Structure of mass gap between two spin multiplets

Takayuki Matsuki [a,*], Toshiyuki Morii [b], Kazutaka Sudoh [c]

[a] *Tokyo Kasei University, 1-18-1 Kaga, Itabashi, Tokyo 173-8602, Japan*
[b] *Graduate School of Science and Technology, Kobe University, Nada, Kobe 657-8501, Japan*
[c] *Institute of Particle and Nuclear Studies, High Energy Accelerator Research Organization, 1-1 Ooho, Tsukuba, Ibaraki 305-0801, Japan*

Abstract

Studying our semirelativistic potential model and the numerical results, which succeeds in predicting and reproducing recently discovered higher resonances of D, D_s, B, and B_s, we find a simple expression for the mass gap between two spin multiplets of heavy-light mesons, $(0^-, 1^-)$ and $(0^+, 1^+)$. The mass gap between chiral partners defined by $\Delta M = M(0^+) - M(0^-)$ and/or $M(1^+) - M(1^-)$ is given by $\Delta M = M(0^+) - M(0^-) = M(1^+) - M(1^-) \approx \Lambda_Q - m_q$ in the limit of heavy quark symmetry, and including $1/m_Q$ corrections, we have $\Delta M \approx \Lambda_Q - m_q + (1.28 \times 10^5 + 4.26 \times 10^2 \cdot m_q)/m_Q$ with $\Lambda_Q \approx 300$ MeV, a light quark mass m_q, and a heavy quark mass m_Q. This equation holds both for D and D_s heavy mesons. Our model calculations for the B and B_s also follow this formula.
© 2007 Elsevier B.V. All rights reserved.

PACS: 12.39.Hg; 12.39.Pn; 12.40.Yx; 14.40.Lb; 14.40.Nd

Keywords: Potential model; Spectroscopy; Heavy mesons

1. Introduction

The discovery of the narrow D_{sJ} particles by BaBar [1] and CLEO [2] and soon confirmed by Belle [3] immediately reminded people an effective theory approach proposed by Nowak et al. and others [4–7]. They constructed an effective Lagrangian for heavy mesons from the Nambu–Jona-Lasinio type four-fermi interactions and combined it with the chiral multiplets so that the mass of heavy mesons can be related to the Higgs scalars of chiral Lagrangian, and they found that two spin multiplets, $j^P = (0^-, 1^-)$ and $(0^+, 1^+)$, are degenerate in the limit in which the chiral symmetry is an exact symmetry of the vacuum and the heavy quark symmetry is exactly realized. From this effective theory, they derived the Goldberger–Treiman relation for the mass gap between chiral partners $0^+(1^+)$ and $0^-(1^-)$ instead of the heavy meson mass

itself and predicted the mass gap between chiral partners of heavy mesons to be around $\Delta M = g_\pi f_\pi \approx 349$ MeV, where g_π is the coupling constant for $0^+ \to 0^- + \pi$ and f_π is the pion decay constant.

Finding that the mass gap between chiral partners $0^+(1^+)$ and $0^-(1^-)$ in the case of D_s agrees well with the experiments (around 350 MeV), people thought that underling physics may be explained by their $SU(3)$ effective Lagrangian [8,9]. However, when $(0^+, 1^+)$ for D meson were found by Belle and FOCUS, and later reanalyzed by CLEO, their explanation needs to be modified even though some people still study in this direction; in fact, the effective Lagrangian approach [8] predicts about 94 MeV smaller mass gap for D mesons than that for D_s mesons, while the experimental mass gap for D mesons is about 70–80 MeV larger than that for D_s mesons [10]. Furthermore, what they originally predicted could not be identified as any of heavy meson multiplets for D, D_s, B, and B_s. In other words, the formula can be applied equally for any of these heavy meson multiplets. Thus, it is required to find the mass gap formula, if it exists, which agrees well with the experiments and explains the physical ground of its formula.

* Corresponding author.
E-mail addresses: matsuki@tokyo-kasei.ac.jp (T. Matsuki), morii@kobe-u.ac.jp (T. Morii), kazutaka.sudoh@kek.jp (K. Sudoh).

