Examples of tasks from different cognitive thinking level for the theme algebraic rational expressions

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Abstract

This work attempts to promote the use of tasks with different cognitive level of thinking during everyday mathematics classes, especially during classes for exercises with the examples of tasks for the theme Algebraic rational expressions. Well-chosen tasks can improve and empower the process of individualization and differentiation during doing mathematics, and can stimulate the process of creative thinking and motivate students in their current learning. Banks of tasks can force and empower self-reflection by students.

1. Types of tasks according to the level of cognitive demands

What students learn does not depend only on the manner of organization of the lesson, performed activities and working conditions, but on the level and type of thinking students are engaged in as well. If we make sure that chosen tasks provide different levels and types of thinking, then the cumulative effect of students’ experience with this kind of tasks will lead to implicit development of ideas for nature and the meaning of mathematics.

In doing so, tasks need to satisfy the following preconditions that teachers must follow as well:

- they must stem from the aims presented in the curriculum for the appropriate grade;
- they must direct student’s attention towards problems that represents a system of complex tasks;
- they must incite creativity;
- they must lead students to ask questions;
- they must create interest for learning new content;
- they must incite exploratory inquisitiveness.

According to the level of cognitive demands, tasks can be divided into:

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Memorization tasks

These tasks require students to memorize and reproduce the studied material. General characteristic of this level is remembering appropriate information, identifying and locating information, recognizing details, comparing reason-consequence relationships, etc.

Procedures without correlation tasks

These tasks require students to use a procedure that is distinctively required, use a procedure for which the teacher has provided previous instruction, or use a procedure that was used previously.

Procedures with correlation tasks

These tasks require students to solve problems or perform more comprehensive writing of logical problems, implement rules, methods, notions, laws, principles and theories. These tasks require skill to use the previously adopted material in new specific situations.

Doing mathematics tasks

These tasks require from students skill to analyze the content into constituent parts and discover mutual correlation between them, and combining elements in order to obtain a unity. Judgment must be based on clear criteria that can be determined by students themselves or from the outside. They provide an opportunity for students to use their entire knowledge and experience in order to come to an original solution.

When we determine the level of cognitive demands of a mathematical task, it is important:

- not to get distracted by the task’s unnecessary characteristics.
- take into account the students for whom the tasks is intended.

**Teachers must respect these two principles when instructing:**

**First principle** – assessment exceeds questioning based on basic memory or memorization

Teaching students is a continuous process that includes entirety of ideas, information, algorithms, steps, procedures and experience. Learning is never an isolated event; it is always a constituent part of life experience and chronology of students’ learning. Whenever we present a subject matter, we will have to incite students to build new connections and views. In addition, we have to help them build connections outside the subject matter, and by doing this we will also incite them to make a comparison between the similarities and differences of this subject matter or text with others they have studied before. We also have to lead students to look at areas under discussion from the subject matter and connect them to other areas from other subject matters or events from their everyday life, and ask them whether those previous experiences influence their current way of thinking.

**Second principle** – the teacher must have a detailed plan to guide student’s thinking

It is important for teachers to start developing a plan for presenting new material or revising which enables students to participate in various thinking processes. However, this plan must serve only as a guide because teachers must react to the flow of discussion in the classroom, changing their questions when necessary because of the reactions in the classroom.

There is another very important matter concerning assessing students’ knowledge. Because teachers are the initiators of the process of examining students’ knowledge, students usually react directly to the teacher. They observe him/her carefully and listen to what he/she says even more attentively than to what their classmates say. If we want to establish a dialog in the classroom, then we have to change this kind of interaction between teachers and students. Namely, during lessons, teachers must change their role of commentators i.e. when students talk, teachers should incite conversation and discussion between students. Instead of one-to-one discussion with the teacher, the teacher
should model the discussion i.e. incite discussion between students in which he/she will take part as a regular participant and not as a central figure.

Likewise, the teacher must not take the role of a dominant assessor who assesses students’ answers. Instead of using phrases like: ‘That is not correct.’; ‘Yes, that is correct.’ or ‘Is that possible?’; teachers should use: ‘Does anyone else want to say anything about this?’, ‘Marko, what do you think?’ etc. these questions help avoid dominant assessment thus enabling students to freely state their ideas.

