A macro-element to simulate 3D soil–structure interaction considering plasticity and uplift

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In structural engineering, soil–structure interaction (SSI) is an important phenomenon that has to be taken into account to reproduce correctly the non-linear behaviour of a structure and thus to be able to predict its relative displacements. Several methods exist: the macro-element approach consists in condensing all non-linearities into a finite domain ("close field") and works with generalised variables (forces and displacements) at the centre of the foundation (Fig. 1, with $\text{H}_x$, $\text{H}_y$, $\text{V}$, horizontal forces, horizontal displacements, vertical settlement). This paper presents a 3D non-linear interface element able to compute SSI based on the "macro-element" concept. The particularity of the macro-element lies in the fact that the movement of the foundation is entirely described by a system of generalised variables (forces and displacements) defined at the foundation centre. The non-linear behaviour of the soil and the uplift mechanism of the foundation are reproduced using the plasticity theory. The failure surface is defined using an adequate overturning mechanism. Coupling of the different mechanisms is straightforward following the theory of multi-mechanisms. The macro-element is able to simulate the 3D behaviour of a rigid shallow foundation of circular, rectangular or strip shape, submitted to cyclic loadings. It is implemented into FEDEASLab, a finite element MATLAB toolbox. Comparisons with experimental results under cyclic loadings show the performance of the approach.

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1. Introduction

In the field of earthquake engineering, soil–structure interaction (SSI) is an important phenomenon that has to be taken into account to reproduce correctly the non-linear behaviour of a structure and thus to be able to predict its relative displacements. Several methods exist: the macro-element approach consists in condensing all non-linearities into a finite domain ("close field") and works with generalised variables (forces and displacements) at the centre of the foundation (Fig. 1, with $\text{H}_x$, $\text{H}_y$, $\text{V}$, horizontal forces, horizontal displacements, vertical settlement). This paper presents a 3D non-linear interface element able to compute SSI based on the "macro-element" concept. The particularity of the macro-element lies in the fact that the movement of the foundation is entirely described by a system of generalised variables (forces and displacements) defined at the foundation centre. The non-linear behaviour of the soil and the uplift mechanism of the foundation are reproduced using the plasticity theory. The failure surface is defined using an adequate overturning mechanism. Coupling of the different mechanisms is straightforward following the theory of multi-mechanisms. The macro-element is able to simulate the 3D behaviour of a rigid shallow foundation of circular, rectangular or strip shape, submitted to cyclic loadings. It is implemented into FEDEASLab, a finite element MATLAB toolbox. Comparisons with experimental results under cyclic loadings show the performance of the approach.

The paper starts with a presentation of the chosen associated dimensionless variables relative to the different shapes of the footing (circular, rectangular and strip). The 3D elastic, plastic and uplift mechanisms are then presented in detail and their coupling according to the theory of multi-mechanisms. The macro-element is implemented into FEDEASLab, a finite element MATLAB toolbox (Filippou and Constantinides, 2004). Numerical results compared with experimental tests under cyclic loadings are provided to show the performance of this new numerical tool.

2. Shape of the foundation and associated dimensionless variables

As usual is the case for a macro-element, it is appropriate to work with generalised (global) variables: the vertical force $V$, horizontal forces $H_x$, $H_y$ and moments $M_x$, $M_y$ and the corresponding displacements, vertical settlement $u_z$, horizontal displacements $u_x$, $u_y$ and rotations $\theta_x$, $\theta_y$. Torque moment ($M_z$) is not taken into account in the current version of the macro-element in order to facilitate the coupling with uplift. However, some recent plasticity models (single surface – isotropic hardening) have been extended to 6 degrees of freedom and calibrated with experiments (Bienen et al., 2006). It would feasible to extend the macro-element following the same ideas. The displacement and force vectors are dimensionless and differ according to the shape (circular, rectangular and strip) of the foundation (in the following the symbol $\cdot$ defines dimensionless variables).
For all the different types of foundations, the reduced forces are denoted as follows:

(i) Reduced horizontal forces: \( H_0 x \), \( H_0 y \).
(ii) Reduced vertical force: \( V_0 \).
(iii) Reduced moments: \( M_0 x \), \( M_0 y \).

and the reduced displacements are:

(i) Reduced horizontal displacements: \( u_0 x \), \( u_0 y \).
(ii) Reduced vertical displacements.
(iii) Reduced rotations: \( h_0 x \), \( h_0 y \).

2.1. Circular footing

The generalised variables for a circular footing are given in Fig. 2. Their associate dimensionless variables are (see Eqs. (1) and (3)):

\[
\mathbf{F} = \begin{bmatrix}
V \\
H_0 \\
M_0 \\
H_0'
\end{bmatrix} = \frac{1}{S q_{\text{max}}} \begin{bmatrix}
V \\
H_x \\
M_x \\
H_x'
\end{bmatrix}
\]

(1)

with \( D_{\text{dm}} \) the diameter and \( S = \left( \pi D_{\text{dm}}^2 / 4 \right) \) the surface area of the foundation. \( q_{\text{max}} \) is the ultimate compression stress of the soil under a vertical centred load (Davis and Booker, 1973; Matar and Salençon, 1979; Philipponnat and Hubert, 2003) but also (Randolph et al., 2004). For a circular footing it takes the following form:

\[
q_{\text{max}} = \frac{0.6}{2} \gamma D_{\text{dm}} N_f + q_0 N_q + 1.3 c N_c
\]

(2)

where \( q_0 \) is the vertical effective stress at the bottom of the foundation, \( N_f \) is the surface term, \( N_q \) a term depending on the depth of the foundation and \( N_c \) the cohesion term. The relations allowing calculating \( N_f, N_q \) and \( N_c \) are given in Caquot and Kérisel (1966) and Randolph et al. (2004). They only depend on the cohesion \( c \) and the friction angle \( \phi \) of the soil. The factors 0.6 and 1.3 are the shape factors of the coefficients \( N_f, N_q \) and \( N_c \) for an axisymmetric foundation.

