

Available online at www.sciencedirect.com**ScienceDirect**

Procedia Engineering 144 (2016) 60 – 67

**Procedia
Engineering**www.elsevier.com/locate/procedia

12th International Conference on Vibration Problems, ICOVP 2015

A Parametric Study on Free Vibration of Multi-Perforated Rectangular Plates*

Keya Ghonasgi^a, Kalpit Bakal^b, Kiran D. Mali^{a,b,*}^{a,b}Mechanical Department, BITS-Pilani K.K. Birla Goa campus, Goa, India, 403726

Abstract

This work involves study of the first three natural frequencies of the perforated plates. The effect of the parameters which influence them have been studied. The parameters considered are the shape of perforations, pattern of the perforations, aspect ratio of the plate, dimensions of the plate, ligament efficiency and the mass remnant ratio (MRR). The study is focused on the effect of the most influencing parameter on the free vibrations.

© 2016 Published by Elsevier Ltd. This is an open access article under the CC BY-NC-ND license (<http://creativecommons.org/licenses/by-nc-nd/4.0/>).

Peer-review under responsibility of the organizing committee of ICOVP 2015

Keywords: Mass Remnant Ratio (MRR); Ligament Efficiency; Aspect Ratio; Perforated Plates; Natural Frequencies

1. Introduction

Perforated metal plates are used for various applications like agriculture, and other applications involving sieving, noise control, heat exchangers, and pressure vessels and so on. Most of these processes involve the vibration of these plates. It is important to study the variation of the properties of the plates during their use. Warburton [1] in his paper 'The Vibration of Rectangular plates' worked on rectangular plates with all the possible combinations of boundary conditions. The frequency is derived by using the frequency expression for all modes of vibration and this expression is derived from the Rayleigh method. Suhm Choi et al [2] performed finite element

Corresponding author. E-mail address: kiranm@goa.bits-pilani.ac.in

modal analysis of perforated plates with square and triangular hole patterns. Dawe [3] computed the natural frequencies of vibration of flat plates having arbitrary shape where the plate is considered as assemblage of elements. Lim and Liew [4] performed a free vibration study of perforated plates with rounded corners. Guminiak [5] studied free vibration analysis of Kirchoff plates by using Boundary Element Method (BEM). Cuenca [6] developed a model for predicting the flexural vibrations of thin polygonal plates in the medium and high frequency range. Mali and Singru formulated analytical models for perforated rectangular plates by using different approaches to represent perforations such as Greatest Integer function, Heaviside functions and negative mass approach [6, 7, 8, 9, 10]. They have also given detailed literature related to the perforated plate vibration. This work aims at understanding the variation of the first three natural frequencies with the variation of some specific parameters. The main parameters which have been considered in this work are, the effects of the mass remnant ratio (MRR), the ligament efficiency, the pattern of the holes and the aspect ratio. The plates considered are thin with the boundary condition, all edges clamped. Three perforation patterns have been considered i.e. the Rectangular pattern, the 45° staggered pattern and the 60° staggered pattern. The thickness of the plate is maintained at 0.002 m for all the simulations that have been considered.

