{tan}} - \left| \frac{dr}{dV} \right|_{\alpha=V_{\text{tan}}} \leq \delta$$  \hspace{1cm} (22)

where subscript “a” means actual trajectory and “Q” QEGC trajectory, $V_{\text{tan}}$ is the critical speed with which the quasi-equilibrium gliding part starts, and $\delta$ a sufficient small positive constant. For the problem in this paper, $\alpha_{\text{tan}}=18.656^\circ$ is the value of the lower boundary for initial descent part and the correspondingly velocity is $V_{\text{tan}}=6884$ m/s. Until $\alpha=18.656^\circ$, the initial descent part is just tangential with heating rate boundary. While $\alpha=20^\circ$, the lowest point of initial descent trajectory is above the heating rate boundary.

2) Quasi-equilibrium gliding part

For the purpose of nominal AOA optimization the bank angle in Eq. (1) is set equal to zero ($\sigma=0$) and the trajectory control parameter is reduced to AOA.

The most part of reentry trajectory is quasi-equilibrium gliding. The path constraints in altitude-velocity plane can be converted to lower boundary constraint of AOA:

$$(1/r-V^2)/r-L(\alpha)\cos\sigma_{\text{QEGC}} = 0$$  \hspace{1cm} (23)

Equation (23) can be rewritten as

$$C_L(\alpha) = (1/r-V^2)/(rK\rho V^2 \cos\sigma_{\text{QEGC}})$$  \hspace{1cm} (24)

where $K=\text{ReSc}_{\text{ad}}(2m)$. The AOA can be obtained from $C_L(\alpha)$. So the condition $r=1$ can be used while finding the AOA lower boundary for path constraints at different velocities. And Eq. (24) can be rewritten as follows:

$$C_L(\alpha) = (1-V^2)/(K\rho V^2 \cos\sigma_{\text{QEGC}})$$  \hspace{1cm} (25)

With $\sigma_{\text{QEGC}}$ increasing, the lower boundary of AOA increases correspondingly. That means the varying range of AOA reduces during quasi-equilibrium gliding. We set $\sigma_{\text{QEGC}}=0$ in this section and the AOA lower boundary (including initial descent part and quasi-equilibrium gliding part) is given in Fig. 6. The optimization problem of nominal AOA will be solved within the range of lower boundary constraint and the maximum trimming AOA.

3.2. Performance indexes

The remarkable high $L/D$ aerodynamic performance for high lifting reentry vehicles allows this class of vehicles to have larger ranging capability. On the other hand, the aerodynamic heating challenge is serious during hypersonic reentry. The thermal loads cause great problems to the design of thermal insulation. The optimization of AOA profile provides theoretical basis for guidance system design and also provides a reference trajectory for attitude control system.

Based on the above consideration, the optimal performance index will be combination of the maximum
downrange and minimum heat load. Through numerical optimization, the result trajectory will be a suboptimal trajectory which satisfies the requirements of both downrange and thermal insulation.

1) The maximum downrange

According to spherical trigonometry of reentry flight (see Fig. 7), $S_d$ is the downrange between reentry point and end point. The maximum downrange is expressed as

$$
S_d = \min \{ \sin \phi_1 \sin \phi_2 \cos \phi_1 \cos \phi_2 \cos (\theta_0 - \theta_1) \} \tag{26}
$$

2) Total heat load

The total heat load $Q$ is modeled by integrating heating rate of stagnation point. It is taken as a state variable (see Section 2.1) which avoids involving integral type performance index. In fact, rather than using total heat load, the heat load with given downrange makes much sense in engineering sight.

$$
\min J_2 = \min (Q + \omega_k | S_d - S_p |) \tag{27}
$$

where $\omega_k$ is the weight of downrange, and $S_p$ the preset downrange for specific mission plan.

3.3. Determination of AOA profile

The AOA optimization is an optimal control problem with control constraints and terminal constraints. The terminal constraints can be written in energy form as follows:

$$
\pi_1 (x_f (\tau_f)) = 1/\tau_f - V_f^2 / 2 - e_f = 0 \tag{28}
$$

where $x_f$ is the vector of final states, $\tau_f$ the final radial distance from center of the Earth to vehicle, $V_f$ the final velocity, and $\tau_f$ the final time. Generally, for direct shooting method $^{[20]}$ the time is divided into subintervals of the same length. And control input accordingly is discrete at each time nodes. With enough time nodes, the result will be very close to the optimal one. However, the convergence of this algorithm is not very well and computationally intensive to find an acceptable solution. Based on the geometry profile of AOA lower boundary in Section 3.1, the nominal AOA in this paper is parameterized by which it will be very practical in engineering application and then the optimal control problem turns to four parameters searching problem. A state-of-art sequential quadratic programming (SQP)$^{[21]}$ is used to solve the problem. The optimization algorithm programming code is self-developed.

