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Phenomenology of the pentaquark antidecuplet

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Abstract

We consider the mass splittings and strong decays of members of the lowest-lying pentaquark multiplet, which we take to be a parity-odd antidecuplet. We derive useful decompositions of the quark model wave functions that allow for easy computation of color-flavor-spin matrix elements. We compute mass splittings within the antidecuplet including spin–color and spin–isospin interactions between constituents and point out the importance of hidden strangeness in rendering the nucleon-like states heavier than the $S = 1$ state. Using recent experimental data on a possible $S = 1$ pentaquark state, we make decay predictions for other members of the antidecuplet.

1. Introduction

Recently, a number of laboratories have announced observation of a strangeness $+1$ baryon [1–3] with a mass of 1540 MeV and a narrow decay width. Such a state cannot be a 3-quark baryon made from known quarks, and it is natural to interpret it as a pentaquark state, that is, as a state made from four quarks and one antiquark, $q^4\bar{q}$. The current example of the strangeness $S = +1$ baryon is positively charged and is called Z^+ in the particle data tables and θ^+ in some recent works [3]. The Z^+ of necessity has an $\bar{s}$ and four non-strange quarks. The parity, spin, and isospin of the experimental state are currently unmeasured.

In this Letter, we study consequences of describing the Z^+ within the context of conventional constituent quarks models, in more focused detail than was done in earlier work [4–6] and with new results. In these models, all quarks are in the same spatial wave function, and spin dependent mass splittings come from either color–spin or flavor–spin exchange. The Z^+ made this way has negative parity. We treat it as a flavor antidecuplet, with spin-1/2 because this state has, at least by elementary estimates, the lowest mass by a few hundred MeV among the Z^+'s that can be made with all quarks in the ground spatial state.

The pentaquark by now has some history of theoretical study. In the context of constituent quark models, it was analyzed relatively early on [4–6], but the subject was not pursued, probably for lack of experimental motivation. (The first of [4] gives a simple estimate of the Z^+ mass of 1615 MeV and then states "There definitely is no

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$Z^*(I=0)$ state at such a low mass".) Much of the effort shifted to studying pentaquarks involving charmed as well as strange quarks [7,8], before the recent flurry of theoretical attention [9].

Pentaquarks have also been studied in the context of the Skyrme model [10,11]. Ref. [11] in particular makes a striking prediction, based on the assumption that the Z^+ is a member of a flavor antidecuplet and that the nucleon-like members of this decuplet are the observed $N^*(1710)$ states, that the Z^+ would have a mass of about 1530 MeV and a width less than 15 MeV. Note that in this case the Z^+ is a positive parity state.

We may elaborate on the Z^+ states and masses in quark models briefly before proceeding. In outline, there are several ways to make a Z^+, and one can obtain Z^+'s which are isospin $0, 1$, or 2. The mass splittings between the states can be estimated using, say, the color-spin interactions described in more detail in the next section. Techniques and useful information may be found in [4,8,12]. The lightest Z^+ state is the isosinglet (in the $\overline{\mathbf{10}}$) with spin-$1/2$. The isosinglet spin-$3/2$ is a few hundred MeV heavier. The heaviest states are the isotensor spin-$1/2$ and (somewhat lighter) spin-$3/2$ states. The mass gap between the lightest and heaviest of the Z^+'s is triple the mass gap between the nucleon and the $\Delta(1232)$, if one does not account for changes in the quarks's spatial wave functions (e.g., due to changes in the bag radius), or the better part of a GeV. The isovector masses lie in between the two limits. These statements are considered in quantitative detail in Ref. [13]

In the next section, we will discuss the color–flavor–spin wave functions of the antidecuplet that contains the Z^+. This is a necessary prelude to a discussion of the mass splittings and decays of the full decuplet, which follows in Section 3. One intriguing result is the roughly equal mass spacing of the antidecuplet, with the Z^+ lightest. Normally one expects the strange state to be heavier that the non-strange one. The explanation of this counterintuitive behavior is hidden strangeness, that is, there is a fairly high probability of finding an $s\bar{s}$ pair in the non-strange state. We also show that there is a markedly different pattern of kinematically allowed decays, depending of whether spin–isospin or spin–color exchange interactions are relevant in determining the mass spectrum. We close in Section 4 with some discussion.