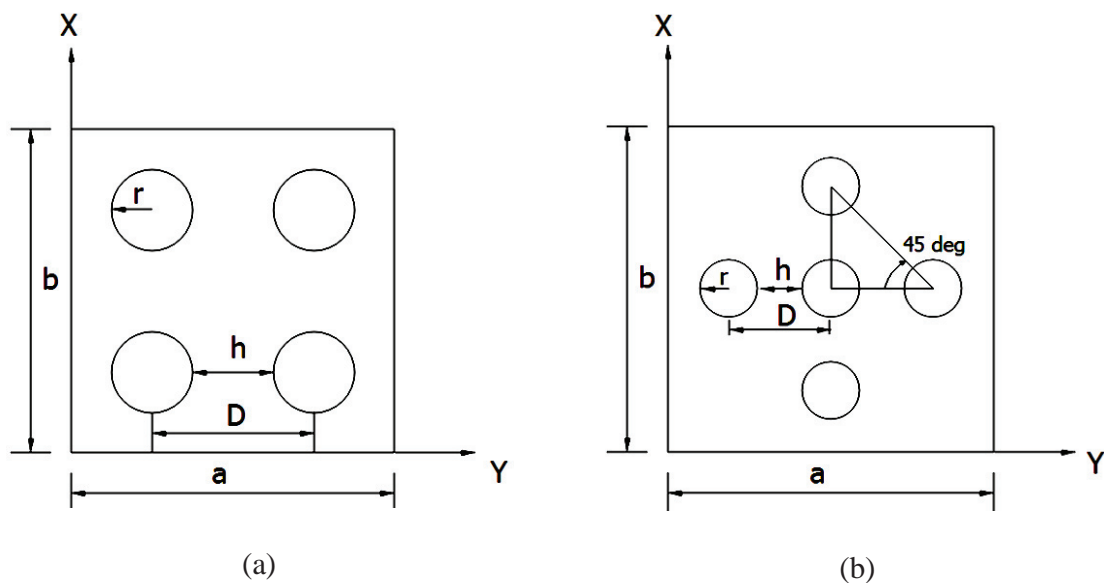


Fig. 1. Description of measures as used for calculation of Ligament efficiency a). Rectangular pattern; b). Triangular pattern

2. Methodology

The Finite Element Analysis has been carried out. The simulations have been conducted using ANSYS 15 APDL, and the element used for this study is Shell63^[1]. The material properties of the mild steel^[2] considered are, the density 7850 kg/m³, the Poisson's ratio 0.3, and the Modulus of elasticity 2.1×10^{11} . The plate has been modelled and the various specimens with different parameter variation have been simulated using ANSYS APDL. Analysis is carried out for the following cases of perforated plate. As the ligament efficiency cannot be kept the same for a plate in the 60° staggered pattern (as the 60° constraint does not allow it), this pattern is considered only for the first case.

1. Constant MRR, no constraints on ligament efficiency and mutual comparison of all 3 patterns.
2. Constant MRR, constant ligament efficiency and varying the aspect ratio for the rectangular and 45° pattern. The 60° pattern cannot be considered.

3. Constant MRR, constant ligament efficiency and varying the dimensions keeping the aspect ratio same for the rectangular and 45° pattern. The 60° pattern cannot be considered.
4. Constant MRR and varying ligament efficiency such as 0.6, 0.68 and 0.75 for the rectangular and 45° pattern. The 60° pattern cannot be considered.

3. Results and Discussions

3.1. Constant MRR, no constraints on ligament efficiency and mutual comparison of the three patterns

Table 1. Results for constant MRR = 0.95, no constraints on ligament efficiency and comparison with 60° triangular pattern for circular holes.
Plate size 1×1

Sr. no.	Pattern	Hole radius (m)	ω_1 (Hz)	ω_2 (Hz)	ω_3 (Hz)	ω_4 (Hz)	El size
1	Rectangular 90°	0.06	17.843	36.533	36.534	54.342	0.01
2	Triangular 45°	0.054	18.084	36.07	36.071	53.049	0.01
3	Triangular 60°	0.045	18.015	35.816	35.929	52.81	0.01

As it is clearly seen in the Table 1) that the lowest first natural frequency is of Rectangular 90° pattern and the highest is of Triangular 45°. The other frequencies show absurd results in the forthcoming analyses. That is the reason they are not being considered. The two patterns having the highest and lowest first natural frequencies are considered for analysis.

3.2. Constant MRR and Ligament Efficiency, varying Aspect Ratio

Table 2. Results for both patterns for varying aspect ratio and ligament efficiency 0.68 for circular holes.