In a similar way we obtain the displacements as follows:

\[
\mathbf{u} = \begin{bmatrix}
u_z \\
u_x \\
\theta_y \\
u_y \\
\theta_x
\end{bmatrix} = \frac{1}{D_{\text{dm}} q_{\text{max}}} \begin{bmatrix}
u_z \\
u_x \\
\theta_y \\
u_y \\
\theta_x
\end{bmatrix}
\]

(3)

The right choice of the form of the vectors \( \mathbf{F} \) and \( \mathbf{u} \) is crucial. They are conjugated in order to calculate the work of forces applied to the foundation (Nova and Montrasio, 1991).

Following Eqs. (1) and (3), the work of the reduced forces \( (W_r) \) for the normalised problem (according to the work of actual forces \( W_a \)) is provided by:

\[
W_r = \frac{1}{D_{\text{dm}} q_{\text{max}}} (V u_z + H_x u_x + M_y \theta_y + H_y u_y + M_x \theta_x)
\]

(4)

In other words, the real work is easily found by multiplying the work of the reduced forces by the constant factor \( D_{\text{dm}} q_{\text{max}} \).

2.2. Rectangular footing

For a rectangular \( A \times B \) footing (Fig. 3), the following adimensional variables are proposed:
In other words, the real work is easily found by multiplying the
work of reduced forces for the normalised problem is this time provided by:

\[ W_r = \mathbf{F} \cdot \mathbf{u} = \frac{\sqrt{A^2 + B^2}}{q_{\text{max}}(AB)^2} (V\dot{u}_z + H_x\dot{u}_x + M_y\dot{\theta}_y + H_y\dot{u}_y + M_x\dot{\theta}_x) \]

\[ = \frac{\sqrt{A^2 + B^2}}{q_{\text{max}}(AB)^2} W_a \]

In other words, the real work is easily found by multiplying the
work of the reduced forces by the constant factor \( q_{\text{max}}(AB)^2 / \sqrt{A^2 + B^2} \).

In case where one of the two lengths (A or B) is very big, this
normalisation is matching with the adimensional variables proposed in Crémer et al. (2001) for a strip footing (see also Section 2.3).

2.3. Strip footing

For a strip footing, it is more appropriate to use dimensionless
variables given for one meter length of foundation (denoted with the
foot script symbol). For example, if the loading is in the plane
\((z, x)\), the vector \( \mathbf{F} \) takes the following form (Crémer et al., 2001):

\[ \mathbf{F} = \begin{bmatrix} V \hline H_x \hline M_y \\ H_y \hline M_x/B \hline M_y/B \end{bmatrix} = \frac{1}{ABq_{\text{max}}} \begin{bmatrix} V \hline H_x \hline M_y/B \hline M_x/A \hline M_y/A \end{bmatrix} \]

and

\[ \mathbf{u} = \begin{bmatrix} u_z \hline u_x \hline \theta_y \hline 0_y \hline 0_x \hline \end{bmatrix} = \frac{1}{B} \begin{bmatrix} u_z \hline u_x \hline \theta_y \hline 0_y \hline 0_x \hline \end{bmatrix} \]

The associated dimensionless vector \( \mathbf{u} \) is found by choosing a very
big value for \( A \).

Reduced forces and displacements are again conjugated. The work of reduced forces for the normalised problem is this time provided by:

\[ W_r = \mathbf{F} \cdot \mathbf{u} = \frac{1}{Bq_{\text{max}}} (V\dot{u}_z + H_x\dot{u}_x + M_y\dot{\theta}_y + H_y\dot{u}_y + M_x\dot{\theta}_x) = \frac{1}{Bq_{\text{max}}} W_a \]

In other words, the real work is easily found by multiplying the
work for one meter length of the reduced forces by the constant factor \( 1/Bq_{\text{max}} \).

3. Mathematical description of the macro-element

3.1. General remarks

The new SSI macro-element takes into account three different
mechanisms: elasticity, plasticity of the soil and uplift of the foundation. The total displacement is thus decomposed as a sum of an
elastic, plastic and uplift part:

\[ \mathbf{u} = \mathbf{u}_e + \mathbf{u}_p + \mathbf{u}_u \]

Uplift is defined as the negative vertical displacement of the centre
of the foundation. It is the result of rocking, i.e. the fact that the
foundation rotates according to \( \theta_x \) or \( \theta_y \) (a part of the foundation
looses contact with the soil), see Fig. 12. In order to compute uplift, the
simple plasticity of the soil is not sufficient and a new non-linear
mechanism must be introduced. The reason is that the plasticity
mechanism of the macro-element cannot take into account non-linear
geometrical effects (i.e. change of geometry of the foundation)
which can lead to important negative vertical displacements
\((u_z < 0)\). The uplift mechanism takes into account this change of geometry following a phenomenological point of view. Another
reason is that considering only the plasticity model there is not possible
contraction of the plasticity surface.

Plasticity and uplift are strongly coupled (Crémer et al., 2001).

More specifically:

(i) Plasticity and uplift mechanisms are taken into account
according to the classical plasticity theory. They are
described independently by failure criteria, loading surfaces,
flow rules and hardening rules. Coupling of the two mechanisms
is considered following the theory of multi-mechanisms.

(ii) 3D loadings can be studied, i.e. loadings according to the two
horizontal and the vertical axis. As mentioned before, the
torque moment \( M_z \) is not taken into account by the
macro-element.
3.2. Elasticity mechanism

The elastic part of the constitutive law is defined in Eq. (12), where the displacement $\mathbf{u}^{el}$ and force vectors $\mathbf{F}$ are dimensionless.

$$\mathbf{F} = \mathbf{K}^{el} \mathbf{u}^{el} = \mathbf{K}^{el} \left( \mathbf{u} - \mathbf{u}^{up} \right)$$

(12)

The elastic stiffness matrix $\mathbf{K}^{el}$ is calculated using the real part of the static impedances (Grange et al., 2008). It is considered diagonal, i.e. there is no coupling between the different directions of the loading. This is an important assumption that allows simplifying the problem. However, as other authors have showed the importance of the off-diagonal terms (Doherty and Deeks, 2003), they could be introduced in a future version of the macro-element.