2. Wave function

There are two useful ways to compose the pentaquark state. One is to build the q^4 state from two pairs of quarks and then combine with the $\bar{q}$. The other is to combine a q^3 state with a $q\bar{q}$ to form the pentaquark. We first represent the pentaquark state in terms of states labelled by the quantum numbers of the first and second quark pairs. Since the antiquark is always in a $(\bar{\mathbf{3}}, \bar{\mathbf{3}}, 1/2)$ (color,flavor,spin) state, we know immediately that the remaining four-quark (q^4) state must be a color $\mathbf{3}$. The flavor of a generic q^4 state can be either a $\mathbf{3}$, $\bar{\mathbf{6}}$, $\mathbf{15}_M$, or $\mathbf{15}_S$ (where S and M refer to symmetry and mixed symmetry under quark interchange, respectively). However, only the $\bar{\mathbf{6}}$ can combine with the $\bar{\mathbf{3}}$ antiquark to yield an antidecuplet. Finally, the spin of the q^4 state can be either 0 or 1 if the total spin of the state is $1/2$. However, it is not difficult to show that any state constructed with the correct quantum numbers using the spin-zero q^4 wave function will be antisymmetric under the combined interchange of the two quarks in the first pair with the two quarks in second pair; this is inconsistent with the requirement that the four-quark state be totally antisymmetric. Thus we are led to the unique choice

$$|(C, F, S)\rangle_{q^4} = |(\mathbf{3}, \bar{\mathbf{6}}, 1)\rangle. \tag{2.1}$$

Fig. 1 shows the possible quark pair combinations that can provide a $(\mathbf{3}, \bar{\mathbf{6}}, 1)$ four-quark state. The symmetry under interchange of quarks 1 and 2, or 3 and 4 is immediate from each of the Young's Tableau shown. The symmetry under interchange of the first and second quark pairs is indicated in brackets next to the tableau. Only three combinations have the right symmetry under quark interchange to form a totally antisymmetric q^4 state, namely

$$|(\bar{\mathbf{3}}, \mathbf{6}, 1)(\bar{\mathbf{3}}, \mathbf{6}, 1)\rangle, \quad \frac{1}{\sqrt{2}}\big(|(\mathbf{6}, \mathbf{6}, 0)(\bar{\mathbf{3}}, \mathbf{6}, 1)\rangle + |(\bar{\mathbf{3}}, \mathbf{6}, 1)(\mathbf{6}, \mathbf{6}, 0)\rangle\big),$$

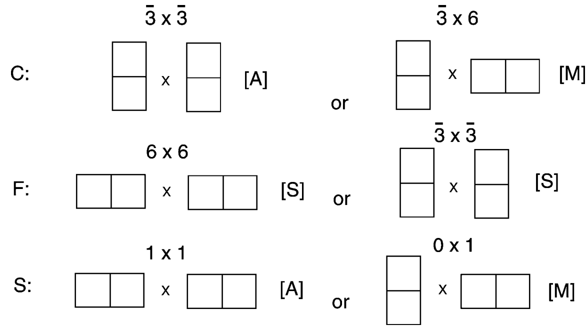

Fig. 1. Quark pair states that can be appropriately combined to yield a total (C,F,S) state $(\mathbf{3},\bar{\mathbf{6}},1)$.

$$\frac{1}{\sqrt{2}}\big(|(\mathbf{6},\bar{\mathbf{3}},1)(\bar{\mathbf{3}},\bar{\mathbf{3}},0)\rangle+|(\bar{\mathbf{3}},\bar{\mathbf{3}},0)(\mathbf{6},\bar{\mathbf{3}},1)\rangle\big).$$