Table 1
Optimal values of parameters

Parameters	α_s^c	α_s^b	a (GeV^{-1})	b (GeV)
	0.261 ± 0.001	0.393 ± 0.003	1.939 ± 0.002	0.0749 ± 0.0020
	$m_{u,d}$ (GeV)	m_s (GeV)	m_c (GeV)	m_b (GeV)
	0.0112 ± 0.0019	0.0929 ± 0.0021	1.032 ± 0.005	4.639 ± 0.005
	# of data	# of parameter	total χ^2/d.o.f	
	18	8	107.55	

In this Letter, using our semirelativistic potential model, we first give our formula for the mass gap between chiral partners $0^+(1^+)$ and $0^-(1^-)$ for *any* heavy meson, D, D_s, B and B_s, among which the known mass gaps, i.e., the ones for D and D_s, agree well with the experiments although there is some ambiguities for D meson data. Next we show how this mass gap depends on a light quark mass m_q for $q = u, d$ and s, where we neglect the difference between u and d quarks. Our formula naturally explain that the mass gap for D is larger than that for D_s and predict the mass gaps for B and B_s.

2. Semirelativistic quark potential model and structure of mass gap

Mass for the heavy meson X with the spin and parity, j^P, is expressed in our formulation as [11]

$$M_X(j^P) = m_Q + E_0^k(m_q) + O(1/m_Q),\qquad(1)$$

where the quantum number k is related to the total angular momentum j and the parity P for a heavy meson as [12]

$$j = |k| - 1 \quad \text{or} \quad |k|, \qquad P = \frac{k}{|k|}(-1)^{|k|+1},$$

$$E_0^k(m_q) = E_0(j^P, m_q).\qquad(2)$$

To begin with, we study the heavy meson mass without $1/m_Q$ corrections so that we can see the essence of the mass gap. States with the same $|k|$ value are degenerate in a pure chiral limit and without confining scalar potential, which is defined as $m_q \to 0$ and $S(r) \to 0$ [13]. We consider the scenario that a chiral symmetry breaking and a confinement take place in two steps. First the degeneracy is broken due to gluon fields when $S(r)$ is turned on and confines quarks into heavy mesons but keeping vanishing light quark mass intact. In fact, in this limit our model gives the mass gap between two spin multiplets $\Delta M \approx 300$ MeV as follows;

$$\begin{aligned}\Delta M &= E_0(1^+, 0) - E_0(1^-, 0)\\&= E_0(0^+, 0) - E_0(0^-, 0)\\&= \begin{cases} 295.1 \text{ MeV} & \text{for } D \text{ and } D_s, \\ 309.2 \text{ MeV} & \text{for } B \text{ and } B_s. \end{cases}\end{aligned}\qquad(3)$$

This gap is mainly due to gluon fields which confines quarks into heavy mesons. It is interesting that obtained values are close to $\Lambda_{\text{QCD}} \approx 300$ MeV. Next, turning on a light quark mass which explicitly breaks a chiral symmetry, we have $SU(3)$ flavor breaking pattern of the mass levels, i.e., mass of D becomes different from that of D_s with the same value of j^P. Since

Table 2
Degenerate masses of model calculations and their mass gap between $0^+(1^+)$ and $0^-(1^-)$ for $n = 1$

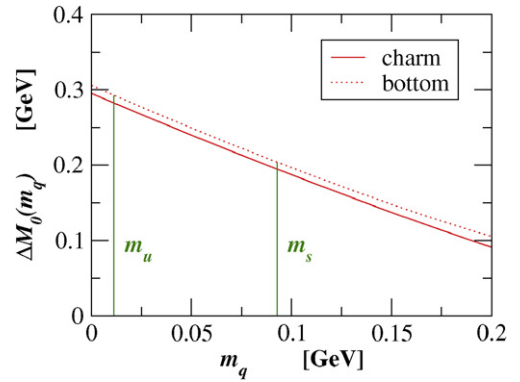|	$M_0(D)$	$M_0(D_s)$	$M_0(B)$	$M_0(B_s)$
$0^-/1^-$	1784	1900	5277	5394
$0^+/1^+$	2067	2095	5570	5598
$0^+(1^+) - 0^-(1^-)$	283	195	293	204

Fig. 1. Plot of the mass gap between two spin multiplets. Light quark mass dependence is given. The horizontal axis is light quark mass m_q and the vertical axis is the mass gap ΔM_0.

