Continuous Transportation Network Design Problem Based on Bi-level Programming Model

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Abstract

A bi-level programming model was widely used to describe the continuous transportation network design problem. The objective function in upper-level model was to make the sum of total impedance and total investment budget minimal in a traffic network. The lower-level model was a user equilibrium assignment model with fixed demand. In order to make the results more realistic, this paper proposed an improved model with stochastic transportation network design, and presented genetic algorithm and Frank-Wolfe (FW) algorithm to find optimal solution. Then a specific network example was given to prove the effectiveness of the model. The results show that an optimal value of existing links capacities for the formulation can be obtained and it can minimize the sum of total impedance and investment budget, which can provide a reasonable reference for the decision-makers.

Keywords: continuous transportation network design; bi-level programming model; genetic algorithm; Frank-Wolfe algorithm

1. Introduction

In recent decades, with the high speed development of social economy and urbanization, the number of motor vehicles has increased sharply, which leads to worse urban road congestion. The existing road capacity has been difficult to satisfy the growing traffic demand, hindering economic development seriously. Many cities began to pay more and more attention to traffic problems. Traffic authorities increase investment in transport infrastructure, build new roads, reconstruct the existing roads to enhance capacities, alleviate traffic pressure and improve traffic condition. Although the traffic congestion has been relieved in recent decades, because of the lack of advanced scientific theory, we still cannot achieve the expected effect. The studies of Zhang H Z [1] have shown that unreasonable road traffic planning methods will likely result in weird phenomena, such as a reduction of traffic network spare capacity, worse road congestion, more serious traffic pollution and so on.

So how to meet people’s needs and optimize the system performance of the entire transport network by building new links or enhancing the capacities of existing links, changing network of institutions to adapt to these changes
under limited expenditure is an interesting problem. This problem is known as the Network Design Problem (NDP).

NDP is usually divided into three forms: one is enhancing capacities of the existing roads, called Continuous Network Design Problem (CNDP); the second is building new roads in the network, namely Discrete Network Design Problem (DNDP); another form is both continuous and discrete, namely Mixed Network Design Problem (MNDP), which can improve existing roads and building new roads at the same time. MNDP is considered to be more effective in solving practical problems, but its solution process is also more complicated.

2. The Bi-level CNDP Model

In urban traffic network system, after being provided with transport facilities, travelers will choose their own trip mode according to current traffic condition. This involves the interaction of government and public, so bi-level programming model is more suitable to solve the urban road network design problem. The upper-level model provided information to the lower-level model according to their own situation, and the lower model will make a choice according to their own preferences after getting information. And then the lower model feedback decisions to the upper, the upper model will make adjustment according to decisions to obtain global optimal decision scheme. Bi-level programming not only considers their own local interests, but also considering the overall interests, which is its advantages and characteristics.

Bi-level programming model [2] gave a description of the continuous network design problem. On the one hand, in terms of travelers, they will always choose the route which can minimize their travel cost when choosing a route, therefore behavior of network users’ route choice are consistent with the user-optimal; on the other hand, from the decision-makers' points, they are aimed at minimizing the sum of total impedance and total investment budget in a traffic network. The model is shown as follows:

\[
(U) \quad \min Z(x, y) = \sum_{a \in A} x_a(y) t_a(x_a, y_a) + \theta \sum_{a \in A} g_a(y_a)
\]

\[s.t. \quad y_a \leq \bar{y}_a \leq \overline{\bar{y}}_a, \forall a \in A\]  

\[
(L) \quad \min z(x, y) = \sum_{a \in A} \int_0^{y_a(y)} t_a(\omega, y_a) d\omega
\]

\[s.t. \quad \sum_{p \in P_{rs}} f_{rs}^p = q_{rs}, \forall r \in R, s \in S\]  

\[x_a = \sum_{y} \sum_{p} f_{rs}^p \delta_{a,p}^{rs}, \forall a \in A\]  

\[f_{rs}^p \geq 0, \forall r \in R, s \in S, p \in P_{rs}\]