The requirement of total antisymmetry of the q^4 wave function, determines the relative coefficients. We find that the properly normalized state is given by

$$\begin{aligned}|(\mathbf{1},\overline{\mathbf{10}},1/2)\rangle &= \frac{1}{\sqrt{3}}|(\bar{\mathbf{3}},\mathbf{6},1)(\bar{\mathbf{3}},\mathbf{6},1)\rangle+\frac{1}{\sqrt{12}}\big(|(\mathbf{6},\mathbf{6},0)(\bar{\mathbf{3}},\mathbf{6},1)\rangle+|(\bar{\mathbf{3}},\mathbf{6},1)(\mathbf{6},\mathbf{6},0)\rangle\big)\\ &\quad-\frac{1}{2}\big(|(\mathbf{6},\bar{\mathbf{3}},1)(\bar{\mathbf{3}},\bar{\mathbf{3}},0)\rangle+|(\bar{\mathbf{3}},\bar{\mathbf{3}},0)(\mathbf{6},\bar{\mathbf{3}},1)\rangle\big),\end{aligned}\tag{2.2}$$

where we have suppressed the quantum numbers of the antiquark, $(\bar{\mathbf{3}},\bar{\mathbf{3}},1/2)$, which are the same in each term. Also tacit on the right-hand side is that each q^4 state is combined to $(\mathbf{3},\bar{\mathbf{6}},1)$. The signs shown in Eq. (2.2) depend on sign conventions for the states on the right-hand side. For the Z^+ component, spin $\uparrow$, we find

$$\begin{aligned}|(\bar{\mathbf{3}},\mathbf{6},1)(\bar{\mathbf{3}},\mathbf{6},1)\rangle &= \frac{1}{24\sqrt{3}}\big(c_1^jc_2^k-c_1^kc_2^j\big)c_3^mc_4^n\bar{c}_k\epsilon_{jmn}\\ &\quad\times\big[(2uudd+2dduu-udud-uddu-duud-dudu)\bar{s}\big]\\ &\quad\times\big[\{\uparrow\uparrow(\uparrow\downarrow+\downarrow\uparrow)-(\uparrow\downarrow+\downarrow\uparrow)\uparrow\uparrow\}\downarrow-(\uparrow\uparrow\downarrow\downarrow-\downarrow\downarrow\uparrow\uparrow)\uparrow\big],\end{aligned}\tag{2.3}$$

$$\begin{aligned}|(\mathbf{6},\mathbf{6},0)(\bar{\mathbf{3}},\mathbf{6},1)\rangle &= \frac{1}{24\sqrt{3}}\big(c_1^jc_2^k+c_1^kc_2^j\big)c_3^mc_4^n\bar{c}_k\epsilon_{jmn}\\ &\quad\times\big[(2uudd+2dduu-udud-uddu-duud-dudu)\bar{s}\big]\\ &\quad\times\left[(\uparrow\downarrow-\downarrow\uparrow)\uparrow\uparrow\downarrow-\frac{1}{2}(\uparrow\downarrow-\downarrow\uparrow)(\uparrow\downarrow+\downarrow\uparrow)\uparrow\right],\end{aligned}\tag{2.4}$$

$$\begin{aligned}|(\mathbf{6},\bar{\mathbf{3}},1)(\bar{\mathbf{3}},\bar{\mathbf{3}},0)\rangle &= \frac{1}{24}\big(c_1^jc_2^k+c_1^kc_2^j\big)c_3^mc_4^n\bar{c}_k\epsilon_{jmn}\big[(ud-du)(ud-du)\bar{s}\big]\\ &\quad\times\left[\uparrow\uparrow(\uparrow\downarrow-\downarrow\uparrow)\downarrow-\frac{1}{2}(\uparrow\downarrow+\downarrow\uparrow)(\uparrow\downarrow-\downarrow\uparrow)\uparrow\right].\end{aligned}\tag{2.5}$$

Here we have written the color wave function in tensor notation for compactness, with $c^i\equiv(r,g,b)$. The remaining component states in Eq. (2.2) can be obtained from Eqs. (2.4) and (2.5) by exchanging the first and second pair of quarks. With these results, one may construct other antidecuplet wave functions by application of $SU(3)$ and isospin raising and lowering operators.