Sr. no.	Pattern	Ligament Efficiency	Radius of the hole (m)	ω_1 (Hz)	ω_2 (Hz)	ω_4 (Hz)	El. Size
Plate Dimensions (m×m) : 0.5×0.5, Aspect Ratio : 1							
1	Rectangular	0.68	0.03	72.025	144.51	214.91	0.005
	Triangular 45°			71.387	145.98	211.15	
	Difference			0.638	-1.47	3.76	
Plate Dimensions (m×m) : 0.5×0.4, Aspect Ratio : 1.25							
2	Rectangular	0.68	0.025	94.039	160.92	209.89	0.005
	Triangular 45°			93.267	161.16	212.41	
	Difference			0.772	-0.24	-2.52	
Plate Dimensions (m×m) : 0.5×0.333, Aspect Ratio: 1.5							
3	Rectangular	0.68	0.023	158.17	215.15	316.12	0.005
	Triangular 45°			151.02	213.75	318.22	
	Difference			7.15	1.4	-2.1	
Plate Dimensions (m×m) : 0.5×0.286, Aspect ratio : 1.75							
4	Rectangular	0.68	0.02133	122.49	184.46	290.17	0.005
	Triangular 45°			119.82	184.14	292.95	
	Difference			2.67	0.32	-2.78	
Plate Dimensions (m×m) : 0.5×0.25, Aspect Ratio : 2							
5	Rectangular	0.68	0.02	199.28	251.79	348.1	0.005
	Triangular 45°			188.3	251.05	349.37	
	Difference			10.98	0.74	-1.27	

Table 3. Results for both patterns for varying aspect ratio and ligament efficiency 0.68 for square holes.

Sr. no.	Pattern	Ligament efficiency	Dimension of the hole (m×m)	ω_1 (Hz)	ω_2 (Hz)	ω_3 (Hz)	Element size
Plate Dimensions (m×m) : 0.5×0.5, Aspect Ratio : 1							
1	Rectangular	0.68	0.053×0.053	71.871	143.05	143.05	0.01
	Triangular 45°		0.047×0.047	71.727	144.41	144.41	
	Difference			0.144	-1.36	-1.36	
Plate Dimensions (m×m) : 0.5×0.4, Aspect Ratio : 1.25							
2	Rectangular	0.68	0.05×0.05	93.924	159.79	208.04	0.01
	Triangular 45°		0.0447×0.0447	93.657	159.83	210.9	
	Difference			0.267	-0.04	-2.86	
Plate Dimensions (m×m) : 0.5×0.333, Aspect Ratio: 1.5							
3	Rectangular	0.68	0.0456×0.0456	122.97	183.79	289.47	0.01
	Triangular 45°		0.0408×0.0408	120.99	182.86	292.06	
	Difference			1.98	0.93	-2.59	
Plate Dimensions (m×m) : 0.5×0.286, Aspect ratio : 1.75							
4	Rectangular	0.68	0.0423×0.0423	157.85	213.67	315.29	0.01
	Triangular 45°		0.0377×0.0377	153.23	212.51	318.57	
	Difference			4.62	1.16	-3.28	
Plate Dimensions (m×m) : 0.5×0.25, Aspect Ratio : 2							
5	Rectangular	0.68	0.0395×0.0395	199.57	250.65	348.07	0.01
	Triangular 45°		0.0354×0.0354	191.2	249.75	350.72	
	Difference			8.37	0.9	-2.65	

The tables 2 and 3 show how the numerical difference between the results of the two patterns changed with change in aspect ratio (a/b) for circular and square holes respectively. It is observed that as the aspect ratio increases from 1, the difference between the first natural frequency increases. For the table with Aspect Ratio 1, the value of ω_2 and ω_3 are the same. Hence the value considered here in place of ω_3 is that of the fourth frequency, i.e. ω_4 .

3.3 Constant MRR, constant ligament efficiency (0.68), constant aspect ratio varying the dimensions.

Table 4. Results for constant MRR, constant ligament efficiency and varying the dimensions keeping the aspect ratio same for circular holes.