In order to simplify the description, the notions used in this study are given following: \(N\) are set of nodes and \(A\) are set of links in the network, \(a \in A \). \(R\) is the set of starting points, \(r \in R \). \(S\) is the set of ending points, \(s \in S \). \(P_{rs}\) is the set of routes connecting all of the \(r\) and \(s \). \(x_a\) is the traffic flow on link \(a\) (lower-level decision variables). \(f_{rs}^p\) is the path flow of path \(p \). \(q_{rs}\) is the traffic demand between \(r\) and \(s \), \(q\) is an OD demand matrix. Where \(\delta_{a,p}^{rs} = 1\) if link \(a\) is used by path \(r\), and \(\delta_{a,p}^{rs} = 0\) otherwise. \(y_a\) is the capacity enhancements of link \(a\) (upper-level decision variables). \(g_a(y_a)\) is the expanding cost function of link \(a \). \(t_a(x_a, y_a)\) is the travel time of link \(a \), which depends on the value of link flow \(x_a\) and link capacity enhancement \(y_a \). \(\theta\) is a conversion coefficient, which is used to finish the conversion of units of expanding cost and travel cost.
Formula (1) is the objective function of the network design problem. Traffic planners choose to expand a few roads in the network to enhance capacities to make the total system impedance minimal with the least investment. $\theta$ is generally 1 or 1.5, and $g_a(y_a)$ generally is calculated from $g_a(y_a) = d_a \cdot y_a$ or $g_a(y_a) = d_a \cdot (y_a)^2$; (2) means that the capacity enhancement should be non-negative, and its range should meet the boundary constraint condition.

Formula (4) shows that sum of traffic flow of each path equals to the total traffic demand between OD pairs; (5) shows the relationship between traffic flow on links and paths. (6) can ensure that the path flow can meet the non-negative constraint condition.

3. Model Improvement

The model mentioned above is the continuous transportation network design model based on a fixed demand, however, Bian C Z et al [3] mentioned that in the actual network, traffic demand may not be represented by a precise value because of many uncertain factors. For example, future travel demand is often assumed to be accurate, but due to the uncertainty, it can only be roughly estimated. Therefore, the following OD traffic demand is assumed to be random variables having a certain probability distribution, and establish a continuous transportation network design model with stochastic demand to get the optimal solution of network design problems.

The uncertainty has attracted many researchers to propose different NDP models under stochastic demand, including Chance-constrained Model, Mean-Variance Model, Probability Model, etc. Further description about the model can be seen in the literature of Anthony Chen et al. [4, 5]. The expected value model was used in this paper to describe the uncertainty.

The expected value model (EVM) is referred to as the most widely used model in solving uncertainty problems. In EVM, the object function is the expected value of the minimum of total investment and impedance in the network, so the upper model can be improved as follows based on mentioned above.

\[
(U) \quad \min_y E[Z(x(y, Q), y)]
\]  

\[
s.t. \quad y_a \leq y_a \leq \bar{y}_a, a \in A
\]

In the formula, $Z(x, y) = \sum_{a \in A} x_a(y)Q(x_a(y, Q), y_a) + \theta \sum_{a \in A} g_a(y_a)$

It is assumed that OD traffic demand is a random number demand having a normal distribution. It can be produced by random sampling. The lower-level model is still the standard UE assignment model.

In order to get random OD traffic demand, Monte Carlo simulation method can be used to generate random numbers. Monte Carlo is a stochastic simulation method based on the law of large numbers. It will link the problem to a certain probability model and realize sampling or statistical simulation with computer to obtain approximate solutions of the problems [6].

In actual operation, probability distribution of OD traffic demand is calculation input. For example, OD traffic demand $q_{rs}$ is obeying normal distribution $N(q_{rs}, \sigma_{rs})$, among which, $q_{rs}$ is an average and $\sigma_{rs}$ is a variance. Also, it can be assumed that traffic demand $q_{rs}$ having a uniform distribution $U(q_{rs}^{min}, q_{rs}^{max})$, among which, $q_{rs}^{min}$ and $q_{rs}^{max}$ are upper bound and lower bound of forecasted traffic demand respectively.

4. Algorithm

In this paper, genetic algorithm is used to obtain the optimal capacity enhancements. It has achieved significant results in solving complex global optimization problems.

Professor Holland [7] first proposed genetic algorithm in 1975, which is a method of achieving the best solution by simulating the process of natural evolution. It conforms to Darwin’s theory about evolution: survival of the fittest, and continually generates new group to make the population keep evolving, and eventually gets the optimal solution. The procedure of genetic algorithm was summarized as follows [8]: firstly, a certain amount of chromosomes are
randomly generated, which form a population. Secondly, evaluate the fitness of each chromosome and select as the value of the fitness. The purpose of selection is to select individuals with good genes in the current population and form a new population. Then, update the new population with crossover and mutation to generate new offspring. Repeat the above process until the best chromosome can generated after a certain number of iterations, which is the optimal solution of the problem.