$$\mathbf{K}^{el} = \begin{bmatrix}
    k_{zz}^{el} & 0 & 0 & 0 & 0 \\
    0 & k_{h_{h}h_{h}}^{el} & 0 & 0 & 0 \\
    0 & 0 & k_{h_{b}h_{b}}^{el} & 0 & 0 \\
    0 & 0 & 0 & k_{h_{b}h_{h}}^{el} & 0 \\
    0 & 0 & 0 & 0 & k_{h_{h}h_{b}}^{el}
\end{bmatrix}$$

(13)

with, for a circular footing:

$$\begin{align*}
    k_{zz}^{el} &= \frac{k_{zz}}{q_{max}} \\
    k_{h_{h}h_{h}}^{el} &= k_{hh}^{el} = \frac{k_{hh}D_{dim}}{q_{max}} \\
    k_{h_{b}h_{h}}^{el} &= k_{hb}^{el} = \frac{k_{hb}v_{max}}{q_{max}} \\
    k_{h_{h}h_{b}}^{el} &= k_{hhb}^{el} = \frac{k_{hhb}v_{max}}{q_{max}}
\end{align*}$$

(14)

for a rectangular footing:

$$\begin{align*}
    k_{zz}^{el} &= \frac{k_{zz}}{q_{max} \sqrt{A^{2} - B^{2}}} \\
    k_{h_{h}h_{h}}^{el} &= k_{hh}^{el} = \frac{k_{hh}}{q_{max} \sqrt{A^{2} + B^{2}}} \\
    k_{h_{b}h_{h}}^{el} &= k_{hb}^{el} = \frac{k_{hb}}{q_{max} \sqrt{A^{2} + B^{2}}} \\
    k_{h_{h}h_{b}}^{el} &= k_{hhb}^{el} = \frac{k_{hhb}}{q_{max} \sqrt{A^{2} + B^{2}}}
\end{align*}$$

(15)

For a strip footing (with $B$ the small dimension and $A$ very big):

$$\begin{align*}
    k_{zz}^{el} &= \frac{k_{zz}}{q_{max}} \\
    k_{h_{h}h_{h}}^{el} &= k_{hh}^{el} = \frac{k_{hh}}{q_{max}} \\
    k_{h_{b}h_{h}}^{el} &= k_{hb}^{el} = \frac{k_{hb}v_{max}}{q_{max}} \\
    k_{h_{h}h_{b}}^{el} &= k_{hhb}^{el} = \frac{k_{hhb}v_{max}}{q_{max}}
\end{align*}$$

(16)

In the case of a strip footing, the others terms are not calculated.

The terms of this stiffness matrix are obtained using the real part of the static impedances defined in Gazetas (1991). The impedance represents the dynamic response of a zero-mass foundation lying on a semi-infinite soil considering its mass.

For a circular foundation:

$$\begin{align*}
    k_{zh} &= \frac{4q_{max}}{\pi} \left( 1 + 0.5 \frac{H}{L} \right) \\
    k_{zz} &= \frac{2q_{max}}{\pi} \left( 1 + 1.28 \frac{H}{L} \right) \\
    k_{ww} &= \frac{q_{max}}{L^{2}} \left( 1 + 0.17 \frac{H}{L} \right)
\end{align*}$$

(17)

For a rectangular foundation we obtain (where $\beta_{h_{h}h_{h}}, \beta_{z}$ and $\beta_{h_{h}b}$ depend on the ratio $\frac{H}{L}$ and they are given in Philipponnat and Hubert (2003)):

$$\begin{align*}
    k_{zh} &= \frac{q_{max}}{L} \beta_{h_{h}h_{h}} \sqrt{AB} \\
    k_{zz} &= \frac{q_{max}}{L} \beta_{zz} \sqrt{AB} \\
    k_{ww} &= \frac{q_{max}}{L} \beta_{h_{h}b} b^{2} \sqrt{AB}
\end{align*}$$

(18)

For a strip footing the following relationships are provided in Gazetas (1991) (for a one meter length foundation):

$$\begin{align*}
    k_{zh} &= \frac{q_{max}}{L} \beta_{h_{h}h_{h}} \sqrt{AB} \\
    k_{zz} &= \frac{0.72q_{max}}{L} \\
    k_{ww} &= \frac{q_{max}}{L} \beta_{h_{h}b} b^{2} \sqrt{AB}
\end{align*}$$

(19)

Eqs. (17)–(19) depend on the geometrical properties of the foundation (diameter $D_{dim}$ or length $A$ and $B$), the elastic stiffness properties of the soil (shear modulus $G_{0}$, Poisson ratio $\nu$) and the depth $H$ (measured from the surface) of the solid bed-rock.

3.3. Plasticity mechanism

The plasticity mechanism is briefly described hereafter. Detailed information can be found in Grange et al. (2008).

The failure criterion of the plasticity mechanism is defined for an overturning mechanism with uplift. It comes from Pecker (1997) and it has been used already in the 2D macro-element for a circular foundation presented in Crémér et al. (2001).

The adaptation of the failure criterion in 3D and for different shapes comes from the following remark (see also Grange et al. (2008)): expressing the failure surfaces found by different authors in the space of the dimensionless variables one can see that their form is rather similar (Fig. 4, where $V_{0}$ represents the bearing vertical capacity of the foundation). The adaptation in 3D consists thus on adding two more terms related with the horizontal force and moment in the other direction and assuming axial symmetry (Eq. (20)).

$$f_{\infty} = \left( \frac{H_{x}}{aV_{0}(1 - V_{0})} \right)^{2} + \left( \frac{M_{x}}{bV_{0}(1 - V_{0})} \right)^{2} + \left( \frac{H_{y}}{aV_{0}(1 - V_{0})} \right)^{2} + \left( \frac{M_{y}}{bV_{0}(1 - V_{0})} \right)^{2} - 1 = 0$$

(20)

Fig. 4. Comparisons between the different failure surfaces given by several authors and plotted with adimensional variables. Strip foundations for Nova and Montrasio (1991) and Pecker (1997), circular for Butterfield and Gottardi (1994).
This is of course a rather harsh simplification but considering the nature of the macro-element and the level of precision that we want to obtain this first level of approximation is acceptable.