It is often convenient for calculational purposes to have a decomposition of the pentaquark wave function in terms of the quantum numbers of the first three quarks, and of the remaining quark–antiquark pair. The quark–antiquark pair can be either in a **1** or **8** of color, which implies that we must have the same representations for the three-quark (q^3) system, in order that a singlet may be formed. As for flavor, the q^3 and $q\bar{q}$ systems must both be in **8**'s: the $q\bar{q}$ pair cannot be in a flavor singlet, since there is no way to construct a $\overline{\mathbf{10}}$ from the remaining three quarks, and the q^3 state must be an **8** since the remaining possibilities (**1** and **10**) do not yield an antidecuplet when combined with the $q\bar{q}$ flavor octet. Finally, the $q\bar{q}$ spin can be either 0 or 1, which implies that the q^3 spin can be either 1/2 or 3/2. The states consistent with q^3 antisymmetry are then

$$\begin{aligned}&|(\mathbf{1},\mathbf{8},1/2)(\mathbf{1},\mathbf{8},0)\rangle,\quad |(\mathbf{1},\mathbf{8},1/2)(\mathbf{1},\mathbf{8},1)\rangle,\quad |(\mathbf{8},\mathbf{8},3/2)(\mathbf{8},\mathbf{8},1)\rangle,\\ &|(\mathbf{8},\mathbf{8},1/2)(\mathbf{8},\mathbf{8},0)\rangle,\quad |(\mathbf{8},\mathbf{8},1/2)(\mathbf{8},\mathbf{8},1)\rangle.\end{aligned}$$

Again, we may find the coefficients by requiring that the total wave function is antisymmetric under interchange of the four quarks. Alternatively, we may take the overlap of any of these states with the wave function that we have already derived in Eqs. (2.2)–(2.5). The details and explicit results will be presented in a longer publication [13]. We find

$$\begin{aligned}|(\mathbf{1},\overline{\mathbf{10}},1/2)\rangle = {}&\frac{1}{2}|(\mathbf{1},\mathbf{8},1/2)(\mathbf{1},\mathbf{8},0)\rangle + \frac{\sqrt{3}}{6}|(\mathbf{1},\mathbf{8},1/2)(\mathbf{1},\mathbf{8},1)\rangle - \frac{\sqrt{3}}{3}|(\mathbf{8},\mathbf{8},3/2)(\mathbf{8},\mathbf{8},1)\rangle\\ &+\frac{1}{2}|(\mathbf{8},\mathbf{8},1/2)(\mathbf{8},\mathbf{8},0)\rangle + \frac{\sqrt{3}}{6}|(\mathbf{8},\mathbf{8},1/2)(\mathbf{8},\mathbf{8},1)\rangle.\end{aligned} \tag{2.6}$$

Our sign conventions may be summarized by noting that each state on the right-hand side of Eq. (2.6) contains the term $udd\bar{s}\uparrow\uparrow\downarrow\uparrow\downarrow rbgr\bar{r}$ with positive coefficient.

Two interesting observations can be made at this point. First, Eqs. (2.2)–(2.5) allow us to compute the expectation value of $S_h = \sum_i |S_i|$, where S_i is the strangeness of the ith constituent. This gives us the average number of quarks in the state with either strangeness $+1$ or -1. For the Z^+ state, the result is obviously 1; using the $SU(3)$ raising operator that changes $d \to s$ and $\bar{s} \to -\bar{d}$, it is straightforward to evaluate the same quantity for members of the antidecuplet with smaller total strangeness. We find

$$\langle Z^+|S_h|Z^+\rangle = 3/3,\qquad \langle N_5|S_h|N_5\rangle = 4/3,\qquad \langle \Sigma_5|S_h|\Sigma_5\rangle = 5/3,\qquad \langle \Xi_5|S_h|\Xi_5\rangle = 6/3, \tag{2.7}$$

where N_5, Σ_5 and Ξ_5 represent the strangeness 0, -1 and -2 members of the $\overline{\mathbf{10}}$, respectively. The non-strange member of the $\overline{\mathbf{10}}$ is heavier than the Z^+ because it has, on average, $m_s/3$ more mass from its constituent strange and antistrange quarks.

We also note that our decomposition in Eq. (2.6) allows us to easily compute overlaps with states composed of physical octet baryons and mesons. For example, the first term in Eq. (2.6) may be decomposed for the Z^+

$$|(\mathbf{1},\mathbf{8},1/2)(\mathbf{1},\mathbf{8},0)\rangle = \frac{1}{\sqrt{2}}\big(pK^0 - nK^+\big). \tag{2.8}$$

The sizes of the coefficients of these terms affect the rate of the "break-apart" decay modes, such as $Z^+ \to NK^+$. We therefore find that the smallness of the observed Z^+ decay width ($\lesssim 21$ MeV) does not originate with small group theoretic factors in the quark model wave function.

