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Angular momentum – area – proportionality of extremal charged black holes in odd dimensions

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ABSTRACT

Extremal rotating cohomogeneity-1 black holes in Einstein–Maxwell theory feature two branches. On the branch emerging from the Myers–Perry solutions their angular momentum is proportional to their horizon area, while on the branch emerging from the Tangherlini solutions their angular momentum is proportional to their horizon angular momentum. The transition between these branches occurs at a critical value of the charge, which depends on the value of the angular momentum. However, when a dilaton is included, the angular momentum is always proportional to the horizon area.

1. Introduction

Although in $D=4$ dimensions the Kerr–Newman solution represents the unique family of stationary asymptotically flat black holes of Einstein–Maxwell (EM) theory, the corresponding $D>4$ charged rotating black holes have not been obtained in closed form yet. Only certain subsets are known: the generalization of the static black hole to higher dimensions pioneered by Tangherlini [1], and the rotating vacuum black holes, obtained by Myers and Perry (MP) [2]. Other subsets could be constructed perturbatively [3–7] and numerically [8,9].[1]

Nevertheless, if additional fields and/or interactions are allowed into the theory, exact higher dimensional charged rotating black holes can be obtained by solution generating techniques. For example, in the simplest Kaluza–Klein (KK) case, a boost is done to the $D+1$ embedding of the uncharged D-dimensional MP black holes along the extra dimension. After dimensional reduction the result is a charged D-dimensional black hole in Einstein–Maxwell-dilaton (EMd) theory. The dilaton coupling constant h for this solution has a particular value, which we denote h_{KK}, that depends on the dimension D [10]. To generate rotating EMd black hole solutions with other values of the coupling constant h, currently perturbative or numerical techniques must be used.

D-dimensional stationary black holes possess, in general, N independent angular momentum J_i associated with N orthogonal planes of rotation [2], where N is the integer part of $(D-1)/2$, corresponding to the rank of the rotation group $SO(D-1)$. As a result, we can distinguish between odd-D and even-D black holes, where the latter have an unpaired spatial coordinate [2]. In the particular case in which all N angular momenta are equal in magnitude, the EMd equations simplify considerably, yielding, for odd dimensions, cohomogeneity-1 equations from which the angular dependence can be extracted analytically. Hence, the equations reduce to a more tractable system of ordinary differential equations.

When the N angular momenta are of equal magnitude, $J=|J_i|$, it is interesting to note that, for extremal MP black holes, the angular momentum J and the horizon area A_{H} are proportional: $J=\sqrt{2(D-3)}A_{\mathrm{H}}$. This is a special case of a more general type of relations for MP black holes in terms of the non-degenerate inner and outer horizon areas of non-extremal black holes [11], and was pointed out in 4 dimensions before [12–18]. In the case of charged black holes, the relation for the product of the horizon areas can be typically written as a sum between the squares of the angular momentum and some power of the charge [11–19]. In this context, several inequalities between angular momenta, area, electric charge and magnetic fluxes were studied for axisymmetric stably outer marginally trapped surfaces, for EMd theory in 4 dimensions [20], and in 5 dimensions [21].

In this Letter we study relations between area, angular momentum and charge for extremal EM and EMd black holes with equal angular momenta. We construct the global solutions numerically and local solutions in the near horizon formalism. The EM case is special since two different branches of charged extremal solutions exist. One branch emerges from the uncharged MP black holes, and the other branch emerges from the static Tangherlini black holes. The area relations are different on each branch: the first branch

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[1] In this Letter we will consider only asymptotically flat black holes with spherical horizon topology.

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retains the proportionality between the angular momentum and the area of the MP solutions. Thus the area of these charged black holes is independent of the charge. In contrast, the second branch exhibits a proportionality between the angular momentum and the horizon angular momentum, while the charge enters into the area relation yielding $A_{\rm H}^2 = C_1 J^2 Q^{-3/2} + C_2 Q^{3/2}$, where C_1 and C_2 are some constants and Q is the electric charge. However, as soon as the dilaton is coupled, the branch structure changes, and only a single branch – similar to the first branch of the EM case – is found. Again along this branch the proportionality between the angular momentum and the area persists for all extremal solutions. We will proceed by first presenting the $D=5$ results and then discussing their generalization to odd $D>5$ dimensions.