Sr. No	Pattern	Dimension of the hole(m)	ω_1 (Hz)	ω_2 (Hz)	ω_4 (Hz)	element size
Plate Dimensions (m×m) : 0.1×0.1						
1	Rectangular	0.006	1801.6	3616.1	3760.7	0.002
	Triangular 45°	0.00536	1783.5	3640.6	5252.6	
	Difference		18.1	-24.5	124.1	
Plate Dimensions (m×m) : 0.2×0.2						
2	Rectangular	0.012	450.39	904.02	1344.2	0.004
	Triangular 45°	0.01072	445.89	910.14	1313.2	
	Difference		4.5	6.12	31	
Plate Dimensions (m×m) : 0.3×0.3						
3	Rectangular	0.018	200.18	401.8	56.44	0.006
	Triangular 45°	0.01608	198.18	404.5	583.61	
	Difference		2	-2.7	-4.17	
Plate Dimensions (m×m) : 0.4×0.4						
4	Rectangular	0.024	112.6	226	336.04	0.008
	Triangular 45°	0.02144	111.47	227.54	328.29	
	Difference		1.13	-1.54	7.75	
Plate Dimensions (m×m) : 0.5×0.5						
5	Rectangular	0.03	72.064	144.65	215.07	0.01
	Triangular 45°	0.0268	71.34	145.61	210.1	
	Difference		0.724	-0.96	4.97	

Table 5. Results for constant MRR, constant ligament efficiency (0.68) and varying the dimensions keeping the aspect ratio same for square holes.

Sr. no.	Pattern	Dimension of the hole (m×m)	ω_1 (Hz)	ω_2 (Hz)	ω_4 (Hz)	Element size
Plate Dimensions (m×m) : 0.1×0.1						
1	Rectangular	0.118×0.118	1799.8	3591.2	5344	0.002
	Triangular 45°	0.01×0.01	1788.7	3616.9	5229.2	
	Difference		11.1	-25.7	114.8	
Plate Dimensions (m×m) : 0.2×0.2						
2	Rectangular	0.022361×0.022361	449.96	897.79	1336	0.004
	Triangular 45°	0.02×0.02	447.18	904.22	1307.3	
	Difference		2.78	-6.43	28.7	
Plate Dimensions (m×m) : 0.3×0.3						
3	Rectangular	0.033541×0.033541	199.99	398.97	593.71	0.006
	Triangular 45°	0.03×0.03	198.75	401.87	581.02	
	Difference		1.24	-2.9	12.69	
Plate Dimensions (m×m) : 0.4×0.4						
4	Rectangular	0.044721×0.044721	112.49	224.45	334	0.008
	Triangular 45°	0.04×0.04	111.8	226.05	326.83	
	Difference		0.69	-1.6	7.17	
Plate Dimensions (m×m) : 0.5×0.5						
5	Rectangular	0.053×0.053	71.952	143.29	210.3	0.01
	Triangular 45°	0.047×0.047	71.838	144.63	208.3	
	Difference		0.569	-1.34	2	

Tables 4 and 5 show the variation of dimensions of the plate by keeping the other parameters same for circular holes and square holes respectively. The elements size is varied along with the plate dimension. It is observed that as the plate dimensions are decreased the difference between the first frequency increases.

3.3. Constant MRR, aspect ratio and dimensions, varying Ligament Efficiency.

Table 6) Result for constant MRR and varying ligament efficiency for circular holes (Plate dimensions 0.5×0.5).

Sr no.	Pattern	Ligament efficiency	Radius of hole (m)	ω_1 (Hz)	ω_2 (Hz)	ω_4 (Hz)	Element size
1	Rectangular	0.6	0.03	72.186	143.91	214.14	0.01
	Triangular 45°		0.0268	72.265	144.64	211.32	
	Difference			-0.079	-0.73	2.82	
2	Rectangular	0.68	0.03	72.064	144.65	215.07	0.01
	Triangular 45°		0.0268	71.47	145.84	210.21	
	Difference			0.594	-1.19	4.86	
3	Rectangular	0.75	0.03	71.859	145.25	216.13	0.01
	Triangular 45°		0.0268	67.839	142.03	210.81	
	Difference			4.02	3.22	5.32	

Table 7) Result for constant MRR and varying ligament efficiency for square holes (Plate dimensions 0.5×0.5).