we assume $m_u = m_d$, there still remains $SU(2)$ iso-spin symmetry. Note that even after chiral symmetry is broken, there is still degeneracy between members of a spin multiplet due to the heavy quark symmetry, i.e., $SU(2)_f \times SU(2)_{\text{spin}}$ symmetry, with $SU(2)_f$ rotational flavor symmetry and $SU(2)_{\text{spin}}$ rotational spin symmetry. By using the optimal values of parameters in Ref. [14], which is listed in Table 1, degenerate masses without $1/m_Q$ corrections for D, D_s and B, B_s mesons are calculated and presented in Table 2. Furthermore, by changing m_q from 0 to 0.2 GeV, we have calculated the m_q dependence of ΔM_0 and have obtained Fig. 1, in which ΔM_0 is linearly decreasing with m_q. From Fig. 1, we find that the mass gap between two spin multiplets for a heavy meson X can be written as

$$\begin{aligned}\Delta M_0 &= M_X(0^+) - M_X(0^-) = M_X(1^+) - M_X(1^-)\\&= g_0 \Lambda_Q - g_1 m_q,\end{aligned}\qquad(4)$$

$\Lambda_Q = 300$ MeV,

$$\begin{cases} g_0 = 0.9836, & g_1 = 1.080, & \text{for } D/D_s, \\ g_0 = 1.017, & g_1 = 1.089, & \text{for } B/B_s, \end{cases}\qquad(5)$$

where the values of g_0, and g_1 are estimated by fitting the optimal line with Fig. 1. Since both g_0 and g_1 are very close to 1,

we conclude that the mass gap is essentially given by

$$\Delta M_0 = \Lambda_Q - m_q. \qquad (6)$$

Though the physical ground of this result is out of scope at present, Eq. (6) is serious, since it is very different from the one of an effective theory approach as mentioned later. This result is exact when $\mathcal{O}(1/m_Q)$ terms are neglected. As we will see later, since $1/m_Q$ corrections are nearly equal to each other for two spin doublets, the above equation (6) between two spin multiplets holds approximately even with $1/m_Q$ corrections.

Let us see how the mass gap can be written in our formulation [11]. Heavy meson mass without $1/m_Q$ corrections can be given by Eq. (1) with an eigenvalue E_0^k being given by the following eigenvalue equation.

$$\begin{pmatrix} m_q + S + V & -\partial_r + \frac{k}{r} \\ \partial_r + \frac{k}{r} & -m_q - S + V \end{pmatrix} \begin{pmatrix} u_k(r) \\ v_k(r) \end{pmatrix} = E_0^k \begin{pmatrix} u_k(r) \\ v_k(r) \end{pmatrix}. \qquad (7)$$

Using this equation, the mass gap between $k = +1$ and $k = -1$, which are corresponding to the spin multiplets $(0^-, 1^-)$ and $(0^+, 1^+)$, respectively, when they are degenerate, is re-expressed as

$$
\begin{aligned}
&\Delta M_0 \\
&= M^1(0^+) - M^{-1}(0^-) = M^1(1^+) - M^{-1}(1^-) \\
&= \int \frac{d^3x}{4\pi r^2} \left\{ \Phi_1^\dagger(r) \begin{pmatrix} m_q + S + V & -\partial_r + \frac{1}{r} \\ \partial_r + \frac{1}{r} & -m_q - S + V \end{pmatrix} \Phi_1(r) \right. \\
&\quad \left. - \Phi_{-1}^\dagger(r) \begin{pmatrix} m_q + S + V & -\partial_r - \frac{1}{r} \\ \partial_r - \frac{1}{r} & -m_q - S + V \end{pmatrix} \Phi_{-1}(r) \right\} \\
&= \int dr \left[\Phi_1^\dagger(r) K_1 \Phi_1(r) - \Phi_{-1}^\dagger(r) K_{-1} \Phi_{-1}(r) \right] \\
&\quad + m_q \int dr \left[\Phi_1^\dagger(r) \beta \Phi_1(r) - \Phi_{-1}^\dagger(r) \beta \Phi_{-1}(r) \right]. \qquad (8)
\end{aligned}
$$

From this equation we can see that the mass gap linearly depends on m_q. Here the radial wave function $\Phi_k(r)$ and the massless free kinetic term K_k with the quantum number k are given by

$$
\begin{aligned}
\Phi_k(r) &= \begin{pmatrix} u_k(r) \\ v_k(r) \end{pmatrix}, \\
K_k &= \begin{pmatrix} S(r) + V(r) & -\partial_r + \frac{k}{r} \\ \partial_r + \frac{k}{r} & -S(r) + V(r) \end{pmatrix}. \qquad (9)
\end{aligned}
$$