2. 5*D* EMd near horizon solutions

In 5 dimensions, the EMd action can be written as

$$I = \int d^5x \sqrt{-g}\mathcal{L} = \int d^5x \sqrt{-g}\left[R - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{4}e^{-2h\phi}F_{\mu\nu}F^{\mu\nu}\right], \tag{1}$$

where R is the curvature scalar, ϕ the scalar dilaton field, h the dilaton coupling constant and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ the field strength tensor, where A_μ denotes the gauge vector potential. The units have been chosen so that $16\pi G = 1$, G being Newton's constant. If we set $h=0$, the pure EM action is recovered, while $h_{\rm KK} = \sqrt{\frac{2}{3}}$ is the KK value.

For cohomogeneity-1 solutions the isometry group is enhanced from $\mathcal{R}\times U(1)^2$ to $\mathcal{R}\times U(2)$, where $\mathcal{R}$ represents time translations. This symmetry enhancement allows to factorize the angular dependence and thus leads to ordinary differential equations.

Following the near horizon formalism [22,23], we now obtain exact near horizon solutions for these extremal EM and EMd black holes. In terms of the left-invariant 1-forms $\sigma_1 = \cos\psi\, d\bar\theta + \sin\psi \sin\bar\theta\, d\varphi$, $\sigma_2 = -\sin\psi\, d\bar\theta + \cos\psi\sin\bar\theta\, d\varphi$, and $\sigma_3 = d\psi + \cos\bar\theta\, d\varphi$, the near horizon metric can be written as

$$ds^2 = v_1\left(\frac{dr^2}{r^2} - r^2\,dt^2\right) + \frac{v_2}{4}\left(\sigma_1^2+\sigma_2^2\right) + \frac{v_2 v_3}{4}(\sigma_3 + 2kr\,dt)^2, \tag{2}$$

where we have defined $2\theta = \bar\theta$, $\varphi_2 - \varphi_1 = \varphi$, $\varphi_1 + \varphi_2 = \psi$, $\theta \in [0,\pi/2]$, $(\varphi_1,\varphi_2)\in[0,2\pi]$. The horizon is located at $r=0$, which can always be achieved via a transformation $r \to r - r_{\rm H}$. Note, that the metric is written in a co-rotating frame.

The metric corresponds to a rotating squashed $AdS_2\times S^3$ spacetime, describing the neighborhood of the event horizon of an extremal black hole. The corresponding Ansatz for the gauge potential in the co-rotating frame reads

$$A_\mu\,dx^\mu = q_1 r\,dt + q_2\sin^2\theta(d\varphi_1 - kr\,dt) + q_2\cos^2\theta(d\varphi_2 - kr\,dt). \tag{3}$$

The dilaton field is simply given by $\phi = u$. The parameters k, v_i, q_i and u are constants, and satisfy a set of algebraic relations, which can be obtained, according to [22,23], in the following way.

Evaluating the Lagrangian density $\sqrt{-g}\mathcal{L}$ for the near horizon geometry (2) and integrating over the angular coordinates yields the function f,

$$f(k, v_1, v_2, v_3, q_1, q_2, u) = \int d\theta\, d\varphi_1\, d\varphi_2\, \sqrt{-g}\mathcal{L}, \tag{4}$$

from which the field equations follow. In particular, the derivatives of f with respect to the parameters vanish except for

$$\frac{\partial f}{\partial k} = 2J, \qquad \frac{\partial f}{\partial q_1} = Q, \tag{5}$$

where J is the total angular momentum and Q is the charge. From these equations a set of algebraic relations for the near horizon expressions (2), (3) is obtained.