Following the same philosophy and for a 3D loading, the loading surfaces take the form (Grange et al., 2008):

\[
f_{V}(x, \rho, \gamma) = \left(\frac{H}{\rho} + \frac{M}{\rho \lambda} \right) - \frac{a}{\rho} = 0
\]

The coefficients \(a, b, c, d, e, f\) define the size of the surface in the planes \((H - M, V)\). The coefficients \(d, e\) and \(f\) define the parabolic shape of the surface in the planes \((V - M)\) and \((V - H)\). These parameters can be fitted to different experimental results. For a semi-infinite space, the following values can be found in the literature (Crémer et al., 2001), Table 1:

<table>
<thead>
<tr>
<th>Purely cohesive soil</th>
<th>Purely frictional soil</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>0.32</td>
</tr>
<tr>
<td>(b)</td>
<td>0.37</td>
</tr>
<tr>
<td>(c)</td>
<td>0.25</td>
</tr>
<tr>
<td>(d)</td>
<td>0.55</td>
</tr>
<tr>
<td>(e)</td>
<td>0.85</td>
</tr>
<tr>
<td>(f)</td>
<td>0.8</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Purely cohesive soil</th>
<th>Purely frictional soil</th>
</tr>
</thead>
<tbody>
<tr>
<td>(b)</td>
<td>0.35</td>
</tr>
<tr>
<td>(c)</td>
<td>1</td>
</tr>
<tr>
<td>(d)</td>
<td>1</td>
</tr>
<tr>
<td>(e)</td>
<td>1</td>
</tr>
<tr>
<td>(f)</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 1: Values of the loading surface parameters for a foundation lying on a semi-infinite space (Crémer et al., 2001).

### 3.4. Uplift mechanism

The mechanism presented hereafter describes in a phenomenological way uplift via a unique state variable \(\delta\). This variable represents the proportion of the surface of the uplifted footing (Crémer et al., 2001) (see also Fig. 6, \(D\) being the total length of the foundation). The macro-element being just a point, this is the only way to introduce the influence of the change of the geometry. We assume hereafter that uplift is not influenced by horizontal forces.

Fig. 7 represents the 2D behaviour of a foundation on a plastic soil during uplift. One can identify the relation between \(M' - \delta\) (\(M'\) being \(M_0\), or \(M_1\)), see also Crémer et al. (2001). More specifically, the principal characteristics of this behaviour are:

- When the foundation undergoes a loading in one direction (for example \(M' > 0\)) the behaviour is elastic until the uplift initiation moment \(M_0^{(0)}\) is reached. After that point, the percentage of uplift \(\delta\) increases creating uplift permanent displacements.
- During unloading (\(M' < 0\)) the permanent displacements become visible (just like in a classical plasticity mechanism). The unloading slope \((\eta q_1 / q_1)\) is increased with respect to the original loading one \((q_1 / q_1)\) and \(\delta\) decreases.
- Although the \(M' - \delta\) unloading curve is linear (Fig. 7), the \(M' - \eta q_1\) curve is not linear (\(\eta q_1\) being the rotation due to uplift, see for example Figs. 10 and 11). Unloading does not follow an elastic linear behaviour and permanent displacements due to uplift can decrease.
- If a new loading cycle takes place, the behaviour can be at the beginning elastic till a new uplift initiation moment \(M_0^{(1)}\) is reached. The size of the elastic domain is thus found reduced.
- The two directions of loading (\(M' > 0\) or \(M' < 0\)) are uncoupled. When the foundation undergoes a loading in the direction \(M' > 0\) the other direction is not affected and the uplift initiation moment for \(M' < 0\) remains unchanged.

\(q_1\) and \(q_2\) are shape factors. For a circular foundation \(q_1 = 6, q_2 = 2\), for a rectangular-strip foundation \(q_1 = 4, q_2 = 1\). The evolution of \(\eta\) is provided in Eq. (22) (Crémer et al., 2001):
\[ \eta = 4 - 3e^{-4V} \]  

(22)

In order to robustly combine the non-linearities coming from the uplift mechanism with the ones coming from plasticity, the classical plasticity formalism is also chosen to describe uplift. This is presented in details hereafter:

3.4.1. Failure criterion

During uplift, failure occurs when the foundation is completely detached of the soil, in other words when \( \delta = 1 \). A simple analysis for different shapes of foundations (circular or rectangular or strip) lying on elastic soil allows finding the relation \( M = V/2 \) between the overturning moment and the given vertical force. This equation can be actually considered as a failure criterion. On a plastic soil, the relationship between the overturning moment and the vertical force is more complicated (Crémer et al., 2001). Moreover, the overturning moment is linked with the shape of the foundation. For a loading in two different directions (for \( M > 0 \) and \( M < 0 \)) we obtain (Grange, 2008):

\[
f_{\infty} \equiv M^2 - \left( \frac{V}{q_1} (e^{-AV} + q_2) \right)^2 = 0
\]

(23)

where \( A = 2.5 \) is a dimensionless parameter.

3.4.2. Loading surfaces

During uplift, residual displacements can be generated at each part of the foundation combined with the plastification of the soil (see Grange (2008) and Fig. 8). Furthermore, uplift is a non-linear, non-reversible mechanism with the unloading slope increased with respect to the original loading one. The evolutions of the loading surfaces have thus to be activated even during unloading.

Fig. 8 shows clearly the need to define two independent mechanisms for each directions of loading. Positive moments leads to uplift \( \delta^+ \) and negative moments to uplift \( \delta^- \). The mathematical expressions of the loading surfaces for the directions \( \odot \) and \( \odot \) are provided in Eq. (24). In order to activate the loading surfaces in loading but also in unloading, they are chosen always positive whatever the sign of the loading.

\[
\begin{align*}
\{ f^+ \equiv M' - \frac{\eta}{q_1} (e^{-AV} + q_2, b^+ \} &= 0 \\
\{ f^0 \equiv M' + \frac{\eta}{q_1} (e^{-AV} - q_2, b^+ \} &= 0
\end{align*}
\]

(24)

For the direction \( \odot \), the corresponding new hardening variable \( b^\odot \) evolves between \( \delta \) and \( \delta_{\max} \) (maximal percent of uplift reached during the loading) and is defined as:

\[
\beta^\odot = \delta_{\max} (1 - \eta) + \eta \delta
\]

(25)

During an initial loading step, \( \delta^\odot = \delta^\odot_{\max} \) and thus \( \beta^\odot = \delta^\odot_{\max} = \delta^\odot_{\max} \). It is only during unloading that \( \beta^\odot \) is different from \( \delta^\odot \). The same equations can be written for the other direction replacing \( \odot \) with \( \odot \).