The detailed algorithm steps are shown below:

- Step 0: Set of input parameters: maximum number of generations, population size, crossover rates, mutation rates and maximum number of random OD sampling.
- Step 1: For each OD demand generated by the Monte-Carlo simulations, calculate the value of upper-level object function with FW algorithm. Repeat generating OD demand, until the maximum number of random OD sampling is reached.
- Step 2: Update.
  - Step 2.1: Calculate the fitness values of all chromosomes based on the value of object function.
  - Step 2.2: Select the superior chromosomes according to the fitness values. The maximum fitness value will have the lowest chance to be selected, and the minimum fitness value will have the highest chance.
  - Step 2.3: Crossover and mutation are carried on to update the chromosomes.
- Step 3: Convergence test. Check if the convergence conditions are satisfied. If yes, stops; otherwise, go to step 1.

FW algorithm was used in 1956 [9] to solve the lower UE model. In following, the specific algorithm is shown:

- Step 0: Initialization. Let travel time \( t_a^0 = t_a(0), \forall a \), and then use all-or-nothing traffic assignment to get link flow \( \{x_a^1\} \). Let \( k = 1 \);
- Step 1: Update. Calculate \( t_a^k = t_a(x_a^k), \forall a \);
- Step 2: Determining direction. Use all-or-nothing traffic assignment on the basis of \( \{t_a^k\} \) to get additional link flow \( \{y_a^k\} \);
- Step 3: Move size. Solve \( \min_{0 \leq x \leq 1} Z(x^k + \lambda(y^k - x^k)) = \sum_a \left[ \int_t^t + \lambda(y_a^k - x_a^k) t_a(\omega)d\omega \right] \) to get optimal move size \( \lambda_a^k \);
- Step 4: Move. Let \( x_a^{k+1} = x^k + \lambda_a^k(y^k - x^k) \), \( \forall a \);
- Step 5: Convergence test. If \( \sqrt{\sum_a (x_a^{n+1} - x_a^n)^2} / \sqrt{\sum_a x_a^n} < \epsilon \) (\( \epsilon \) has been given before), stops, otherwise, let \( k = k + 1 \), and return to step 1.

5. Numerical Results

The network in Fig. 1 is used to enhance the efficiency of the algorithm. The network consists of 4 starting points (1, 2, 3, 4), 3 ending points (7, 8, 9), 14 links. OD traffic demand is assumed to have a normal distribution, average \( \mu \) is 150 and variance \( \sigma^2 \) is 30.

The travel time function is \( t_a(x_a, y_a) = A + B(x_a + 1 + y_a)^2 \); object function is \( Z(x, y) = \sum_{a \in A} x_a(y) + 0.5(y_a^2 + 0.5(y_a^3)) \).

Parameters A, B, K are shown in table 1.

The following parameters were used in this paper: population size is set to be 40, the maximum number of generations is 100, crossover probability is 0.8, mutation probability is 0.1, the number of OD sampling is 50.

Matlab was used in this paper to program the FW algorithm and genetic algorithm. Results are shown in Table 2. From the results, it can be obtained that what value the capacity enhancement of link is, the sum of total impedance and total investment budget in the network can be minimal, so that the performance of the transport network can be in best state, which can provide a reasonable plan for the decision-makers in government. It is also proved that FW algorithm and genetic algorithm are feasible to solve the problem.
6. Conclusions

This paper presented a continuous network design problem based on stochastic demand. And a bi-level model for the formulation was proposed. The bi-level model was solved by genetic algorithm, and FW algorithm was used for the lower-level UE model to find the optimal results.

Links, with different travel time, need to expand with different methods. This quantitative analysis formulation can provide scientific basis for network design problem, and we can get proper optimal solution, where the sum of impedance and investment budget can be minimal.

Continuous transportation network design problem is more complex, so the methods and results in this paper have limitations. For further research, we can study other algorithms to solve this model and compare their efficiency. The FW algorithm is not fast enough for solving the continuous network design problem, so the move
size and descent direction should be improved.

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