The entropy function is obtained by taking the Legendre transform of the above integral with respect to the parameter k, associated with both angular momenta, $J_1 = J_2 = J$, and with respect to the parameter q_1, associated with the charge Q,

$$\mathcal{E}(J,k,Q,q_1,q_2,v_1,v_2,v_3,u) = 2\pi\big(2Jk + Qq_1 - f(k,v_1,v_2,v_3,q_1,q_2,u)\big). \tag{6}$$

Then the entropy associated with the black holes can be calculated by evaluating this function at the extremum, $S = \mathcal{E}_{extremal}$.

The horizon angular momenta $J_{{\rm H}(k)}$ of the black holes are obtained from the Komar expressions associated with the corresponding Killing vector fields $\eta_{(k)} \equiv \partial_{\varphi_k}$

$$J_{{\rm H}(k)} = \int_{\mathcal{H}} \beta_{(k)}, \tag{7}$$

where $\mathcal{H}$ represents the surface of the horizon and $\beta_{(k)\mu_1\mu_2\mu_3} \equiv \epsilon_{\mu_1\mu_2\mu_3\rho\sigma}\nabla^\rho\eta^\sigma_{(k)}$. Note that $J_{{\rm H}(1)} = J_{{\rm H}(2)} \equiv J_{\rm H}$, since we are considering solutions with equal angular momenta.

For the discussion of the solutions we need to consider the EM and the EMd case separately. In the EM case, i.e. for $h=0$, the system of equations yields two distinct solutions, depending on two parameters. These two solutions of the near horizon geometry have been found independently by Kunduri and Lucietti in [24].[2] Here we now calculate the charges and entropies associated with these two branches. The solution containing the MP limit, and thus the first branch, has $v_2 = 4v_1$, $v_3 = 2 - \frac{q_2^2}{v_1}$, $q_1 = 0$, $k = \frac{1}{2}$ and

$$J = 32\pi^2 v_1\sqrt{2v_1 - q_2^2}, \qquad Q = -32\pi^2 q_2\sqrt{2v_1 - q_2^2},$$
$$S = 2\pi J, \qquad J_{\rm H} = 16\pi^2\left(2v_1 - q_2^2\right)^{3/2}, \tag{8}$$

while the solution containing the Tangherlini limit, and thus the second branch, has $v_2 = 4v_1$, $v_3 = \frac{1}{4k^2+1}$, $q_1 = -\frac{(2k+1)(2k-1)\sqrt{3}}{2}\times\sqrt{\frac{|v_1|}{4k^2+1}}$, $q_2 = -2\sqrt{3}k\sqrt{\frac{|v_1|}{4k^2+1}}$ and

$$J = 128\pi^2 k\left(\frac{|v_1|}{4k^2+1}\right)^{3/2}, \qquad Q = 32\sqrt{3}\pi^2\frac{v_1}{4k^2+1},$$
$$S = 64\pi^3\frac{|v_1|^{3/2}}{\sqrt{4k^2+1}}, \qquad J_{\rm H} = J/4, \tag{9}$$

where $J_{\rm H}$ is the horizon angular momentum. The two solutions match at $k = 1/2$, where $q_1 = 0$. At this critical point the angular momentum can be written as

$$J = \frac{1}{2\sqrt{2\pi}}\frac{1}{3^{3/4}}Q^{3/2}. \tag{10}$$

Thus we have the surprising result that along the first branch, the proportionality of the angular momentum and the area known for the MP black holes, continues to hold in the presence of charge until the critical point is reached. In contrast, on the second branch we have proportionality of the angular momentum and the horizon angular momentum.

[2] We thank Hari Kunduri for pointing this out to us.

In the case of the EMd black holes, only one solution is found. It can be obtained by replacing $q_2 \to q_2 e^{-hu}$, $\bar{Q} = Q\, e^{hu}$ in the first branch solution of the EM case. Hence, as long as $h \neq 0$, the angular momentum and the area are always proportional, independent of h and Q. In particular, this includes the KK case, where the full solution is known analytically.