3. Antidecuplet masses and decays

Using the observed mass and width of the Z^+, one may make predictions for the decay widths of other members of the antidecuplet. Here we consider the decays $\overline{\mathbf{10}} \to BM$ where B (M) is a ground state octet baryon (meson). We assume exact $SU(3)_F$ symmetry in the decay amplitudes, but take into account $SU(3)_F$ breaking in the mass spectra. Mass splittings within the antidecuplet obey an equal spacing rule when the strange quark mass is the only source of $SU(3)_F$ breaking. We compute these splittings within the framework of the MIT bag model [14,15], using the original version for the sake of definiteness, including effects of single gluon exchange interactions between the constituents. (See also [16,17]; these works show how the overall mass level of a multiquark or gluonic state may be shifted, with only small changes in the predictions for ground state baryons and for spin-dependent splittings.) We also consider the possibility of dominant spin–isospin constituent interactions, which would be expected if non-strange pseudoscalar meson exchange effects are important [18]. The predicted spectra differ significantly and yield distinguishable patterns of kinematically accessible decays.

In the bag model, the mass of a hadronic state is given by

$$M = \frac{1}{R}\left\{\sum \Omega_i - Z_0 + \alpha_s C_I\right\} + B\frac{4\pi R^3}{3}, \tag{3.1}$$

where Ω_i/R is the relativistic energy of the ith constituent in a bag of radius R,

$$\Omega = \left(x^2 + m^2 R^2\right)^{1/2}, \tag{3.2}$$

and x is a root of

$$\tan x = \frac{x}{1 - mR - \Omega}. \tag{3.3}$$

The parameter Z_0 is a zero-point energy correction, and B is the bag energy per unit volume. In the conventional bag model, $Z_0 = 1.84$ and $B^{1/4} = 0.145$ GeV. The term $\alpha_s C_I$ represents the possible interactions among the constituents. We first take into account the color–spin interaction originating from single gluon exchange, so that

$$\alpha_s C_I = -\frac{\alpha_s}{4}\langle \mathbf{1}, \overline{\mathbf{10}}, 1/2| \sum_{i<j} \mu(m_i, m_j)\lambda_i \cdot \lambda_j \sigma_i \cdot \sigma_j |\mathbf{1}, \overline{\mathbf{10}}, 1/2\rangle, \tag{3.4}$$

where $\alpha_s = 2.2$ is the value of the strong coupling appropriate to the bag model, and $\mu(m_i, m_j)$ is a numerical coefficient that depends on the masses of the ith and jth quarks. For the case of two massless quarks, $\mu(0,0) \approx 0.177$; the analytic expression for arbitrary masses can be found in Ref. [15].

We take into account the effect of $SU(3)$ breaking (i.e., the strange quark mass) in both Ω_i and in the coefficients $\mu(m_i, m_j)$. To simplify the analysis, we break the sum in Eq. (3.4) into two parts, quark–quark and quark–antiquark terms, and adopt an averaged value for the parameter μ in each, μ_{qq} and $\mu_{q\bar{q}}$. Using the wave function in Eqs. (2.2)–(2.5)) we find that the relevant spin–flavor–color matrix elements are given by

$$\begin{aligned} &\langle \mathbf{1}, \overline{\mathbf{10}}, 1/2| \sum_{i<j\neq 5} \lambda_i \cdot \lambda_j \sigma_i \cdot \sigma_j |\mathbf{1}, \overline{\mathbf{10}}, 1/2\rangle = 16/3, \\ &\langle \mathbf{1}, \overline{\mathbf{10}}, 1/2| \sum_{i<j=5} \lambda_i \cdot \lambda_j \sigma_i \cdot \sigma_j |\mathbf{1}, \overline{\mathbf{10}}, 1/2\rangle = 40/3, \end{aligned} \tag{3.5}$$

where $j = 5$ corresponds to the antiquark. This evaluation was done by group theoretic techniques [13], as well as brute-force symbolic manipulation [19]. To understand how we evaluate the coefficients μ_{qq} and $\mu_{q\bar{q}}$ let us consider a nucleon-like state in the antidecuplet, the p_5. The probability of finding an $s\bar{s}$ pair in the p_5 state is $2/3$. In this case, $1/2$ of the possible qq pairs will involve a strange quark. On the other hand, the probability that the p_5 will contain five non-strange constituents is $1/3$. Thus, we take

$$\mu_{qq}(p_5) = \frac{2}{3}\left[\frac{1}{2}\left(\mu(0,0) + \mu(0,m_s)\right)\right] + \frac{1}{3}\mu(0,0). \tag{3.6}$$