Sr. no.	Pattern	Ligament efficiency	Dimension of hole (m ×m)	ω_1 (Hz)	ω_2 (Hz)	ω_4 (Hz)	Element size
1	Rectangular	0.6	0.0559×0.0559	72.143	142.33	211.79	0.01
	Triangular 45°		0.05×0.05	72.414	142.38	210.55	
	Difference			-0.271	-0.05	1.24	
2	Rectangular	0.68	0.0559×0.0559	71.991	143.65	213.78	0.01
	Triangular 45°		0.05×0.05	72.174	143.69	210.22	
	Difference			-0.183	-0.04	3.56	
3	Rectangular	0.75	0.0559×0.0559	71.543	145.43	216.27	0.01
	Triangular 45°		0.05×0.05	70.514	144.61	208.61	
	Difference			1.029	0.82	7.66	

Table 6 and 7 show the variation of frequencies with the ligament efficiency. It clearly shows that as the ligament

efficiency is increased the difference between the first frequencies of the patterns also increases. The first frequency of rectangular 90° pattern is seen to be greater than triangular 45° pattern only after the ligament efficiency is seen to cross 0.68.

4. Conclusion

This study deals with vibration of rectangular perforated plates. Different perforation patterns are considered in this study and their effect on first three natural frequencies is examined. In almost all the cases the rectangular 90° pattern has a higher first natural frequency than the triangular 45° pattern. The most influencing parameter is the aspect ratio. There are observable differences in the frequency of rectangular 90° pattern and triangular 45° pattern as the aspect ratio is increased.

Appendix A. No. of Nodes and elements for converged values as shown in above tables.

Table 8. Results for constant MRR = 0.95, no constraints on ligament efficiency and comparison with 60° triangular pattern for circular holes.

Pattern	El size	No. of nodes	No. of elements
Rectangular 90°	0.01	9926	9656
Triangular 45°	0.01	9965	9689
Triangular 60°	0.01	10051	9750

Table 9. Results for both patterns for varying aspect ratio and ligament efficiency 0.6 for circular holes.

Pattern	Ligament Efficiency	El. Size	No. of nodes	No. of elements	Aspect ratio
Rectangular	0.6	0.005	5117	4911	2
Triangular 45°			4792	5001	
Rectangular			5834	5618	1.75
Triangular 45°			5569	5785	
Rectangular			6686	6454	1.5
Triangular 45°			6453	6690	
Rectangular	7882	7639	1.25		
Triangular 45°	7653	7903			

Table 10. Results for both patterns varying aspect ratio and ligament efficiency

Sr. no.	Pattern	Ligament efficiency	Dimension of the hole (m × m)	Element size	No. of nodes	No. of elements	Aspect ratio	Dimension of the plate
1	Rectangular	0.6	0.05×0.05	0.01	2115	1991	1.25	0.5×0.4
	Triangular 45°		0.0447×0.0447		2150	2017		
2	Rectangular		0.0456×0.0456		1782	1664	1.5	0.5×0.333
	Triangular 45°		0.0408×0.0408		1822	1695		
3	Rectangular		0.0423×0.0423		1591	1477	1.75	0.5×0.286
	Triangular 45°		0.0377×0.0377		1563	1451		
4	Rectangular	0.0395×0.0395	1321	1218	2	0.5×0.25		
	Triangular 45°	0.0354×0.0354	1383	1273				

Table 11. Results for constant MRR, constant ligament efficiency and varying the dimensions keeping the aspect ratio same for circular holes.

Sr. No	Dimension of the plate (m × m)	Pattern	No. of nodes	No. of elements	element size
1	0.1×0.1	Rectangular	2631	2490	0.002
		Triangular 45°	2600	2457	
2	0.2×0.2	Rectangular	2637	2485	0.004
		Triangular 45°	2596	2455	
3	0.3×0.3	Rectangular	2570	2426	0.006
		Triangular 45°	2539	2396	
4	0.4×0.4	Rectangular	2642	2501	0.008
		Triangular 45°	2611	2471	
5	0.5×0.5	Rectangular	2594	2450	0.01
		Triangular 45°	2563	2420	

Table 12. Results for constant MRR, constant ligament efficiency (0.68) and varying the dimensions keeping the aspect ratio same for square holes.