3. 5*D* EMd black hole solutions

We now need to consider the full solutions, which we obtain by numerical integration. For the metric we employ the parametrization

$$ds^2 = -f\,dt^2 + \frac{m}{f}\left(dr^2 + r^2\,d\theta^2\right) + \frac{n}{f} r^2 \sin^2\theta \left(d\varphi - \frac{w}{r}\,dt\right)^2 + \frac{n}{f} r^2 \cos^2\theta \left(d\psi - \frac{w}{r} dt\right)^2 + \frac{m-n}{f} r^2 \sin^2\theta \cos^2\theta (d\varphi - d\psi)^2, \quad (11)$$

for the gauge potential we use

$$A_\mu\, dx^\mu = a_0\, dt + a_k\left(\sin^2\theta\, d\varphi + \cos^2\theta\, d\psi\right), \quad (12)$$

while the dilaton field is described by the function $\phi(r)$.

The resulting set of coupled ODEs then consists of first order differential equations for a_0 and n, and second order differential equations for f, m, n, ω, a_k and ϕ. The equation for a_0 allows to eliminate this function from the system.

To obtain asymptotically flat solutions, the metric functions should satisfy the following set of boundary conditions at infinity, $f|_{r=\infty} = m|_{r=\infty} = n|_{r=\infty} = 1$, $\omega|_{r=\infty} = 0$. For the gauge potential we choose a gauge such that $a_0|_{r=\infty} = a_\varphi|_{r=\infty} = 0$. For the dilaton field we choose $\phi|_{r=\infty} = 0$, since we can always make a transformation $\phi \to \phi - \phi|_{r=\infty}$.

In isotropic coordinates the horizon is located at $r_H = 0$. An expansion at the horizon yields $f(r) = f_4 r^4 + f_\alpha r^\alpha + o(r^6)$, $m(r) = m_2 r^2 + m_\beta r^\beta + o(r^4)$, $n(r) = n_2 r^2 + n_\gamma r^\gamma + o(r^4)$, $\omega(r) = \omega_1 r + \omega_2 r^2 + o(r^3)$, $a_0(r) = a_{0,0} + a_{0,\lambda} r^\lambda + o(r^2)$, $a_k(r) = a_{k,0} + a_{k,\mu} r^\mu + o(r^2)$, $\phi(r) = \phi_0 + \phi_\nu r^\nu + o(r^2)$. Interestingly, the coefficients α, β, γ, λ, μ and ν are non-integer. Only ω has an integer expansion.

To construct the solutions numerically, we employ a compactified radial coordinate, $x = r/(r+1)$. We then reparametrize the metric in terms of the functions $f = \hat{f} x^2$, $m = \hat{m}$, $n(r) = \hat{n}$, $\omega(r) = \hat{\omega}(1-x)^2$, $a_k = \hat{a}_k / x^2$, and $\phi = \hat{\phi}/x^2$ to properly deal with the non-integer coefficients in the horizon expansion, eliminating possible divergences in the integration of the functions.

We employ a collocation method for boundary-value ordinary differential equations, equipped with an adaptive mesh selection procedure [25]. Typical mesh sizes include 10^3–10^4 points. The solutions have a relative accuracy of 10^{-10}. The estimates of the relative errors of the global charges and other physical quantities are of order 10^{-6}.

Fig. 1 exhibits the ratios J/A_H and J/J_H versus the charge Q/M for extremal 5*D* EM ($h = 0$) and KK ($h = h_{KK}$) black holes. It clearly reveals the two branches of the extremal EM solutions, together with their matching point. This is in contrast to the single branch of the EMd solutions, shown here for the KK case.

We exhibit in Fig. 2 the domain of existence of the EM and EMd black holes for dilaton coupling constants $h = 0$, 2, 0.5 and h_{KK}. Here we display the area $A_H/M^{3/2}$ versus the charge Q/M for extremal and static 5*D* black holes. All black holes of the respective theories can be found within these boundaries. Again we note the different structure for the EM case. The EM static extremal solution has finite area, whereas for $h \neq 0$ the static extremal solution is singular with vanishing area.