By similar reasoning,

$$\mu_{q\bar{q}}(p_5) = \frac{1}{3}\mu(0,0) + \frac{1}{2}\mu(0,m_s) + \frac{1}{6}\mu(m_s,m_s). \tag{3.7}$$

We also use the averaged kinetic energy terms

$$\frac{2}{3R}\big[3\Omega(0)+2\Omega(m_s)\big]+\frac{1}{3R}[5\Omega(0)]. \tag{3.8}$$

The bag mass prediction is then obtained by numerically minimizing the mass formula with respect to the bag radius R. Applying this procedure to the p_5 and Z^+ states, we find the antidecuplet mass splitting

$$\Delta M_{\overline{\mathbf{10}}} \approx 52\ \text{MeV}. \tag{3.9}$$

We use the observed Z^+ mass, 1542 MeV, and the splitting $\Delta M_{\overline{\mathbf{10}}}$ to estimate the masses of the p_5, Σ_5, and Ξ_5 states; we find 1594, 1646, and 1698 MeV, respectively. Decay predictions from $SU(3)$ symmetry are summarized in Table 1.

While we used the bag model as a framework for evaluating the mass spectra above, we believe our results are typical of any constituent quark model.

We adopt a simpler approach in evaluating the effect of spin–isospin constituent interactions,

$$\Delta M_{\mathrm{SI}} = -C_\chi \langle \mathbf{1}, \overline{\mathbf{10}}, 1/2| \sum_{i<j} \tau_i \cdot \tau_j \sigma_i \cdot \sigma_j |\mathbf{1}, \overline{\mathbf{10}}, 1/2\rangle. \tag{3.10}$$

In this case the flavor generators τ are Pauli matrices, and the coefficient $C_\chi = 25$–30 MeV is determined from the $N-\Delta$ mass splitting; we use 30 MeV [18]. The dimensionless matrix element can be computed using Eqs. (2.2)–(2.5), and we find 10, 20/3, 25/9 and $-5/3$ for the Z^+, p_5, Σ_5 and the Ξ_5, respectively. The mass splitting due to the strange quark constituent mass can be estimated from our previous bag model calculation, by excluding the spin–color interactions, yielding $\Delta M_s \approx 55$ MeV. Again fixing the Z^+ mass at 1542 MeV, we then find 1697, 1869, and 2058 MeV for the p_5, Σ_5, and Ξ_5 mass, respectively. Decay results for this mass spectrum are also presented in Table 1. Note that a number of the decay modes that were kinematically forbidden before are allowed if spin–isospin interactions dominate, due to the larger

Table 1
$SU(3)$ decay predictions for the highest isospin members of the antidecuplet. A_0 and Γ_0 are the amplitude and partial decay width for $Z^+ \to NK^+$, respectively; SC and SI indicate antidecuplet mass spectra assuming dominant spin–color or spin–isospin constituent interactions

Decay	$\lvert A/A_0\rvert^2$	Γ/Γ_0 (SC)	Γ/Γ_0 (SI)
$Z^+ \to pK^0$	1	0.99	0.99
$p_5 \to \Lambda K^+$	1/2	–	0.49
$p_5 \to p\eta$	1/2	0.50	0.68
$p_5 \to \Sigma^+ K^0$	1/3	–	0.12
$p_5 \to \Sigma^0 K^+$	1/6	–	0.06
$p_5 \to n\pi^+$	1/3	0.63	0.68
$p_5 \to p\pi^0$	1/6	0.32	0.34
$\Sigma_5^+ \to \Xi^0 K^+$	1/3	–	0.30
$\Sigma_5^+ \to \Sigma^+\eta$	1/2	–	0.62
$\Sigma_5^+ \to \Lambda\pi^+$	1/2	0.89	1.11
$\Sigma_5^+ \to p\bar{K}^0$	1/3	0.45	0.63
$\Sigma_5^+ \to \Sigma^+\pi^0$	1/6	0.27	0.36
$\Sigma_5^+ \to \Sigma^0\pi^+$	1/6	0.27	0.36
$\Xi_5^+ \to \Xi^0\pi^+$	1	1.47	2.37
$\Xi_5^+ \to \Sigma^+\bar{K}^0$	1	0.36	1.99