3.4.3. Elastic zone

Unless the loading is important, an initial elastic domain exists. The mathematical expression of the surface defining the elastic limit zone is:

\[
\begin{align*}
\{ f^\odot_{\odot} \equiv M - \frac{\eta}{q_1} q_2 b_{\max} (1 - \eta) - \frac{\eta}{q_1} e^{-AV} &= 0 \\
\{ f^\odot_{\odot} \equiv M - \frac{\eta}{q_1} q_2 b_{\max} (1 - \eta) + \frac{\eta}{q_1} e^{-AV} &= 0
\end{align*}
\]

(26)

The loading surfaces being always positive, the following tests allow knowing which mechanism (elastic or uplift) is active:

\[
\begin{align*}
\{ f^\odot_{\odot} (M, V) &< 0 \quad \text{or} \quad f^\odot (M, V, b^\odot) = 0 \quad \Rightarrow \text{elasticity} \\
\{ f^\odot_{\odot} (M, V) &> 0 \quad \text{and} \quad f^\odot (M, V, b^\odot) > 0 \quad \Rightarrow \text{uplift}
\end{align*}
\]

(27)

The same equations are of course valid for the other direction replacing \( \odot \) with \( \odot \).

If residual uplift occurs on the \( \odot \) side of the foundation, the elastic domain entirely disappears. The mechanisms \( \odot \) and \( \odot \) can in principle be activated simultaneously. The graphical representation of all the surfaces is finally given in Fig. 9 at a given time \( t \) and is explained hereafter:

---

Fig. 7. Moment–\( \delta \) relationship of a foundation on a plastic soil.

Fig. 8. Residual displacements on a plastic soil during uplift.
3.4.4. Kinematic hardening laws

While the loading point is situated outside the elastic domain, behaviour is non-linear during loading and unloading. Furthermore, displacements due to uplift decrease during unloading. The kinematic hardening laws have thus to be activated while on the monotonic loading curve and for loading–unloading. This is done with the following equation (presented here for the \( \oplus \) direction):

\[
\bar{\beta}^\oplus = \frac{\beta^\oplus}{\beta^\oplus_{\text{max}}} \left(1 - \frac{z_{\text{up}}}{2(z_{\text{up}} - z_{\text{max}})}\right)^2 \quad \text{if } \beta^\oplus = \beta^\oplus_{\text{max}} \quad (\text{monotonic loading curve})
\]

\[
\bar{\beta}^\oplus = \frac{\beta^\oplus}{\beta^\oplus_{\text{max}}} \left(1 - \frac{z_{\text{up}}}{2(z_{\text{up}} - z_{\text{max}})}\right)^2 \quad \text{if } \beta^\oplus \leq \beta^\oplus_{\text{max}} \quad (\text{loading–unloading})
\]

(28)

3.4.5. Flow rule

The flow rule is found through geometrical considerations, assuming that the centre of rotation of the foundation stays always at the middle of the non-uplifted segment (Fig. 12, Grange (2008)).

The uplift vertical displacement generated by the uplift rotation is given as follows:

\[
dz_{\text{up}} = \left(\frac{D}{2} - \frac{D(1 - \delta)}{2}\right) \frac{d\delta_{\text{up}}}{2} = -\frac{D\delta}{2} d\delta_{\text{up}}
\]

(29)

Eq. (29) leads to the velocity (without dimension):

\[
z_{\text{up}} = -\frac{\delta}{2} d\delta_{\text{up}}
\]

(30)

As the uplift mechanism does not generate any other displacements (e.g. horizontal displacements), the flow rule is completely described by Eq. (31).

\[
\frac{\partial \gamma}{\partial \phi} = \frac{\delta}{2} \frac{\partial \gamma}{\partial \phi}
\]

(31)

A new function \( f_g \) is introduced to define the sign of the term \( \frac{\partial \gamma}{\partial \phi} \) considering that:

- \( \frac{\partial \gamma}{\partial \phi} \geq 0 \) if the loading point moves from inside to outside the loading surface \( f \) (Fig. 9).
- \( \frac{\partial \gamma}{\partial \phi} \leq 0 \) if the loading point moves from outside to inside the loading surface \( f \).

\( f_g \) is built from the function \( f \) with the difference that \( f_g \) is only positive outside and negative inside. It is calculated as:

\[
f_g \equiv M^2 \frac{V^2}{q_1} \left(1 + q_1 \delta\right) = 0
\]

(32)

The following equations finally link the uplift rotation and the uplift vertical displacement of the foundation:

\[
\left\{ \begin{array}{l}
\frac{\partial \gamma}{\partial \phi} = \frac{f_g}{q_1} = \text{sign}(f_g) \\
\frac{\partial \gamma}{\partial \phi} = -\frac{f_g}{q_1} = -\frac{1}{2} \text{sign}(f_g)
\end{array} \right.
\]

(33)

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\frac{\partial \gamma}{\partial \phi} = -\frac{f_g}{q_1} = -\frac{1}{2} \text{sign}(f_g)
\end{array} \right.
\]

(33)
3.5. 3D behaviour

For a 3D loading, a simplified way to model uplift is to define uplift mechanisms in two horizontal directions similar to the mechanism presented before. The mechanisms are then coupled considering a projection in the principal direction of solicitations (Grange (2008), Fig. 13).