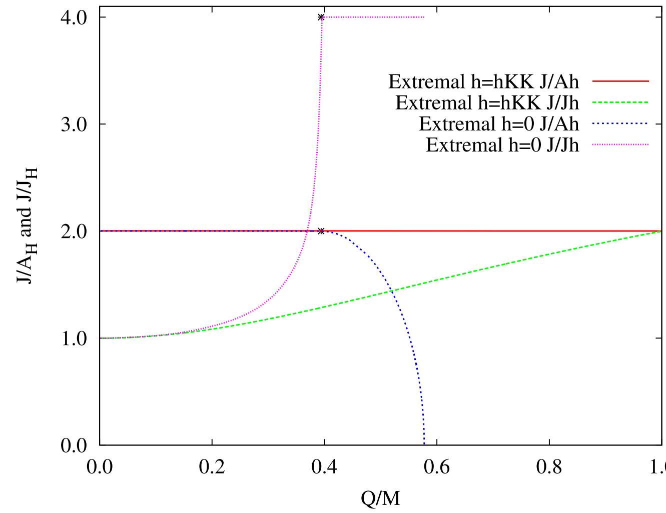

Fig. 1. The ratios J/A_H and J/J_H are shown versus the charge Q/M for extremal 5*D* EM ($h = 0$) and KK ($h = h_{KK}$) black holes. The asterisks mark the matching point of the two EM branches.

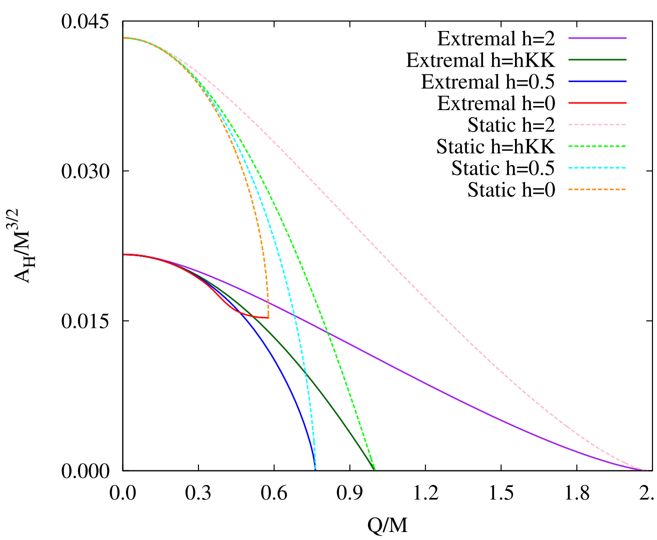

Fig. 2. The area $A_H/M^{3/2}$ is shown versus the charge Q/M for extremal and static 5*D* black holes for $h = 0$, 0.5, h_{KK} and 2.

4. EMd black holes in odd *D* > 5

In a straightforward generalization the near horizon solutions can be constructed for arbitrary odd dimensions $D > 5$. In the EM case we retain two branches of solutions, the MP branch with

$$J = \sqrt{2(D-3)}\,A_H, \quad (13)$$

and the Tangherlini branch with

$$J = (D-1)\,J_H. \quad (14)$$

In the EMd case the near horizon solutions possess only a single branch corresponding to the first branch, with $J = \sqrt{2(D-3)}A_H$.

We have performed the respective set of numerical calculations in 7*D* and in 9*D*, and obtained results that are analogous to the 5*D* case.

5. Comparison with other theories

Let us now compare these results with those of two theories whose extremal black holes also exhibit a branch structure with two distinct branches: the rotating dyonic black holes of 4-dimensional KK theory [26], and the 5-dimensional black holes of Einstein–Maxwell–Chern–Simons (EMCS) theory (minimal $D = 5$ supergravity) [27].

In the first example the 4-dimensional black holes are characterized by their mass M, angular momentum J, electric charge Q and magnetic charge P.[3] In the extremal case, only three of these charges are independent and two distinct surfaces, **S** and **W**, are found. The restriction to $P = Q$ then yields two distinct branches. The **S** branch, $J > PQ$, emerges from the extremal Kerr solution, and presents all the normal characteristics of charged rotating solutions, such as an ergo-region and non-zero angular velocity. On the other hand, the **W** branch, $J < PQ$, possesses no ergo-region and has vanishing horizon angular velocity, although the angular momentum of the black holes along this branch does not vanish. At the matching point of both branches, $J = QP$, the horizon area is zero and the configuration is singular.