predicted splitting within the antidecuplet. (For a smaller choice of $C_\chi \approx 25$ MeV, the ΣK modes are still inaccessible.)

The Skyrme model also has predictions [11] for the masses and decays of the antidecuplet. The mass splittings there were about 180 MeV between each level of the decuplet (with the Z^+ still the lightest), considerably larger splittings than we find in a constituent quark model where the mass splittings come from strange quark masses and from color–spin interactions. Mass splittings using isospin–spin interactions were, on the other hand, more comparable to the Skyrme model results.

Decays of the antidecuplet into a ground state octet baryon and an octet meson involve a decay matrix element and phase space. Ratios of decay matrix elements for pure antidecuplets, such as we show in Table 1, are fixed by $SU(3)_F$ symmetry. They are the same in any model, as may be confirmed by comparing Table 1 to results in [11]. We have neglected mixing; Ref. [11] does consider mixing but does not find large consequences for the decays. The differences between relative decay predictions are then due to differences in phase space, and the differences are due to masses and due to parity. Negative parity states decaying to ground state baryon and pseudoscalar meson have S-wave phase space, while positive parity states have P-wave phase space. Note also that $SU(3)_F$ symmetry does not allow decays of antidecuplets into decuplet baryons plus octet mesons.

4. Discussion

In this Letter we have shown how to construct the quark model wave functions for members of the pentaquark antidecuplet, the flavor multiplet that we argue is most likely to contain the strangeness one state recently observed in a number of experiments [1–3]. We present two decompositions of the $\overline{\mathbf{10}}$ wave function that are useful for computing spin–flavor–color matrix elements, and that reveal the hidden strangeness in each component state. In addition, we have presented the Z^+ wave function in explicit form. We use these results to estimate the effect of spin–color and spin–isospin interactions on the pentaquark mass spectrum. In the first case, we use the MIT bag as a representative constituent quark model to compute the equal spacing between antidecuplet states that differ by one unit of strangeness; we estimate a splitting of 52 MeV. The observed Z^+ mass and $SU(3)$ symmetry then allows us to make decay predictions. Notably, if only color–spin interactions are present, decays of the p_5 and Σ_5 to final states in which both decay products have non-zero strangeness are kinematically forbidden. In addition, the Ξ_5 states are narrower than those in Ref. [11], so that experimental detection might be possible and dramatic. If instead, spin–isospin interactions dominate, all the decays in Table 1 become kinematically accessible.

The work summarized here sets the groundwork for further investigation. Of particular interest to us is the relation between bag model predictions for the absolute pentaquark mass (rather than the mass splittings considered here) and the mass of other multiquark exotic states. The conventional MIT bag predicts a Z^+ mass that is too large relative to the experimental value (we find that a prediction of about 1700 MeV is typical); however, these numbers can be easily reconciled by allowing bag model parameters to float [16,17]. An appropriate analysis requires a simultaneous fit to pentaquark and low-lying non-exotic hadron masses, and consideration of center-of-mass corrections. Whether such fits simultaneously allow for sufficiently heavy six-quark states, given a choice of constituent interactions, is an open question. Our analysis also gives insight into other pentaquark states. For example, there are nucleon-like states in the pentaquark octet (states in the same spin–color representation as the Z^+) which are potentially light. However, we find that these states also have hidden strangeness, placing them within one-third of the strange quark mass below the Z^+, if no other effects are considered, and at or above the Z^+ mass if spin–isospin interactions are taken into account. This is one example of the value of extending our present analysis to other pentaquark multiplets. A more detailed discussion of these topics, as well as of the group theoretical issues described here will be presented in a longer publication [13].

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