The loading surfaces, elastic zones and failure criteria of the two mechanisms are presented hereafter (where $u$ is the angle defining the principal direction of the solicitation). For the direction $x$:

$$f_x^o = M - \left[ \frac{\nu}{\eta} \frac{q_x}{\tilde{q}_x} \right] \left( 1 - \eta \right) + \frac{\nu}{\eta} e^{\alpha_x} \cos u = 0$$
$$f_y^o = \left| M - \frac{\nu}{\eta} \left( e^{\alpha_x} \cos u + q_x b_x^o \right) \right| = 0$$
$$f_z^o = M - \left[ \frac{\nu}{\eta} \frac{q_z}{\tilde{q}_z} \right] \left( 1 - \eta \right) + \frac{\nu}{\eta} e^{\alpha_z} \cos u = 0$$
$$f_{z,0}^o = M^2 - \left[ \frac{\nu}{\eta} \left( e^{\alpha_x} \cos u + q_x \right) \right] ^2 = 0$$

For the direction $y$:

$$f_x^y = M - \left[ \frac{\nu}{\eta} \frac{q_y}{\tilde{q}_y} \right] \left( 1 - \eta \right) - \frac{\nu}{\eta} e^{\alpha_y} \sin u = 0$$
$$f_y^y = \left| M - \frac{\nu}{\eta} \left( e^{\alpha_y} \sin u + q_y b_y^o \right) \right| = 0$$
$$f_z^y = M - \left[ \frac{\nu}{\eta} \frac{q_z}{\tilde{q}_z} \right] \left( 1 - \eta \right) - \frac{\nu}{\eta} e^{\alpha_z} \sin u = 0$$
$$f_{z,0}^y = M^2 - \left[ \frac{\nu}{\eta} \left( e^{\alpha_y} \sin u + q_y \right) \right] ^2 = 0$$

Consider that the hardening variables of the two uplift mechanisms should tend respectively to $\beta_{ux} = \| \cos u \|$ and $\beta_{uy} = \| \sin u \|$, they are calculated as follows (see also Eq. (28)): for the $\odot$ mechanism:

$$\beta_x^\odot = \frac{\eta}{\tilde{q}_x} \left( \frac{2 \beta_{ux}}{\beta_{ux} - \beta_{ux}^{(i)}} \right)^2 \quad \text{if } \beta_x^\odot = \beta_{ux}^{(i)} \quad \text{(monotonic loading curve)}$$
$$\beta_y^\odot = \frac{\eta}{\tilde{q}_y} \left( \frac{2 \beta_{uy}}{\beta_{uy} - \beta_{ux}^{(i)}} \right)^2 \quad \text{if } \beta_y^\odot = \beta_{ux}^{(i)} \quad \text{(load-unload)}$$

For the $\otimes$ mechanism:

$$\beta_x^\otimes = \frac{\eta}{\tilde{q}_x} \left( \frac{2 \beta_{ux}}{\beta_{ux} - \beta_{ux}^{(i)}} \right)^2 \quad \text{if } \beta_x^\otimes = \beta_{ux}^{(i)} \quad \text{(monotonic loading curve)}$$
$$\beta_y^\otimes = \frac{\eta}{\tilde{q}_y} \left( \frac{2 \beta_{uy}}{\beta_{uy} - \beta_{ux}^{(i)}} \right)^2 \quad \text{if } \beta_y^\otimes = \beta_{ux}^{(i)} \quad \text{(load-unload)}$$

Remark 1. With this formalism, if $\beta^o, \beta^c$ are the “fictive” hardening variables in the principal direction of loading, the following equations are every time verified:

$$\beta_x^o = \beta_{ux}^o \beta_x^\odot$$
$$\beta_y^o = \beta_{uy}^o \beta_y^\otimes$$
$$\beta_x^c = \beta_{ux}^c \beta_x^\odot$$
$$\beta_y^c = \beta_{uy}^c \beta_y^\otimes$$

Remark 2. In this problem, two uplift variables are calculated (the first along the direction $x$ and the second along the direction $y$). Is there a risk to generate too high vertical displacements? The answer is no because the two hardening variables tend respectively to $\beta_{ux}$ and $\beta_{uy}$.

The demonstration follows:

Let’s consider a radial loading resulting to a rotation $\theta_u$, $\delta_u \in [0,1]$ is the corresponding percentage of uplift. The decomposition of the uplift rotation into the coordinate system of the foundation is:

$$\theta_u = \cos \theta_u \theta_u^o + \sin \theta_u \theta_u^c$$

$$\theta_u = \cos \theta_u \theta_u^o + \sin \theta_u \theta_u^c$$

The vertical displacement is calculated thanks to the flow rule defined for the uplift mechanism (Eq. (30)). For each direction we obtain:

$$u_x^{up} = -\frac{1}{2} \theta_u \theta_u^o$$
$$u_y^{up} = -\frac{1}{2} \theta_u \theta_u^c$$

Therefore, the total uplift generated by the two mechanisms is:

$$u^{up}_x = u_x^{up} + u_y^{up}$$

The same equations are obtained for the direction $y$ by replacing $x$ by $y$. 

**Fig. 11.** Moment–rotation, moment–uplift, moment–$\delta$ relationships, case 2: no elastic zone. $t$ corresponds to the time step of Fig. 9(b).
Furthermore, at each step the uplift variables verify:

\[ \begin{align*}
\delta_x &= |\cos u| \delta_u \\
\delta_y &= |\sin u| \delta_u
\end{align*} \]  

(42)

By introducing Eqs. (39), (40), (42) into Eq. (41) we obtain (because \( |\cos u|^2 + |\sin u|^2 = 1 \)):

\[ u_{up}^{plm} = -\frac{\delta_u}{2} \]  

(43)

4. Coupling of the two non-linear mechanisms: plasticity and uplift

Coupling of the plasticity and uplift mechanisms is done following the classical theory of multi-mechanisms (Simo and Hughes, 1998; Grange, 2008). A representation of the superposition of the different surfaces is given in Fig. 14.