The AOA profile is parameterized as (solid line in Fig. 8)

$$
\alpha = \frac{\alpha_\tan + \Delta \alpha}{2} \begin{cases} 
\alpha_\tan, & V_{\tan} < V < V_0 \\
\alpha_2, & V_0 < V < V_{\tan} \\
\frac{\alpha_1 - \alpha_2}{V_2 - V_1} (V-V_1) + \alpha_2, & V_1 < V < V_2 \\
\alpha_1, & V_2 < V \leq V_{\tan}
\end{cases} \tag{29}
$$

The first section corresponds to initial descent part with velocity ($V_{\tan} < V < V_0$) and the other three sections represent quasi-equilibrium gliding part with velocity ($V_1 < V < V_{\tan}$).

Since in initial descent part there is a lower boundary $\alpha_{\tan}$ for the constraint of heating rate, the AOA for initial descent part can be set as $\alpha_{\min} + \Delta \alpha$ (where $\Delta \alpha \geq 0$). We have known that $\alpha_{\tan} = 18.656^\circ$ and the maximum trimming AOA is $\alpha_{\max} = 20^\circ$. This means the searching space is very limited and the AOA is set to the maximum value $\alpha_{\min} + \Delta \alpha = 20^\circ$ for simple.

For quasi-equilibrium gliding part, there is also a lower boundary for AOA. The AOA profile is represented by three linear functions. It can satisfy AOA lower boundary constraint and at the same time reduce dimensions for parameter searching.

Through the above parameterization, the optimal problem for AOA is then turned to a parameter searching problem for points ($V_1$, $\alpha_1$) and ($V_2$, $\alpha_2$). SQP is efficient to find the optimal value of these parameters.

In order to enforce lower boundary constraints, the penalty functions $J_3$ and $J_4$ for optimization are expressed as

$$
J_3 = \begin{cases} 
0, & \alpha_1 \geq g(V_1) \\
g(V_1) - \alpha_1, & \alpha_1 < g(V_1)
\end{cases} \tag{30}
$$

$$
J_4 = \begin{cases} 
0, & \alpha_2 \geq g(V_2) \\
g(V_2) - \alpha_2, & \alpha_2 < g(V_2)
\end{cases} \tag{31}
$$

where $g(\cdot)$ is lower boundary function of AOA in altitude-velocity plane. Because the function $g(\cdot)$ is not monotonously decreasing with respect to $V$, the only
constraint at points \((V_1, \alpha_1)\) and \((V_2, \alpha_2)\) cannot ensure the AOA constraint. So there is an additional condition \(\alpha_2 \geq \alpha_{\text{ext}}\) (\(\alpha_{\text{ext}}\) corresponds to the extreme of \(g(\cdot)\)). The details are shown in Fig. 8.

In this paper, the performance index is a combination index of downrange, total heat load and penalty functions. The combination index is

\[
J = \omega_1 J_1 + \omega_2 J_2 + \omega_3 J_3 + \omega_4 J_4
\]

where \(\omega_i\) \((i = 1, 2, 3, 4)\) are the weights. While neither \(\omega_1\) nor \(\omega_2\) is 0, \(\omega_3\) in \(J_3\) needs to be set to 0.

The four performance indexes \(J_1, J_2, J_3\) and \(J_4\) in Eq. (32) represent different physical meanings. So the orders of magnitude of these four performance indexes are different. For different missions, the most concerned performance is different too. The choice of weights are based on the following two respects: A) balance the orders of magnitude of these performance indexes to avoid stiff problem in numerical computation; B) the weight for the performance index which is concerned most in a specific mission is chosen with relatively larger value, while the others are chosen with relatively smaller values.

### 3.4. Impact of nominal AOA on footprint

In Fig. 9, the \(S_A\) section is descent part and \(S_B\) and \(S_C\) sections are gliding parts. The designed AOA for the maximum downrange should be closed to the lower boundary in \(S_A\) and \(S_B\) sections. While the AOA for the maximum \(L/D\) is higher than the lower boundary (with velocity \(V_1\) as the critical point), the larger one will be chosen as nominal value for AOA.

While the vehicle reenters with deigned AOA for the maximum downrange index, it could not be controlled by bank angle during section \(S_A\) and \(S_B\). Because once the bank angle is not zero, the reentry trajectory will violate constraint definitely. At this condition, the footprint is the area covered by the dashed line shown in Fig. 10.