Teacher’s delivery of speech and prepared questions play an important role in the lesson. Teachers should use the standard language, the questions need to be concise and prompt students to think. The questions should be more like: ‘Why do you think that?’, ‘How can we…?’ etc, and teachers should avoid questions like: ‘Let’s see what you have learnt.’; ‘What did you learn for today?’; ‘Who can tell me …?’ etc. Teachers must avoid using closed questions which students can only answer with ‘Yes’ or ‘No’ and examining the student who first raised his/her hand to answer questions posed by the teacher.

Another important question that we have to consider is the necessary amount of time teachers allocate after presenting the task. Research shows that there is a direct connection between the time teachers wait after they present the task and the students’ level of thinking. The research points out that if teachers extend the allocated waiting-time, the level of thinking increases and the number of students who react rises as well. It is logical that if we pose tasks with high-level cognitive demands, then students need more time to think.

It is important to incite participation from all students in the stage of implementation of tasks. In order to achieve this, teachers must call by name less extrovert students, and sometimes even ignore students who think that they should answer every question. As soon as students begin to get used to real discussion, in which every idea is respected and considered important and there is not only one correct answer, they will want to express their thoughts and listen to other students’ ideas. When students reach this level of mutual interaction in the classroom, guiding the discussion in which everyone takes part, becomes easier for the teacher and more natural for the students.

In order to decrease nervousness, anxiety and fear when choosing tasks that have to be presented during lessons, it is desirable to have teaching manuals for the professors, as well active forums for teachers, where they can discuss educational issues, leave their remarks and suggestions regarding specific issues and tasks. There is also an increasing need of math collections that shall not be adjusted to the needs of the ‘average student’. Instead of that, such books shall provide tasks divided into complexity levels, in order to promote more effectively the individual development of every student.

If “teachers” are not the only “perpetrators”, but they are important creators, organizers and “conductors” of the singing musical choir of students, therefore their behavior has a significant influence over the process of adopting new knowledge and abilities.

The fact of the matter is that for conscientious and permanent acquisition of new knowledge while studying the topic rational algebraic expressions, the teacher should not be focused towards eliminating the errors that have already been made, but towards seeking and finding preventive measures which do not allow a lot of errors. The measures are reduced to discovering the basic definitions and characteristics (theorems and statements) in the theory where the mentioned errors occur. Definitions and characteristics in quantitative relation are fewer less in comparison to the large number of concrete errors. Therefore, the basic result lies not solely in the solving of large number of tasks in order to mechanically remember large number of technical activities assigned as exercises on non-basic algorithms, but during exercise to provide arguments for the execution of the appropriate steps through definitions or characteristics they are based on. Those definitions are:

a) for natural numbers  
\[ a + a + \ldots + a = na \quad \text{and} \quad a \cdot a \cdot \ldots \cdot a = a^n \]

b)  
\[ \frac{a}{k} \pm \frac{b}{k} = \frac{a \pm b}{k} \quad ; \quad \frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d} \quad ; \quad \frac{a}{b} : \frac{c}{d} = \frac{a \cdot d}{b \cdot c} \]

(\[ \frac{a}{k} \pm \frac{b}{k} = \frac{a \pm b}{k} \quad ; \quad \frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d} \quad ; \quad \frac{a}{b} : \frac{c}{d} = \frac{a \cdot c}{b \cdot d} \])
For example, for \( a \cdot c \pm b \cdot c = (a \pm b) \cdot c \) we can write the following tasks:

Present the following as product:

a) \( k \cdot k \)  
b) \( (\pm k) \cdot (\pm k) \cdot (\pm k) \cdot (\pm k) \)  

c) \( \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \)  

d) \( (a \pm b) \cdot c = a \cdot c \pm b \cdot c ; \quad (a \cdot c \pm b \cdot c = (a \pm b) \cdot c) \)  

What's more, these definitions and characteristics can be practiced through appropriate systems of tasks where the solutions for each task are obtained by solving the previous components (constituents).

For example, for \( a \cdot c \pm b \cdot c = (a \pm b) \cdot c \) we can write the following tasks:

Present the following as product:

a) \( ax^2 + bx^2 \)  
b) \( a(x + 2) + b(x + 2) \)  
c) \( a(x + 2) + x + 2 \)  
d) \( ax + 2a \)  
e) \( ax + 2a + b(x + 2) \)  
f) \( a(x + 2) + bx + 2b \)  
g) \( ax + 2a + bx + 2b \)  
h) \( ax + 2a + x + 2 \)  
i) \( a(x + 2) + b(x + 2) \)  
j) \( a(x + 2) - bx - 2b \)  
k) \( a(x + 2) - bx - 2b \)  
l) \( ax + 2a - bx - 2b \)  
m) \( a(x + 2) - x - 2 \)  
n) \( a(x - 2) + x - 2 \) etc.,

but we cannot do the following:

a) \( ax + 2a + bx + 2b \)  
b) \( b(x + 2) + ax + 2a \)  
c) \( ax + 2a + x + 2 \)  
d) \( ax + 2a - bx - 2b \)  
e) \( ax + 2a - x - 2 \)  
f) \( a(x + 2) + b(x + 2) \)  
g) \( a(x + 2) - b(x + 2) \)  
h) \( a(x + 2) + x + 2 \)  
i) \( b(x + 2) - x - 2 \)  
j) \( a(x + 2) - bx - 2b \)  
k) \( ax^2 + bx^2 \)  
l) \( ax + 2a \)

If an error occurs, we should not mention that this is not the way it is done, but explain how it is done, and by explaining the correct we should mention the common mistake that is noticed during solving that type of tasks.

Further on, we provide examples for appropriate tasks for the beginning (first of this theme) lecture together with the serious discussion among the students:

**Task 1.1.** Write down the following product as a power:

a) \( k \cdot k \)  
b) \( (-k) \cdot (-k) \cdot (-k) \cdot (-k) \)  
c) \( \cdot \cdot \cdot \cdot \cdot \cdot \cdot \)  

d) \( \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \)  

**Task 1.2.** Write down the power with base \(-10\) and exponent 6 as:

a) power  
b) product  

**Task 1.3.** Determine the sign of the power:

a) \( (-7)^7 \)  
b) \( 7^6 \)  
c) \( -77^{100} \)  
d) \( x^2 \), if \( x \neq 0 \).

**Task 1.4.** Determine the real number \( x \), if:

a) \( x^2 = 4 \)  
b) \( x^4 = 3^4 \)  
c) \( x^5 = 2^5 \). Explain why?

**Task 1.5.** Evaluate:

a) \( \frac{3^5 \cdot 3^2}{3^2} \)  
b) \( \frac{2^{22}}{2^{12} \cdot (2^3)^2} \)  
c) \( \frac{(y + 1)^y \cdot (y + 1)^y}{(y + 1)^y} \)  

\( y \neq -1 \)
d) \( \left( \frac{x^2 \cdot x \cdot x^4}{x^2} \right)^5 \)  

e) \( \left( \frac{a - 3}{a - 3} \right)^{x}, a \neq 3 \)  

\(| \left( \frac{3}{4} \right)^{17} \cdot \left( \frac{3}{4} \right)^{17} \)

**Task 1.6.** Write down the following expressions as a power with \( x \) as a base,

a) \( (x^3 \cdot x^4)^2 \)  
b) \( \frac{x^7 \cdot (x^4 \cdot x^3)^2}{x^{11}} \)  
c) \( \left( \frac{x^3 \cdot x \cdot x^4}{x^2} \right)^5, x \neq 0 \)  
d) \( \left( \frac{x^4 \cdot x^{10} \cdot x^3}{x^{11} \cdot (x^3)^2} \right)^3, x \neq 0 \)

**Task 1.7.** Write down the following expressions as a power with \( a \) as a base:

a) \( a^n \cdot a^{n+1} \)  
b) \( \frac{a^{n+2}}{a^n}, a \neq 0 \)  
c) \( (a^{n+1})^2 \)

**Task 1.8.** Raise to the power:

a) \( (a^2 \cdot b)^4 \)  
b) \( (a^2 b c^4)^4 \)  
c) \( (3p^3 z^2)^3 \)

d) \( \left( \frac{1}{3} \right)^2 \)  
e) \( \left( \frac{y}{2} \right)^4 \)

f) \( \left( \frac{3xa^2}{2c^4} \right)^2 \)

**Task 1.9.** Evaluate:

a) \( \frac{81\text{ }^{13}}{9^5} \)  
b) \( \frac{2442}{1241 \cdot 239} \)  
c) \( \frac{99^{99} \cdot 5^{100}}{33^{98} \cdot 15^{101}} \)

Namely, this paper distributed to teachers in primary and secondary schools represents a sound starting point to improve the obtained knowledge and its permanence during studying this subject. Simultaneously, it should encourage us for better cooperation so that our students obtain conscientious and long-term knowledge.

**Reference**


Igor Dimovski, Mathematics for the first class secondary school, 2002, Skopje