In practice there are not two but five mechanisms to link: the plasticity mechanism and the two uplift mechanisms (\( \equiv \) and \( \equiv \)) for each direction (x and y). Each mechanism generates residual displacements. Let’s define \( \mathbf{u}^{plm} \) the contribution of a mechanism \( m \) with \( m \in [1, M] \) and with \( M \in [1, S] \) the number of the activated mechanism. The total plastic velocity can thus be written as follows:

\[ \mathbf{u}^{pl} = \sum_{m=1}^{M} \mathbf{u}^{plm} \]  

(44)

Due to the normality rule we have:

\[ \mathbf{u}^{pl} = \sum_{m=1}^{M} J_m \mathbf{g}_m^{pl} \]  

(45)

Let’s also define \( f^1(\mathbf{F}, \mathbf{q}^1); f^2(\mathbf{F}, \mathbf{q}^2); f^3(\mathbf{F}, \mathbf{q}^3); f^4(\mathbf{F}, \mathbf{q}^4) \) and \( f^5(\mathbf{F}, \mathbf{q}^5) \) the loading surfaces of the five mechanisms. \( \mathbf{F} \) represents the loading vector and \( \mathbf{g}_m^{pl} \) represents the hardening variable array of the mechanism \( m \).

Following the classical plasticity theory the Kuhn–Tucker conditions have to be verified:

\[ \dot{\mathbf{g}}_m^{-} \geq 0 \quad f_m^{-} \leq 0 \quad \dot{\mathbf{g}}_m^{-} f_m^{-} = 0 \]  

(46)

The consistency condition is checked for each mechanism in order to calculate the corresponding plastic potential \( \mathbf{X}_m \). This condition translates the fact that the loading point has always to be on the loading surfaces. In other words, for the mechanism \( m \), the relationships \( f_m^1 = 0 \) and \( f_m^2 = 0, m \in [1, M] \) have always to be checked.

The first condition allows calculating the plastic multiplier (considering that \( \mathbf{F} - \mathbf{K}^{el}(\mathbf{u} - \mathbf{u}^{pl}) \)) as follows:

\[ \dot{\mathbf{f}} = 0 \quad \Leftrightarrow \quad \frac{\partial f_j}{\partial \mathbf{F}} \mathbf{F} + \sum_{j=1}^{M} \frac{\partial f_j}{\partial \mathbf{g}_m^{pl}} \mathbf{g}_m^{pl} = 0 \]

\[ \Leftrightarrow \quad \frac{\partial f_j}{\partial \mathbf{F}} \mathbf{F} + \mathbf{K}^{el} \mathbf{u} - \sum_{j=1}^{M} \dot{\mathbf{g}}_m^{-} \frac{\partial f_j}{\partial \mathbf{g}_m^{-}} - \dot{\mathbf{g}}_m^{-} \frac{\partial f_j}{\partial \mathbf{g}_m^{-}} \mathbf{h}_m = 0 \]  

(47)

The previous \( M \) equations are coupled and the plastic multipliers \( \dot{\mathbf{g}}_m^{-} \) are given hereafter:

\[ \begin{bmatrix} \dot{\mathbf{g}}_1^- \\ \dot{\mathbf{g}}_2^- \\ \vdots \\ \dot{\mathbf{g}}_M^- \end{bmatrix} = \mathbf{H}_p^{-1} \mathbf{H}_p^{el} \]  

(48)

where \( \mathbf{H}_p^{el} \) is the diagonal matrix of plastic multiplier \( H_{pp}^{el} = \delta_i^j \frac{\partial f_i}{\partial \mathbf{g}_m^{pl}} \mathbf{h}_m \) without sum according to \( i \) and \( j \) the Kronecker \( \delta \). \( \mathbf{H}_p \) is the matrix defined by the terms \( H_{pp}^{el} = \frac{\partial f_i}{\partial \mathbf{g}_m^{pl}} \mathbf{K}^{el} \frac{\partial f_j}{\partial \mathbf{g}_m^{pl}} \).
If we consider
\[ H^e = \left( H^e_d + H^e_u \right)^{-1} \]  
we finally obtain for \( M \) coupled mechanisms:
\[ F = \left( K^e_d - \sum_{j=1}^{M} \sum_{l=1}^{M} \left( K^e_d \cdot \frac{\partial^2}{\partial x_l^2} \right) \right) \cdot \mathbf{u} \]  
The macro-element is implemented into FEDEASLab, a finite element Matlab toolbox (Filippou and Constantinides, 2004). The return mapping algorithm (Simo and Hughes, 1998) is used for the plasticity and uplift mechanisms.

5. TRISEE: experimental campaign and numerical simulations

The numerical performance of the macro-element and the influence of plasticity and uplift on the behaviour of a rectangular foundation are studied hereafter using the experimental results of the European program TRISEE (TRISEE, 1998).

5.1. Experimental set-up and loading

Within the European program TRISEE, several experiments were performed on a shallow rectangular foundation lying on a low density sand (LD) sand and a high density sand (HD) (TRISEE, 1998). Horizontal cyclic solicitations were applied at the top of a vertical beam resting on the foundation, while a vertical force was kept constant throughout the tests (Fig. 15). More specifically:

- The dimensions of the foundation were 1 m \( \times \) 1 m.
- The vertical beam was 0.9 m high.
- The dimensions of the sand box were 4.6 m \( \times \) 4 m \( \times \) 4 m.
- The constant vertical force \( V \) was equal to 100 kN for the LD sand and 300 kN for the HD sand.
- The horizontal cyclic solicitations were divided in three phases:
  1. Phase I, small sine-shaped horizontal force cycles.
  3. Phase III, sine-shaped horizontal displacement cycles of increasing amplitude.

5.2. Numerical model

The new SSI macro-element able to couple plasticity and uplift is used to simulate the foundation. An elastic beam reproduces the upper structure. The node at the base of the macro-element is considered fixed, whereas the horizontal and vertical loadings are applied at the upper end of the elastic beam.

The parameters of the numerical model are presented in Tables 2 and 3. They have been calibrated using the experimental moment–rotation and horizontal force–horizontal displacement diagrams. They are divided into two groups:

(1) The ones that change between the three phases (Table 2). During the different experimental phases the soil actually settled something that led to an increase of the ultimate bearing capacity of the foundation and to a decrease of the elastic stiffnesses particularly in rotation. A curve showing the decrease of the elastic parameters at the beginning of the different phases is provided into TRISEE (1998). Based on this experimental evidence we have tuned numerically the initial elastic stiffness for the different phases.