Numerically the coefficient of m_q becomes negative, while the first term in Eq. (8) is approximately given by 300 MeV, which is nearly equal to the scale parameter of QCD, Λ_{QCD}. That a coefficient of m_q becomes negative in Eq. (8) can be explained or we can intuitively understand this result in our formulation as follows. The quantum numbers $k = -1$ and $k = 1$ correspond to $\ell = 0$ and $\ell = 1$ respectively, where ℓ is the angular momentum of a light antiquark relative to a heavy quark as can be seen from Table 1 of Ref. [14]. An excited state with $\ell = 1$ ($k = +1$) is more relativistic compared with the one with $\ell = 0$ ($k = -1$), which means a lower component $v_1(r)$ becomes larger than $v_{-1}(r)$ since they are normalized as $(u_k)^2 + (v_k)^2 = 1$. Hence $(u_1)^2 - (v_1)^2 = \Phi_1^\dagger(r) \beta \Phi_1(r)$

becomes smaller than $\Phi_{-1}^\dagger(r) \beta \Phi_{-1}(r)$. Thus the coefficient of m_q becomes negative. As a matter of fact, linear m_q dependence of ΔM is not yet definite since radial wave functions u_k and v_k are also dependent on m_q. However, looking at Eq. (4) or Fig. 1 which are the numerical calculation of our model, we can say that implicit dependence on m_q of these wave functions is numerically small. Thus the above physical and intuitive interpretation of linear m_q dependence of ΔM_0 is correct.

3. Interpretation due to chiral effective theory

The above result suggests that the physical ground of chiral symmetry breakdown or generation of mass for heavy mesons occurs differently from what people in [4–6] originally considered. Let us briefly explain the mechanism that these authors considered as a generation of the mass gap, which is due to the paper [5]. The Lagrangian for the chiral multiplets, which couples to the heavy quark sector, can be written as follows.

$$
\begin{aligned}
\mathcal{L}_{\text{chiral}} = \bar{\psi}(i\slashed{\partial} - m_q)\psi &- g\bar{\psi}_L \Sigma \psi_R \\
&- g\bar{\psi}_R \Sigma^\dagger \psi_L - \frac{1}{2}\Lambda^2 \text{Tr}(\Sigma^\dagger \Sigma), \qquad (10)
\end{aligned}
$$

where ψ is the chiral quark field with three flavors and Σ is the 3×3 complex auxiliary field which are given by

$$\psi^T = (u, d, s), \qquad \Sigma = \frac{1}{2}\sigma I_3 + i\pi^a \frac{\lambda^a}{2}. \qquad (11)$$

When this Lagrangian is combined with the effective theory for heavy hadrons, the effective mass of a constituent quark is given by $\langle \sigma \rangle + m_q$. Then the mass gap is given by

$$\Delta M_0 = g_\pi (\langle \sigma \rangle + m_q), \qquad (12)$$

where g_π is the Yukawa coupling constant between the heavy meson and a chiral multiplet and is taken to be $g_\pi = 3.73$ in [8], and $\langle \sigma \rangle = f_\pi$. This expression is obtained in the heavy quark symmetric limit and should be compared with our Eq. (6). Instead of minus sign for the term m_q that we obtained, the authors of [5] obtained plus sign as shown in the above equation. The same result is obtained even if we use the nonlinear Σ model [8].

4. $1/m_Q$ corrections

Next let us study the case when $1/m_Q$ corrections to the mass gap are taken into account. Part of the results is given in [15]. In Table 3, we give our numerical results in the cases of $n = 1$ and $n = 2$ (radial excitations). Values in brackets are taken from the experiments. Our values seem to agree with the experimental ones though the fit is not as good as the case for the absolute values of heavy meson masses. We assume the form of the mass gap with the $1/m_Q$ corrections as follows.

$$\Delta M = \Delta M_0 + \frac{c + d \cdot m_q}{m_Q}. \qquad (13)$$

Using Eq. (4) for D and D_s mesons, i.e. $\Delta M_0 = g_0 \Lambda_Q - g_1 m_q = 295.1 - 1.080 m_q$, we obtain the values of the parameters c and d for D/D_s mesons given in Table 3, which are