The varying range of bank angle is determined by AOA profile. If the vehicle reentry with AOA is designed for the maximum downrange (see Fig. 10), the point \(b\) has a velocity of \(V_1\). After point \(b\), the vehicle can do crossrange maneuvering slightly. In this case the footprint has the maximum downrange, but the area of footprint is small. To increase the area footprint covers, the vehicle should not use the AOA for the maximum downrange. Instead, a small value is added to the AOA and shown in Fig. 9 as the dashed line. This augment can improve crossrange maneuverability greatly since it allows the bank angle a larger control range. The region covered by solid line in Fig. 10 is the correspondingly footprint.

For reentry flight, vehicle often needs to satisfy waypoint and no-fly zone constraints. These all require crossrange maneuverability. Take the case in Fig. 10 as an example, if the vehicle reenters with AOA for the maximum downrange, the vehicle will not be able to do crossrange maneuvering between reentry point and point \(b\).

To sum up, the design of nominal AOA profile should be a tradeoff considering all the factors like footprint, thermal insulation, waypoint, no-fly zone and other trajectory performance. Furthermore, the maximum downrange and the maximum crossrange cannot be achieved at the same time. The crossrange maneuverability must come from the sacrifice of downrange, and vice versa.

### 4. Simulation and Verification

To verify AOA optimization method proposed in Section 3, we select two performance indexes: the maximum downrange denoted by “Max D” and the minimum heat load with prescribed downrange de-
noted by “Min H”. The simulation condition is the same as that in Table 1. For different missions the weights are chosen different. In order to find out the maximum downrange, the weight term of heating load has been set to zero. And when it comes to the minimum total heat load, the weight term of \( J_1 \) which represents downrange has been set to zero.

1) The maximum downrange

The optimal parameters are found through SQP. \( \alpha_1=6.14^\circ, V_1=1 268 \text{ m/s}; \alpha_2=12.76^\circ, V_2=6 800 \text{ m/s}. \)

The solid line in Fig. 11 is the optimal AOA for the maximum downrange and the dash-dot line is the maximum heating boundary. The total heat load is 11.008 GJ/m². Downrange is 15 620 km and the final velocity is 3 027 m/s.

![Fig. 11 Optimal AOA with Max D and Min H.](image)

2) The minimum heat load

The final downrange is set as 12 756 km. The optimal parameters are found through SQP. \( \alpha_1=14.57^\circ, V_1=1 000 \text{ m/s}; \alpha_2=14.49^\circ, V_2=1 000 \text{ m/s}. \)

The dashed line in Fig. 11 is the optimal AOA for minimum heat load. The total heat load is 5.513 GJ/m². Downrange is 12 756 km and the final velocity is 3 002 m/s.

The optimization results can be helpful to better understand the inherent physical mechanism and provide a significant foundation for nominal AOA design. As Fig. 11 shows, to reduce total heat load, the vehicle should fly with larger nominal AOA profile. Meanwhile, to increase downrange, the vehicle should fly with AOA profile corresponding to \((L/D)_{\text{max}}\). Actually, the above optimization results are for extremum cases which are illustrative and often used for reference and evaluation.

3) Footprints with different AOAs

In engineering experience, the design of nominal AOA is a very challenging work which involves specific mission scenarios. Here we give a proved nominal AOA to investigate the impact of AOA on footprint. The nominal AOA is shown in Fig. 12.

Figures 14(a)-14(c) are bank angles for the minimum heat load AOA, nominal AOA and the maximum downrange AOA respectively. The bank angle for the maximum downrange during [5 547, 6 733] m/s approximates to zero. That means the vehicle has very little crossrange maneuverability because of the maximum downrange objective.

Figures 15(a)-15(c) are \( r-V \) trajectories for the minimum heat load AOA, nominal AOA and the maximum downrange AOA respectively. The \( r-V \) trajectory of the minimum integrated heat load is much higher than the other two cases. The \( r-V \) trajectories for the maximum downrange ride on the heating rate boundary before velocity reaching \( V_1 \) (see Fig. 9). These results just demonstrate physical mechanism of reentry flight.

![Fig. 12 Designed nominal AOA.](image)

![Fig. 13 Ground tracks for different AOAs.](image)
5. Conclusions

A method for generation of footprint is developed in this paper, and analysis of footprint problem with the effects of AOA is discussed based on skilled parametric optimization of AOA. The proposed methodology is an ideal tool for reentry mission planning, analysis, and tradeoff studies on the ground. Simulations with different AOA profiles are presented to demonstrate that the effects of AOA on footprint are significant. Higher AOA profile can decrease the total heat load while the maximum $L/D$ profile can increase downrange. But these AOA strategies are extreme cases. The design of AOA should be a comprehensive work involving thermal insulation, ranging capability and attitude maneuvering. Future work will add the effects of ascent flight, environment dispersion and modeling uncertainties.

References


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