(2) The ones that stay constant during the three phases (Table 3). The coefficients of the loading surface presented in Table 3 have been chosen in order to fit the experimental curves and particularly the position of their horizontal plateaux. They are different from the ones provided in Table 1, which are valid for a semi-infinite soil (Crémer et al., 2001). This shows clearly that the assumption of semi-infinite soil cannot be applied in this case and that is difficult to obtain a priori the right values for a cohesive or a frictional soil.

5.3. Experimental vs. numerical results

Numerical and experimental results for the phases I, II and III and for the LD and HD sands are given in Figs. 16–18. We present each time the moment–rotation, horizontal force–displacement and vertical displacement–time curves.

5.3.1. Phase I

Fig. 16 shows the behaviour of the foundation on the LD and HD sands during phase I. This phase generates only small non-linearities. The numerical model reproduces correctly the behaviour of the foundation in terms of horizontal displacements and rocking angles (which are slightly underestimated). Important differences appear however on the vertical displacements. A possible explanation could the fact that phase I is actually a “set-up” phase for the foundation on the soil whereas the amplitude of the load is small.

Table 2

<table>
<thead>
<tr>
<th>Phase</th>
<th>HD</th>
<th>LD</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>( k^H_{zz} = 52 \text{ MNm/m} )</td>
<td>( k^H_{zz} = 25 \text{ MNm/m} )</td>
</tr>
<tr>
<td></td>
<td>( k^H_{zz} = 65 \text{ MNm/m} )</td>
<td>( k^H_{zz} = 65 \text{ MNm/m} )</td>
</tr>
<tr>
<td>I</td>
<td>( q_{max} = 0.58 \text{ MPa} )</td>
<td>( q_{max} = 0.14 \text{ MPa} )</td>
</tr>
<tr>
<td>II</td>
<td>( k^H_{zz} = 52 \text{ MNm/m} )</td>
<td>( k^H_{zz} = 15 \text{ MNm/m} )</td>
</tr>
<tr>
<td>IV</td>
<td>( k^H_{zz} = 40 \text{ MNm/m} )</td>
<td>( k^H_{zz} = 40 \text{ MNm/m} )</td>
</tr>
<tr>
<td>III</td>
<td>( q_{max} = 0.58 \text{ MPa} )</td>
<td>( q_{max} = 0.14 \text{ MPa} )</td>
</tr>
</tbody>
</table>

Table 3

<table>
<thead>
<tr>
<th>Plasticity parameters</th>
<th>HD</th>
<th>LD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a )</td>
<td>0.93</td>
<td>1.1</td>
</tr>
<tr>
<td>( b )</td>
<td>0.8</td>
<td>0.9</td>
</tr>
<tr>
<td>( c )</td>
<td>1</td>
<td>0.8</td>
</tr>
<tr>
<td>( d )</td>
<td>1</td>
<td>0.8</td>
</tr>
<tr>
<td>( e )</td>
<td>1</td>
<td>0.8</td>
</tr>
<tr>
<td>( f )</td>
<td>1</td>
<td>0.8</td>
</tr>
</tbody>
</table>
5.3.2. Phase II

Fig. 17 shows the behaviour of the foundation on the LD and HD sands during phase II. Plasticity is now more pronounced. The loops of the force–displacement relationships are wide-opened and vertical settlements become important (of the order of centimetre). It can be also noted that no uplift occurs during this phase.

Numerical results are again satisfactory in terms of horizontal displacements and rocking angles in both directions. The size of the loops is correctly reproduced, indicating that the model dissipates energy in a similar manner than the experiment. Finally, it is also important to notice that vertical displacements are well reproduced by the macro-element, particularly for the LD sand.
Note: For the LD case, due to numerical reasons, we applied displacements and not forces on the top of the beam.

5.3.3. Phase III

Fig. 18 shows the behaviour of the foundation on the LD and HD sands for phase III. Important non-linearities are developed during this displacement controlled phase. As can be seen from the S-shaped curve, the influence of uplift is important for the HD sand. For the LD sand instead, only plasticity is developed and the vertical settlements are important. The experimental horizontal force–moment–rotation curve shifts towards the negative direction of the horizontal displacements although the loading and the geometry of the mock-up are symmetric.

Except for the non-symmetric curve, numerical results reproduce satisfactorily the behaviour of the foundation in terms of horizontal forces and moments in both directions. For the LD case, the model generates more plasticity (bigger loops) than the experiment, something that can explain the higher numerical vertical settlements. For the HD case, the model reproduces correctly the influence of uplift (S-shaped curve).

In order to quantify the influence of uplift, we proceed to the same calculation deactivating the uplift components of the macro-element, see Fig. 19. One can clearly identify the influence of uplift looking at the S-shaped moment–rotation curve for the HD sand. Rotations due to the plasticity of the soil are found almost equal to the ones coming from uplift. In other words, uplift and plasticity of the soil have similar contributions on the moment–rocking angle curve for the HD sand. Finally another interesting remark is that if uplift is not taken into account, the waves present on the settlement curve are not reproduced (HD case).

6. Conclusions

The 3D SSI macro-element developed within this work is able to simulate the non-linear behaviour of shallow rigid foundations of circular, rectangular or strip shape on an infinite space submitted to cyclic loadings. It takes into account the plasticity of the soil and the uplift of the foundation. Using global variables it has the advantage of inducing low computational costs (couple of minutes for each simulation).

The paper presents the three mechanisms considered in the macro-element (elasticity, plasticity and uplift) and their coupling. Uplift is formulated using the classical plasticity theory. The numerical performance of the element is finally validated using the experimental results of the European program TRISSE.

An interesting result coming from the numerical validation of the macro-element is that for a foundation on a high density sand, rotations due to the plasticity of the soil can be equal to the ones coming from uplift. In other words, uplift and plasticity of the soil can have similar contributions on the moment–rocking angle curve. Furthermore, the uplift mechanism is necessary in order to reproduce the waves often present on the moment–settlement curve. It is obvious that in certain cases uplift has significant influence on the behaviour of a foundation and thus it cannot be neglected.

The 3D behaviour of the element has not yet been validated due to the difficulty to find experimental results with loadings in two horizontal directions. This point should constitute the subject of a future work.

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References