Table 3

Model calculations of the mass gap. Values in brackets are taken from the experiments. Units are MeV

Mass gap ($n = 1$)	$\Delta M(D)$	$\Delta M(D_s)$	$\Delta M(B)$	$\Delta M(B_s)$
$0^+ - 0^-$	414 (441)	358 (348)	322	239
$1^+ - 1^-$	410 (419)	357 (348)	320	242
Mass gap ($n = 2$)	$\Delta M(D)$	$\Delta M(D_s)$	$\Delta M(B)$	$\Delta M(B_s)$
$0^+ - 0^-$	308	274	206	160
$1^+ - 1^-$	350	327	216	171

Table 4

D/D_s meson mass spectra for both the calculated and experimentally observed ones. Units are MeV

$^{2s+1}L_J(J^P)$	$M_{\text{calc}}(D)$	$M_{\text{obs}}(D)$	$M_{\text{calc}}(D_s)$	$M_{\text{obs}}(D_s)$
$^1S_0(0^-)$	1869	1867	1967	1969
$^3S_1(1^-)$	2011	2008	2110	2112
$^3P_0(0^+)$	2283	2308	2325	2317
"$^3P_1$"(1^+)	2421	2427	2467	2460

Table 5

B/B_s meson mass spectra for both the calculated and experimentally observed ones. Units are MeV

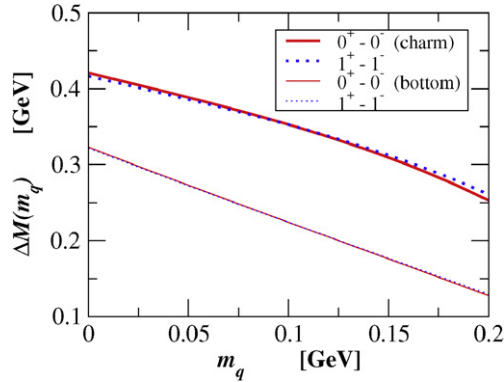| $^{2s+1}L_J(J^P)$	$M_{\text{calc}}(B)$	$M_{\text{obs}}(B)$	$M_{\text{calc}}(B_s)$	$M_{\text{obs}}(B_s)$
$^1S_0(0^-)$	5270	5279	5378	5369
$^3S_1(1^-)$	5329	5325	5440	–
$^3P_0(0^+)$	5592	–	5617	–
"$^3P_1$"(1^+)	5649	–	5682	–

Fig. 2. Plot of the mass gap between two spin multiplets. Light quark mass dependence is given. The horizontal axis is light quark mass m_q and the vertical axis is the mass gap ΔM.

given by

$$c = 1.28 \times 10^5 \text{ MeV}^2, \qquad d = 4.26 \times 10^2 \text{ MeV}. \tag{14}$$

The term c/m_Q lifts the constant $g_0 \Lambda_Q$ about 100 MeV and the term d/m_Q gives deviation from -1 to the coefficient for m_q in the case of D/D_s.

Applying this formula, Eq. (13), to the case for B/B_s with $m_Q = m_b$, we obtain the mass gap as follows.

$$B(0^+) - B(0^-) \approx B(1^+) - B(1^-) \approx 322,$$
$$B_s(0^+) - B_s(0^-) \approx B_s(1^+) - B_s(1^-) \approx 240 \text{ MeV}, \tag{15}$$

which should be compared with our model calculations, 321 and 241 MeV, in Table 3. Thus the linear dependence of the mass gap on m_q is also supported in the case where the $1/m_Q$ corrections are taken into account. The calculated m_q dependence of ΔM with $1/m_Q$ corrections is presented in Fig. 2, for $0 < m_q < 0.2$ GeV. The values in Table 3 are calculated using those in Tables 4 and 5.

5. Miscellaneous phenomena

Global Flavor SU(3) Recovery. Looking at the mass levels of 0^+ and 1^+ states for the D and D_s mesons, one finds that mass differences between D and D_s becomes smaller compared with those of the 0^- and 1^- states. This can be seen from Table 4 and was first discussed in Ref. [16] by Dmitrašinović. He claimed that considering D_{sJ} as a four-quark state, one can regard this phenomena as flavor $SU(3)$ recovery. However, in our interpretation, this is not so as we have seen that this is

caused by the mass gap dependency on a light quark mass, m_q, as shown in Fig. 1. That is, when the mass of D meson is elevated largely from the $0^-/1^-$ state to the $0^+/1^+$ state, the mass of D_s meson is elevated by about 100 MeV smaller than that of $0^-/1^-$ as one can see from Fig. 1. In our interpretation, the $SU(3)$ is not recovered since the light quark masses of $m_u = m_d$ and m_s do not change their magnitudes when the transition from $0^-/1^-$ to $0^+/1^+$ occurs, and their values remain to be $m_{u(d)} = 11.2$ MeV and $m_s = 92.9$ MeV, respectively, as presented in Table 1.