Sr. no.	Pattern	Dimension of the plate (m × m)	Dimension of the hole (m × m)	Element size	No. of nodes	No. of elements
1	Rectangular	0.1×0.1	0.118×0.118	0.002	2636	2496
	Triangular 45°		0.01×0.01		2605	2463
2	Rectangular	0.2×0.2	0.022361×0.022361	0.004	2642	2500
	Triangular 45°		0.02×0.02		2605	2463
3	Rectangular	0.3×0.3	0.033541×0.033541	0.006	2651	2511
	Triangular 45°		0.03×0.03		2594	2451
4	Rectangular	0.4×0.4	0.044721×0.044721	0.008	2643	2501
	Triangular 45°		0.04×0.04		2605	2463

Table 13. Result for constant MRR and varying ligament efficiency for circular holes.

Sr no.	Pattern	Ligament efficiency	Radius of hole (m)	Element size	No. of nodes	No. of elements
1	Rectangular	0.6	0.03	0.01	2527	2391
	Triangular 45°		0.0268		2533	2391
2	Rectangular	0.68	0.03	0.01	2522	2387
	Triangular 45°		0.0268		1212	1121
3	Rectangular	0.75	0.03	0.01	2527	2432
	Triangular 45°		0.0268		2619	2477

Table 14. Result for constant MRR and varying ligament efficiency for square holes.

Sr. no.	Pattern	Dimension of hole (m × m)	Element size	No. of nodes	No. of elements
1	Rectangular	0.0559×0.0559	0.01	2636	2496
	Triangular 45°	0.05×0.05		2566	2424
2	Rectangular	0.0559×0.0559	0.01	2642	2501
	Triangular 45°	0.05×0.05		2637	2499
3	Rectangular	0.0559×0.0559	0.01	2521	2422
	Triangular 45°	0.05×0.05		2607	2466

References

- [1] G.B. Warburton, The Vibration of Rectangular Plates, *Proceedings of the Institution of Mechanical Engineers*. 168 (1954) 371-384.
- [2] Suhn Choi, Kyeong Hoon Jeong, TaeWan Kim, KangSoo Kim, KeunBae Park, Free vibration analysis of perforated plates using equivalent elastic properties, *Journal of the Korean Nuclear Society*. 30 (1998) 416-423.
- [3] D.J. Dawe, A finite element approach to plate vibration problems, *Journal of Mechanical Engineering*. 7 (1965) 28-32.
- [4] C.W. Lim, K.M. Liew, Vibration of perforated plates with rounded corners, *Journal of Engineering Mechanics*. 121 (1995) 203-213.
- [5] M. Guminiak, Free vibration analysis of thin plates by the boundary element method in non- singular approach, *Scientific research of the Institute of Mathematics and Computer Science*. 6 (2007) 75-90.
- [6] K.D. Mali, P.M. Singru, An analytical model to determine fundamental frequency of free vibration of perforated plate by using greatest integer functions to express non homogeneity, *Advanced Materials Research*. 622 (2013) 600-604.
- [7] K.D. Mali, P.M. Singru, Analytical model to determine fundamental frequency of free vibration of perforated plate by using unit step functions to express non-homogeneity, *Journal of Vibroengineering*. 14 (2012) 1292-1298.
- [8] K.D. Mali, P.M. Singru, An analytical model to determine fundamental frequency of rectangular plate having rectangular array of circular perforations, *Journal of Vibroengineering*. 15 (2013) 588-596.
- [9] K.D. Mali, P.M. Singru, On the Use of Greatest Integer Function to Express Material Property Variation in Free Vibration Problem of Simply Supported Square Plates with Square Array of Circular Perforations, *Procedia Materials Science*. 6 (2014) 409-416.
- [10] K.D. Mali, P.M. Singru, Determination of the fundamental frequency of perforated rectangular plates: concentrated negative mass approach for the perforation, *Advances in Acoustics and Vibration* 2013 (2013).