Nevertheless, the area-angular momentum relation for these extremal solutions can be written as

$$A_{\rm H}^2 = 64\pi^2 \left| J^2 - Q^2 P^2 \right|. \tag{15}$$

Note, that the electric and magnetic charges are entering the relation for both branches, and that the only difference in the area relation is an overall sign in the expression, depending on whether we are on the **S** ($J > PQ$) or on the **W** ($J < PQ$) branch.

The second example exhibits rather analogous features. Here we consider 5-dimensional black holes in EMCS theory for the supergravity value of the CS coupling constant, $\lambda = 1$ (in an appropriate parametrization). In the extremal case, when both angular momenta possess equal magnitude, the black holes are parametrized by the angular momentum J and the charge Q.[4] Again two branches of extremal black holes are present. The first branch has $J^2 > -\frac{4}{3\sqrt{3}\pi} Q^3$ and is the ordinary branch with an ergo-region, while the second branch has $J^2 < -\frac{4}{3\sqrt{3}\pi} Q^3$ and is ergo-region free with vanishing horizon angular momentum. The area-angular momentum relation for both branches reads

$$A_{\rm H}^2 = 64\pi^2 \left| J^2 + \frac{4}{3\sqrt{3}\pi} Q^3 \right|. \tag{16}$$

At the matching point of both branches the horizon area is again zero and the solution is singular, and again there is a change of sign in the area-angular momentum relation depending on the branch.

Thus in these cases, both charge and angular momentum are entering the area relation. Moreover, the relations (15) and (16) are in accordance with the general expressions obtained in [11], which also depend on both, the charges and the angular momenta.

6. Further remarks

It is interesting to note that for the extremal rotating black holes in EM theory with equal angular momenta, a branch structure with two distinct branches is found, where for one of the branches – the one emerging from the MP solution – the area is independent of the charge of the configuration. Along this branch of solutions, the area remains proportional to the angular momentum and the charge is not entering the relation. This is different from other charged black holes considered before.

However, once the critical extremal EM solution[5] is passed, the charge enters again into the area relation, yielding the expression

$$A_{\rm H} = C_1 J^2 Q^{-3/2} + \frac{1}{16 C_1} Q^{3/2}, \tag{17}$$

where $C_1 = \frac{3^{1/4}\pi}{\sqrt{2}}$ in our normalization.

In contrast to the two branches of global extremal EM black hole solutions, the two branches of EM near horizon solutions do not end at the critical solution. Thus a study of only near horizon solutions is insufficient to clarify the domain of existence of extremal solutions, as was first observed for the extremal dyonic black holes of $D = 4$ Gauß–Bonnet gravity [28].

Interestingly, in the general EMCS theory (with CS coupling constant $\lambda \neq 1$ [29,30]), there appear even more than two branches of extremal black holes for sufficiently large CS coupling [31]. As in the case discussed above, however, the area of these branches of rotating charged black holes always depends on both, the charge and the angular momentum.

Whereas the branch structure of these extremal black holes is very intriguing, their relation with the corresponding near horizon solutions is surprising as well. In particular, a given near horizon solution can correspond to (i) more than one global solution, (ii) precisely one global solution, or (iii) no global solution at all. It would be interesting to perform an analogous study for the general EMd theory (with dilaton coupling constant $h \neq h_{\rm KK}$ [32]), since the analogy between the known black holes of both theories suggests that a similar more complex branch structure would be present for sufficiently large dilaton coupling.

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[3] Note that cohomogeneity-1 is not possible in this theory since it is an even-dimensional one.

[4] The asymmetry in the electric charge of the Chern–Simons term gives rise to different properties for solutions with opposite signs of the electric charge.

[5] Note, that in contrast to the critical solutions of the 4D EMd and 5D EMCS theories discussed above, the critical EM solution is not singular.

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