Mass Gap of Heavy Baryons. When we apply our formula to the heavy-light baryons which include two heavy quarks, (ccs), (ccu), (bcs), (bcu), (bbs), and (bbu), mass gaps between two pairs of baryons, like (ccs) and (ccu), will be given by Eq. (6) in the heavy quark symmetric limit and by Eq. (13) with $1/m_Q$ corrections where we have to replace m_Q with $m_{Q_1} + m_{Q_2}$. Here the isospin symmetry is respected since in our model $m_u = m_d$. This speculation is legitimized since QQ pair can be considered to be 3^* expression in the color $SU(3)$ space so that the baryon like QQq can be regarded as a heavy-light meson and our arguments expanded in this Letter can be applied [17,18].

References

[1] BaBar Collaboration, B. Aubert, et al., Phys. Rev. Lett. 90 (2003) 242001.
[2] CLEO Collaboration, D. Besson, et al., Phys. Rev. D 68 (2003) 032002.
[3] Belle Collaboration, P. Krokovny, et al., Phys. Rev. Lett. 91 (2003) 262002;
    Y. Mikami, et al., Phys. Rev. Lett. 92 (2004) 012002.
[4] M.A. Nowak, M. Rho, I. Zahed, Phys. Rev. D 48 (1993) 4370.
[5] W.A. Bardeen, C.T. Hill, Phys. Rev. D 49 (1994) 409.
[6] D. Ebert, T. Feldmann, R. Friedrich, H. Reinhardt, Nucl. Phys. B 434 (1995) 619;
    D. Ebert, T. Feldmann, H. Reinhardt, Phys. Lett. B 388 (1996) 154.
[7] A. Deandrea, N. Di Bartolomeo, R. Gatto, G. Nardulli, A.D. Polosa, Phys. Rev. D 58 (1998) 034004.
[8] W.A. Bardeen, E.J. Eichten, C.T. Hill, Phys. Rev. D 68 (2003) 054024.
[9] M. Harada, M. Rho, C. Sasaki, Phys. Rev. D 70 (2004) 074002.
[10] Belle Collaboration, K. Abe, et al., Phys. Rev. D 69 (2004) 112002.
[11] T. Matsuki, T. Morii, Phys. Rev. D 56 (1997) 5646;

See also T. Matsuki, Mod. Phys. Lett. A 11 (1996) 257;
T. Matsuki, T. Morii, Aust. J. Phys. 50 (1997) 163;
T. Matsuki, T. Morii, K. Seo, Trends Appl. Spectroscopy 4 (2002) 127;
T. Matsuki, T. Morii, K. Sudoh, AIP Conf. Proc. 814 (2005) 533;
A talk given by T. Matsuki at the "XI International Conference on Hadron Spectroscopy" (Hadron05), held at Rio De Janeiro, Brazil, 21–26 August 2005, hep-ph/0510269.

[12] T. Matsuki, K. Mawatari, T. Morii, K. Sudoh, hep-ph/0408326.

[13] T. Matsuki, K. Mawatari, T. Morii, K. Sudoh, Phys. Lett. B 606 (2005) 329.

[14] T. Matsuki, T. Morii, K. Sudoh, Prog. Theor. Phys. 117 (2007) 1077, hep-ph/0605019.

[15] T. Matsuki, T. Morii, K. Sudoh, Eur. Phys. J. 31 (2007) 701;
A talk given by T. Matsuki at the IVth International Conference on Quarks and Nuclear Physics (QNP06), held at Madrid, Spain, 5–10 June 2006, hep-ph/0610186.

[16] V. Dmitrašinović, Phys. Rev. Lett. 94 (2005) 162002.

[17] M. Savage, M.B. Wise, Phys. Lett. B 248 (1990) 177.

[18] T. Ito, T. Morii, M. Tanimoto, Z. Phys. C 59 (1993) 57.